Topographic Mapping of the EEG: An Examination of Accuracy and Precision

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Summary: The accuracy and precision of topographic maps depicting scalp potentials and scalp potentials squared have been examined. Electrode placement was that specified by the International 10-20 System and the methods of interpolation bilinear and bicubic splines. The results indicate that, for these interpolation methods, the maximum error expected between the measured scalp quantities and those predicted by interpolation is positively correlated to the root-mean-square value of the measured quantity. Both interpolation methods produce accurate estimates of the interelectrode quantities. Both methods produce precise estimates of the scalp potential in the delta, theta and alpha frequency bands but only poor estimates in the beta band. The precision of the estimates of the scalp potentials squared is poor in all frequency bands. This result indicates that another look at the now common practice of topographically mapping the power-spectral components of the EEG is in order. In general, the bilinear and bicubic spline methods of interpolation perform about equally. This result is used to suggest that because of its additional computational complexity, use of the bicubic method for potential mapping may not be warranted. Advantages of the bicubic method, particularly in radial-current mapping, are however discussed.

Introduction

Topographic mapping has become a popular method for the presentation of the EEG (Duffy, 1986). Virtually any feature in the EEG with a spatial dependency can be usefully mapped and the usual approach is to extract the magnitude of this feature from each of a finite number of electrodes attached to the scalp and to interpolate to fill in the regions between. The important question then is how accurately and precisely do interpolated values represent the actual magnitudes of the feature at the sites where there are no electrodes?

The features of brain electrical activity most commonly mapped are the instantaneous magnitude of the scalp potential and its mean-squared value. The former is most widely considered when analyzing event-related potentials while the latter is usually of interest when studying the power spectrum of the background EEG. As the determination of the spectral power in a waveform requires, amongst other things, a nonlinear transformation (squaring) of instantaneous magnitudes, it is possible that a density of electrodes satisfactory for magnitudes will not necessarily be adequate for the interpolation of spectral powers.

Most published maps of brain electrical activity have utilized the electrode density specified by the International 10-20 System. This standard results in a mean inter-electrode distance on a normal-sized head (10 cm radius) of about 8 cm so that the areas filled by interpolation are about 64 cm^2. Because of its simplicity, the usual method of interpolation has been the 3 point linear type (Duffy et al. 1979). With this method, triangular areas formed by the electrodes are filled by linear combinations of the quantities measured at the electrode sites. More exotic methods of interpolation such as the bilinear (Naitoh, P. and Walter, 1969), the bicubic (Koles et al. 1988), nearest 4 neighbors (Buchsbaum et al. 1982), low-pass filtering (Ueno and Matsuoka 1976), unbiased polynomials (Ashida et al. 1984) and natural splines (Perrin, Pernier et al. 1987) have also been used.

It has been suggested that the 3-point linear interpolation method is adequate for the construction of topographic maps of both the instantaneous magnitudes of event-related potentials and the average power present in a spectral band of the background EEG (Duffy 1982). However, we are not aware of any definitive and quantitative data which corroborates this. Also, it is of significant concern that all methods of linear interpola-

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tion result in the location of the maxima and minima of mapped features at the electrode sites. In addition, on one hand, Gevins (1984) has suggested that the point-spread function for activity generated on the cerebral cortex would be about 2.5 cm on the scalp indicating that some restrictions are necessary on the size of the generators if the 10-20 System is to be utilized. On the other hand, Epstein and Brickley (1985) have shown, for example, that the magnitude of the alpha band EEG recorded with bipolar electrodes placed 1 cm apart is only about 20% of the maximum recordable with more widely spaced electrodes. This implies, although does not prove, that alpha activity within regions extending several cms is highly correlated and therefore interpolation is sometimes justified.

For these and other reasons, we have decided that a critical look at some aspects of the current thinking regarding the topographic mapping of the EEG is warranted. Specifically, given the spatial sampling density afforded by the 10-20 System, whether there are restrictions on the magnitude and spectral content of the EEG which limit the accuracy and precision of the interpolated values in topographic mapping. We have used only the background EEG for this study and considered actual and squared values for the rhythms in the traditional spectral bands. Our intention here was to obtain some insight into the topographic mapping of the instantaneous magnitude and the average power content of scalp potentials. Also, because of the erroneous location of maxima and minima by linear methods of interpolation, we have chosen to consider both the methods of bilinear and bicubic splines. The bilinear method is considered to be very similar to the more common triangular method in that both are linear and neither requires any assumptions about boundary conditions. The difference is only that the former method is applied to rectangular areas whereas the latter is applied to triangular areas. Because the bilinear method could be applied to the same areas as the bicubic method, it was chosen over the triangular method for this study.

Methods

EEGs were recorded from 4 normal volunteers (aged 25 to 51) using the 19 electrode locations specified by the International 10-20 System. Additional electrodes were placed at locations FP_z and O_z and between FP_1 and F_7, P_3 and O_1, FP_2 and F_8, P_4 and O_2 to facilitate the interpolation methods. This augmented 10-20 montage is shown in Figure 1 with the four additional corner electrodes designated as X_1, X_2, X_3 and X_4. Topographic maps were constructed for the area enclosed by the 5 x 5 array of electrodes. The electrodes were located at what were referred to as the grid points. Six additional electrodes were placed within the mapped area to serve as test points for the accuracy and precision of the

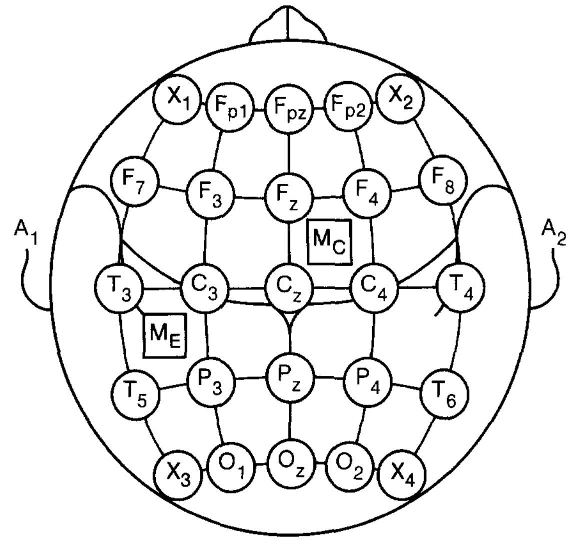

Figure 1. Grid and test electrode locations. The electrodes X_1, X_2, X_3, X_4, FP_z and O_z were added to the 10-20 System to complete a 5 x 5 recording grid. Test sites within F_3-F_4-P_3-P_4 were designated M_C for central and those outside M_E for edge.

topographic maps. Test electrodes within the area F_3-F_4-P_3-P_4 were designated M_C for central and outside this area as M_E for edge.

Recordings of the EEG were obtained using two Grass Model 16 Amplifiers with respect to a common left-ear reference. Amplified potentials were digitized to 12 bits at the rate of 120 samples per second from each of the 31 electrodes. To prevent temporal aliasing at digitization, the high and low pass filters on the Grass amplifiers were set at .5 cps and 35 cps respectively.

The interpolating function used to compute inter-electrode quantities from measured quantities at the grid points was

$$u(x,y) = \sum_{k=1}^{n} \sum_{l=1}^{n} a_{ijkl} \bullet (x - x_i)^{k-1} \bullet (y - y_j)^{l-1}$$

where $u(x,y)$ = the interpolated quantity at x, y, in either uV or uV^2;

x, y = the lateral and sagittal coordinates respectively of a point on the map;

i,j = a section on the map formed by 4 adjacent electrodes

x_i, y_j = the origin of section i, j;

n = 2 for bilinear splines and 4 for bicubic splines;

a_{ijkl} = the spline coefficients in section i, j, which provide the appropriate units to the right-hand side of the equation.

The map origin (x,y = 0) was defined to be at the X_1 electrode and the section origins at the respective upper-left corner electrodes. Sections were numbered i = 1,4 left

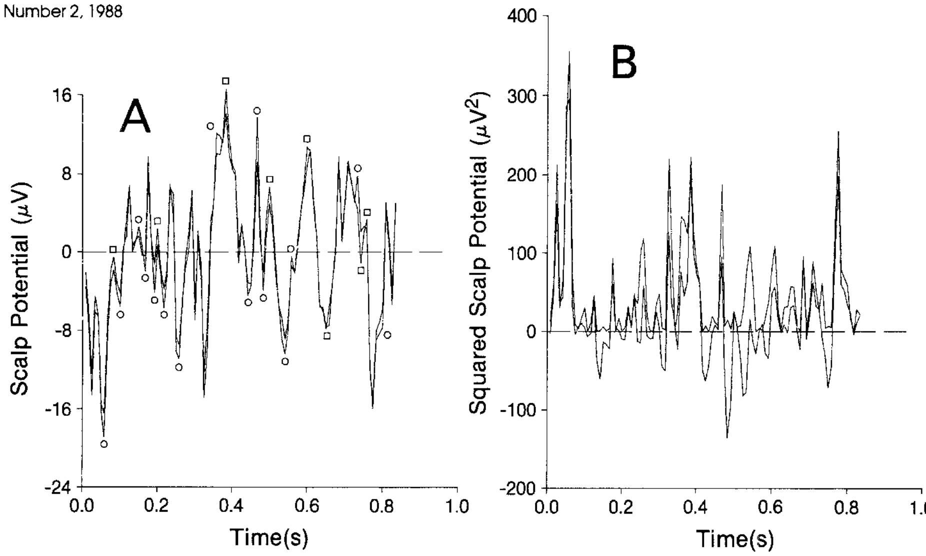

Figure 2. A . Traces show an example of the correspondence between the measured potential variations at a test site and those predicted by bicubic-spline interpolation. Peak excursions where the measured value exceeds the predicted value of the potential are marked with a square, while peak excursions where the predicted value of the potential exceeds the measured value are marked with an circle. B. Traces show an example of the correspondence between the measured scalp potentials squared at a test site and those predicted by bicubic-spline interpolation. The waveform with the negative excursions is the one predicted by interpolation.

to right and j = 1,4 front to back. All sections were assumed to be square, of equal area, and to consist of 25 x 25 map elements. Therefore for each map, values for u(x,y) were computed for the integer values of x and y in the range $0 \leq x, y \leq 100$ and the values of x_i and y_j determined by the section in which u(x,y) was computed. For example, if x=55 and y=23, then $x_3=50$ and $y_1=0$ would apply as would a_{31kl}.

Interpolation of the measured quantities at the grid points involved the determination of the values for the 4 bilinear coefficients a_{ijkl} k,l = 1,2 and for the 16 bicubic coefficients a_{ijkl} k,l = 1,4 for each of the 16 sections in the mapped area. In order to do this, the coefficients were chosen so that the splines u(x,y) matched the measured quantities at the grid points x_i, y_j i,j=1,5. In addition, for the bicubic splines, the coefficients were chosen so that the first and second partial derivatives of u(x,y) with respect to x and y were continuous across the section boundaries. A unique determination of the coefficients for the bicubic splines also required additional assumptions to be made about the interpolated surface at the boundaries of the mapped area. The assumption made was that the slope of the surface in the direction perpendicular to the boundary (as expressed by the first partial derivative) was everywhere zero and that the crossed partial derivatives were zero at the four corners. A detailed description of the method of bicubic- spline interpolation including the boundary problem can be found in Spath (1974).

Prior to interpolation, the potential variations from the 31 electrodes were digitally filtered into the delta (1-3 cps), the theta (3-8 cps), the alpha (8-13 cps) and the beta (13-25 cps) bands. The magnitudes of each of these components from the grid electrodes at successive instants in time were then interpolated to determine values for this quantity at the test electrode sites. This procedure was repeated for a second quantity, the magnitude of each component squared.

The accuracy of both interpolated quantities was assessed using the measure:

$$A_c = \frac{\sum_{i=1}^{N} (m_i - p_i)}{N}$$

where m_i = the quantity measured by the test electrodes;
p_i = the quantity predicted by interpolation;
N = number of consecutive samples in the epoch

The expected value of A_c for an accurate interpolation was 0.

The precision of the interpolated quantities was assessed using the measure

$$P_r = \frac{\sum_{i=1}^{N} (m_i - \overline{m}) \bullet (p_i - \overline{p})}{\left[\sum_{i=1}^{N} (m_i - \overline{m})^2 \bullet \sum_{i=1}^{N} (p_i - \overline{p})^2 \right]^{1/2}}$$

where $\overline{m} = \frac{1}{N} \sum_{i=1}^{N} m_i$ and $\overline{p} = \frac{1}{N} \sum_{i=1}^{N} p_i$

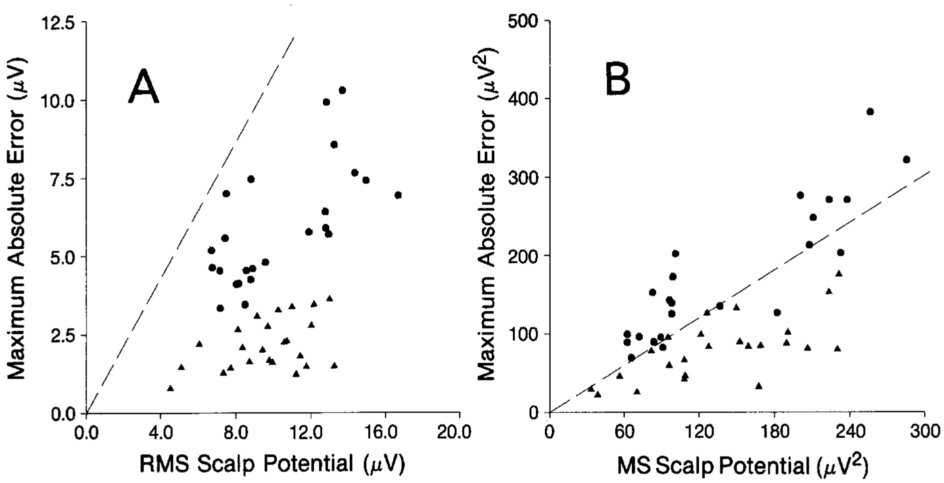

Figure 3. A Relationship in one subject between the maximum error in the bicubic-spline interpolation of the alpha-band potentials and the rms value of the voltage at the test site. Maximum error was computed in epochs of N=64. A triangle indicates a central site and a circle an edge site, the dashed line is of unit slope. B. Relationship in the same subject between the maximum error in the bicubic-spline interpolation of the alpha-band potentials squared and the ms value of the voltage at the test site. Central and edge sites indicated as in part A, the dashed line is of unit slope.

P_r is simply the coefficient of correlation between the measured and predicted quantities.The expected value of P_r for a precise interpolation was 1. Clearly, if A_c =0 and P_r =1, then the measured and predicted quantities follow each other exactly.

The measures A_c, P_r and

$$\max_{i=1}^{N}\left[m_i - p_i\right]$$

were calculated with N=64 (.5 sec) for numerous epochs from each individual. Values for these measures were plotted against the root- mean-square (rms) value of the magnitude of the quantity at the test site over the epoch to elicit any functional relationship.

Results

An example of the temporal variations in the magnitude of the scalp potential at a test electrode located centrally in the mapped area of one of the subjects and that predicted by bicubic- spline interpolation is shown in Figure 2A. Peak excursions from zero potential where the measured value exceeds in absolute value the predicted value of the potential are marked with a square while peaks where the excursion of the predicted value of the potential exceeds in absolute value the measured value are marked with a circle. The circles appear to outnumber the squares overall particularly on the negative excursions.

Figure 2B shows the magnitude-squared values of the same potential variations shown in Figure 2A. The measured values are, as expected, always positive while the values predicted by the bicubic-spline interpolation method are often negative. This result is expected since the surface obtained using bicubic splines is not restricted to the range of values present at the grid sites. The extent to which the predicted surface is negative and generally deviates from the measured values is, however, surprising.

The maximum absolute difference between the measured values of the scalp potential, filtered into the alpha band, and the corresponding values predicted by bicubic-spline interpolation in numerous epochs of length N=64 for one of the subjects is shown in Figure 3. This measure of error in each epoch is plotted against the rms value of the measured quantity at the test site in the same epoch. Figure 3A shows the maximum error in the magnitude of the interpolated potential while Figure 3B shows the maximum error in the magnitude squared. In each case the error is shown for one of the test sites located centrally in the mapped area (M_C in Figure 1) and for one of those located near the edge (M_E in Figure 1). Figure 3 suggests a linear relationship between maximum error and rms value for both the centrally located and edge test sites. The slope of this relationship is greater for the edge site both when the magnitude of the potential is interpolated and when the squared value of the magnitude is interpolated. The dashed lines in Figure 3 are of unit slope.

Figures 4A and B show the accuracy (A_c) of the interpolated magnitude and magnitude-squared bicubic potentials at the central and edge test sites for one of the subjects. Once again, A_c is plotted against the rms value of the measured quantity at the test site. Figure 4A appears to indicate that A_c is independent of the value at the test site and that the average value of all of its deter-

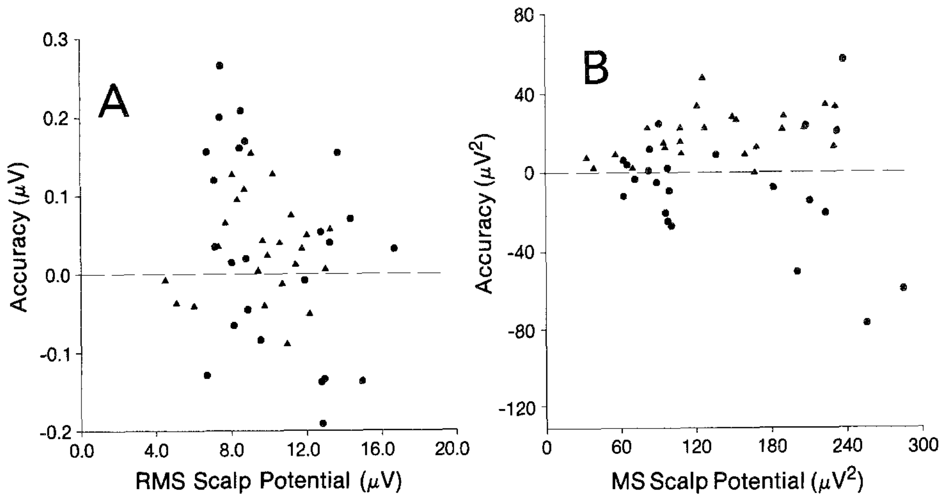

Figure 4. A. Values of the accuracy measure A_C calculated from epochs of length N=64 for bicubic-spline interpolation of the alpha-band potentials. Results from one subject are shown, a triangle indicates a central site and a circle an edge site. B. Values of the accuracy measure A_C calculated from epochs of length N=64 for bicubic-spline interpolation of the alpha-band potentials squared. Central and edge sites are indicated as in part A.

minations is zero. That is, it appears that the method of interpolation produces an estimate of the potential at the test site which is unbiased with respect to the measured value. The difference between the central and edge test sites appears to be that the variance of the estimate is less at the center than it is at the edge. Figure 4B appears to indicate that the method of interpolation produces an estimate of the squared value of the potential at the test site which is biased with respect to the measured value at the test site. At the central test site the interpolation method tends to underestimate the measured value while at the edge site it tends to overestimate the measured value. The variance of the estimates at the edge site appears to be larger than that at the central site.

The precision (P_r) of the magnitude and magnitude-squared bicubic potentials with respect to the rms value of the measurements at the test sites is shown in Figure 5 for one of the subjects. Again, precision at the edge sites appears to be less then that at the central sites and estimates of the magnitude of the potential appear generally to be more precise than those of the magnitude squared. Precision also appears to be a function of the measured value at the test sites with low values at the test sites leading to lower values of P_r. The proportion of the values obtained for P_r above some threshold value (say .95) increases with the magnitude of the quantity at the test sites in all cases and particularly so when the magnitude-squared value of the potential at the test site is estimated.

A summary of the results comparing the 4 frequency bands in the EEGs collected from the 4 subjects (including all 6 test electrode sites) and comparing the methods of bilinear and bicubic interpolation is shown in Tables I and II. Table I relates to scalp potentials and Table II to scalp potentials squared. In each frequency band, the bilinear and bicubic splines are contrasted using 3 performance measures and these are shown for interpolation near the edge of the mapped area (M_E) and near the center of the mapped area (M_C) Measure 'A' is the average slope of the relationship between the maximum absolute error in the quantity predicted by interpolation and the rms value of the quantity in the epoch. It was computed for each test site (as in Figure 3) using the mean-squares minimization approach and these averaged over the sites and subjects to obtain the entries for the Tables. The corresponding standard deviations over the subjects are contained in brackets. Measure 'B' is the average coefficient of variation (with standard deviation) obtained from the 4 subjects for the accuracy measure A_C (illustrated in Figure 4). Once again, the coefficient of variation of A_C was computed for each subject and these values averaged to obtain the entries for 'B'. Measure 'C' is the overall measure of precision for the interpolated quantities and is simply the proportion of all epochs which yielded a precision measure P_r greater than an arbitrarily chosen value of .95 (see Figure 5). The rms values of the quantities at the test sites over all of the epochs used to compute the performance measures are also included in the tables.

Examination of the values of measure 'A' in Tables I and II suggests that a significant positive correlation exists between maximum absolute error and the rms value of the quantity in the epoch (see Figure 3). The values for 'A' in Table I are generally less than those in Table II however because of the large relative standard deviation of its values over the subjects, it is not possible to say that this difference is statistically significant. This situation also applies across frequency bands, across methods of interpolation and across test sites. However, it is clear that all values of 'A' are significantly different from zero indicating that the larger the rms value of the interpolated quantity, the larger the expected value of the

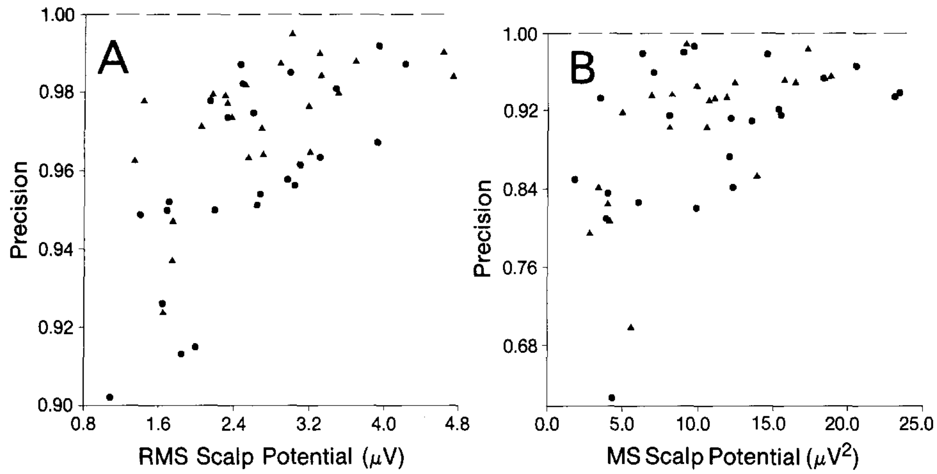

Figure 5. A. Values of the precision measure P_r calculated from epochs of length N=64 for bicubic-spline interpolation of the alpha-band potentials. Results are from one subject, a triangle indicates a central site and a circle an edge site. B. Values of the precision measure P_r calculated from epochs of length N=64 for bicubic-spline interpolation of the alpha-band potentials squared. Central and edge sites are indicated as in part A.

maximum error in the interpolation. Measure 'B' indicates that the methods of bilinear and bicubic spline interpolation produce accurate estimates of inter-electrode scalp potentials and scalp potentials squared. In all but a few cases (two in Table I, one in Table II) the standard deviation of 'B' over the 4 subjects is greater than its mean value. This is taken to mean that the average difference between the measured quantities at inter-electrode sites and those predicted by interpolation is not significantly different from zero with either interpolation method or in any frequency band. In isolated cases where bias is indicated, the precision of the predicted quantities is also low.

Measure 'C' in Table I suggests that the precision of the interpolated values of scalp potential is a function of the location of the test site, the method of interpolation and the frequency band. An examination of Table I reveals that the precision of interpolated scalp potentials is greatest (.95) in the alpha band when using bicubic splines at a central test site. Precision is lowest (.44) in the beta band, also with bicubic splines, but at the edge sites. It is noteworthy, however, that in the former case the rms value of the alpha activity considered was 4.4μ while in the latter case the rms value of the beta activity considered was only 1.6 μ. Whether the loss of precision is due to the reduction in the magnitude of the activity or to its frequency is not known. However, the latter is suggested since in the theta band, at the edge sites, the activity level was only 1.5 m but the precision remained high at .87. It is also noteworthy that in each frequency band the poorest precision was obtained with bicubic splines at the edge sites. Bilinear splines also performed more poorly at edge than at central sites The values for the precision measure 'C' in Table II are generally lower than those in Table I. The highest value is .65 in the theta band using bicubic splines at central sites. The lowest value is .12 in the beta band also with bicubic splines but at edge sites. Once again, this table suggests that precision is more a function of the frequency of the activity than its magnitude. Also, both interpolation methods, though particularly the bicubic splines, perform more poorly at edge sites than at central sites. As with Table I, there is little to suggest that one of the methods of interpolation is superior to the other, however, most of the measures of precision for the bilinear splines are higher than the corresponding ones for the bicubic splines.

Discussion

We have examined the accuracy and precision of the methods of bilinear and bicubic spline interpolation of scalp potentials and scalp potentials squared for producing topographic maps of the EEG. The results were presented separately for each of the traditional frequency bands in the EEG in an attempt to determine if the character of the EEG in these bands is a factor in this process. Our results indicate that the bicubic method of interpolation offers little overall to indicate that it is the method of choice for topographic mapping. There are indications that it can reduce the maximum error between the interpolated surface and the measured quantities at the test sites (performance measure 'A') but this does not appear to be significant. Both methods appear to yield unbiased estimates of the inter-electrode quantities (performance measure 'B') and both are about equally precise (performance measure 'C'). The overall impression is that the bicubic splines are better when the interpolated quantity is scalp potential and the sites are central but that they are worse everywhere else particularly when the interpolated quantity is scalp potential squared. The undershooting indicated in Figure 2B is probably a good indication of why this is so. In any case, the size of our subject population (n=4) prevents more definitive conclusions from being made with regard to the method of choice. Our results are sufficient however to indicate that any advantages would at best be marginal particularly in light of the additional computational burden represented by the bicubic method.

The most important conclusions to be drawn from this work are related to the performance of the interpolating functions in the beta frequency band and their performance with respect to scalp potentials squared. Table I indicates that the precision of both interpolators drops markedly when beta band activity is considered. This effect cannot be attributed to the lower magnitude of the beta activity since theta band activity of even lower magnitude could be interpolated with much greater precision. The conclusion then is that topographic maps of the instantaneous magnitudes of beta band activity can only be viewed as imprecise indications of the actual scalp topography in this frequency band.

Regarding the interpolation of scalp potentials squared, both the bilinear and bicubic spline methods produce relatively imprecise estimates of the inter-electrode values. This is particularly troublesome since most of the topographic maps produced today of the background EEG depict spectral power content. Spectral power is the mean-squared value of the activity in the various frequency bands and these values, computed at electrode sites, are interpolated to produce topographic maps. Our results therefore apply to these spectral maps. One possible explanation of these results is the low magnitude of the EEG used. It is however suggested that this is not the case since the largest value for precision of the interpolators was obtained in the theta band where the ms value of the activity was only 7.8 μ (table II, bicubic central) and this precision actually dropped when the ms value of the activity was 45.5μ in the alpha band.

The result that the precision of the interpolated quantities dropped with bicubic splines when the test site was changed from the central to edge location is not surprising. The assumption of zero slope at the boundary is probably responsible for this. However, the fact that the precision of the bilinear interpolations also dropped at the edge sites is surprising. We are at a loss to explain this since no assumptions related to boundary conditions were required to implement this technique. The only factor possible would seem to be the recording reference (the left ear) which was located either very near or very far from the edge sites. It may be that the spatial correlation between electrodes at these extremes is less than at central sites. This is most likely to occur near the reference site since scalp potentials there would be of lower magnitude and noise (say muscle potentials) of higher proportion. In any case, the loss of precision at edge sites

Frequency Band	Performance Measure	Bilinear Central	Bilinear Edge	Bicubic Central	Bicubic Edge
Delta	A.	.93 (66)	.83 (.86)	.89 (.72)	.82 (.84)
	B.	.04 (.11)	-.11 (.21)	.05 (.10)	.-11 (.22)
	C.	.83	.79	.82	.74
	rms	2.7 μV	2.3 μV	2.7 μV	2.3 μV
Theta	A.	1.17 (.82)	.83 (.73)	1.03 (.98)	.80 (75)
	B.	-.06 (.14)	-.01 (.07)	-.05 (.12)	-.03 (.12)
	C.	.90	.87	.92	.78
	rms	2.1 μV	1.5 μV	2.1 μV	1.5 μV
Alpha	A.	1.05 (.82	1.02 (.81)	.98 (.89)	1.04 (.77)
	B.	.03 (.08)	.08 (.11)	.03 (.10)	).07 (.10)
	C.	.90	.85	.98	.75
	rms	4.4 μV	4.9 μV	4.4 μV	4.9 μV
Beta	A.	1.09 (.70)	1.02 (.68)	.99 (.80)	.98 (.77)
	B.	.01 (.02)	.04 (.06)	-.07 (.03)	.06 (.03)
	C.	.63	.53	.57	.44
	rms	1.6 μV	1.6 μV	1.6 μV	1.6 μV

Table I. Measures of the performance of the methods of bilinear and bicubic spline interpolation as a function of the test site and the frequency component in the background EEG. The interpolated quantity is the instantaneous magnitude of the scalp potential. Measure 'A' is the average (sd) of the least mean- squared slopes obtained from the Figure 3 data for each subject (n=4); measure 'B' is the average (sd) of the coefficients of variation of the accuracy measure A_C for each subject and measure 'C' is the proportion of the values of the precision measure P_r from all the epochs from all of the subjects with a value greater than .95. The rms values of the potentials at all test sites over all epochs used to calculate these measures are also given.

Frequency Band	Performance Measure	Bilinear Central	Bilinear Edge	Bicubic Central	Bicubic Edge
Delta	A.	1.16 (.33)	1.40 (1.39)	1.15 (.49)	1.66 (1.85)
	B.	.49 (.94)	.30 (.65)	.77 (.60)	.41 (.62)
	C.	.60	.46	.49	.43
	ms	12.2 μV^2	10.9 μV^2	12.2 μV^2	10.9 μV^2
Theta	A.	1.43 (.49)	1.28 (1.03)	1.22 (.67)	1.47 (1.36)
	B.	-.67 (.89)	.02 (.71)	.78 (.79)	.15 (.64)
	C.	.62	.54	.65	.45
	ms	7.8 μV^2	4.1 μV^2	7.8 μV^2	4.1 μV^2
Alpha	A.	1.2 (.94)	1.46 (.96)	1.15 (1.01)	1.52 (.91)
	B.	.85 (.94)	.43 (.83)	1.00 (.94)	.35 (.87)
	C.	.64	.34	.64	.34
	ms	45.5 μV^2	53.8 μV^2	45.5 μV^2	53.8 μV^2
Beta	A.	1.37 (.95)	1.43 (1.43)	1.24 (.98)	1.29 (1.41)
	B.	.71 (1.02)	.14 (1.24)	.90 (1.18)	.11 (1.25)
	C.	.38	.16	.27	.12
	ms	4.7 μV^2	4.5 μV^2	4.7 μV^2	4.5 μV^2

Table II. Measures of the performance of the methods of bilinear and bicubic spline interpolation as a function of the test site and the frequency band in the background EEG. The interpolated quantity is the instantaneous magnitude of the scalp potential squared. The measures 'A', 'B' and 'C' are defined in the caption to table I. The ms values of the potentials at all test sites over all epochs used to calculate these measures are also given.

is less with bilinear than with bicubic splines. However this pattern could probably be reversed if the bicubic splines were clamped to the linearly predicted slopes rather than to zero as they were.

Perrin, Pernier et al. 1987 have also studied the errors in estimated inter-electrode scalp potentials using different methods of interpolation. Specifically, they have used simulations to compare the 4 nearest-neighbor methods of orders 1, 2 and 3 with that of natural splines of orders 2, 3 and 4. As in our case, the former methods produce a surface with extrema always at the electrode locations and require no assumptions about the boundary conditions for the mapped area while the latter methods do not produce a surface with the extrema constrained to the electrode locations but do require assumptions about the boundary conditions. The results of their work indicate that the spline methods produce generally less maximum error and less rms error (similar to our measure P_r). Interestingly, errors in the spline methods increase rapidly as the spatial-frequency content of the potential surface increases and approach those in the nearest-neighbor methods. It may be that natural splines of higher order would restore the previous balance, however the danger inherent in higher orders is the possible instability of the method used to determine the parameters of the interpolant. It may be that our results, which indicate little to choose between bilinear and bicubic splines, are due to the presence of high spatial frequencies in the background EEG. Since the highness of a spatial frequency is related to the density at which it is sampled, it may be that the sampling density afforded by the 10-20 System was not adequate.

Some improvement in our results may have been realized by a method of interpolation based on the exact locations of the electrodes on a curved surface more closely resembling the shape of the head. This approach, however, is much more complex computationally and would not, in our opinion, have changed any of the conclusions. For example, we can see no way in which our assumption of sections with square and equal areas could affect the conclusion that the interpolations were accurate but lost precision with higher temporal frequencies. The answer to all questions lies, of course, with more electrodes and as the density of electrodes relative to the spatial frequencies present increases, the assumptions become more valid and all methods of interpolation should perform better. The gap between lower and higher-order methods, not withstanding errors due to instability, will be less.

As bicubic splines perform no better (and probably worse with scalp potentials squared) their value for the topographic mapping would seem to be in doubt particularly in view of the additional computational complexity that they represent. However, the bicubic spline interpolation of scalp potentials has the distinct advantage over bilinear interpolation that estimates of the radial current density can be obtained from the interpolated surface. This can be done by applying the Laplacian operator analytically to the functional form of the surface. The bilinear surface does not possess the curvature necessary to enable this to be done. The application of the Laplacian operator analytically to the functional form of a surface obtained by interpolation has already been demonstrated by Perrin, Bertrand and Pernier (1987). Radial current maps have the advantages over potential maps that they are more highly focused representations of the generator activity within the brain and that they are independent of the recording reference used to obtain the EEG.

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