### SCHURZ ON HYPOTHETICO-DEDUCTIVISM

### 1. INTRODUCTION $\mathbf{1}$

Schurz (1991) demonstrates two derivations of unacceptable conclusions, (UC1) and (UC2), drawn from a standard version of hypothetico-deductivism (H-D1). The first derivation proceeds via recourse to a principle (P1) proposed by Schurz which he claims to be "a reasonable principle of every 'logic of confirmation'". The second derivation proceeds via a principle (P2) proposed by Hempel. To avoid (UC1) and (UC2) Schurz suggests that we reject (H-D1) in favor of an alternative version of hypothetico-deductivism (H-D2). In this note I shall demonstrate that

(1) Schurz's (P1) is by no means a reasonable principle of every logic of confirmation, and (P2) is, at least from some perspectives, highly questionable,

(2) Schurz's (H-D2), like (H-D1), combined with (P1) and (P2) has unacceptable consequences,

(3) While Schurz's (H-D2) successfully avoids a tacking problem noted in Gemes (1990) and Grimes (1990) it can only avoid the type of tacking problem noted in Glymour (1980) and (1980a) at the price of denying a canonical equivalence principle,

(4) Schurz's (H-D2) by itself has highly counter-intuitive consequences stemming from its utilization of the notion of relevant consequence.

## 2. SCHURZ'S FIRST DERIVATION AND THE QUESTIONABLE PRINCIPLE OF STRENGTHENING THE CONFIRMANS

Note, for convenience of exposition we (i) take theories to be sentences, possibly long conjunctive sentences, rather than sets of sentences, and (ii) freely move without preamble from applications dealing with sentences of ordinary language to applications dealing with sentences of more formal languages.

Here is the version of hypothetico-deductivism considered by Schurz,

### **KEN GEMES**

(H-D1) A sentence S confirms a theory T if (i) S is contentful  $(\not + S)$ , (ii) T is consistent  $(T \not + (p \& \neg p), (iii) S$  is true [or 'rationally acceptable'], and (iv)  $T \vdash S$ . (Schurz, 1991, p. 394)

The alleged reasonable principle, which Schurz calls "the condition of strengthening the confirmans", utilized in Schurz's first derivation is

(P1) If S confirms T and S\* logically implies S and is consistent with T, then S\* confirms T also. (Schurz, 1991, p. 394)

The unacceptable conclusion is

(UC1) Every consistent theory T is confirmed by every contentful and true [rationally acceptable] sentence S provided only S is consistent with T and  $\sim S$  with  $\sim T$ . (Schurz, 1991, pp. 394-395)

Schurz is correct in claiming that (UC1) follows from (H-D1) and (P1).

**Proof:** Let T by any consistent theory and S be any true [rationally acceptable] contentful sentence such that S is consistent with T and  $\sim S$  with  $\sim T$ . Now under these conditions clearly  $(S \lor T)$  is contentful, S is consistent and  $S \vdash (S \lor T)$ . So, by (H-D1),  $(S \lor T)$  confirms T. So, under these conditions,  $(S \lor T)$  confirms T, S logically implies  $(S \lor T)$ , and S is consistent with T, so by (P1), S confirms T.

(UC1) is clearly unacceptable since, applied to standard first order quantificational languages, it has such unacceptable consequences as that 'Bs', if true [rationally acceptable], confirms 'Bp'. Under an obvious interpretation this may be read as the claim that 'Sydney has a harbor bridge', if true [rationally acceptable], confirms 'Paris has a harbor bridge.' More generally (UC1), applied to standard first order quantification languages, has the totally unacceptable consequence

(UCC) For any (variable free) atomic sentences  $\alpha$  and  $\beta$ , if  $\alpha$  is true [rationally acceptable],  $\alpha$  confirms  $\beta$ .

Since the argument from (H-D1) and (P1) is valid, to avoid the conclusion (UC1) we must deny at least one of its premises. Schurz, assuming that (P1) is a "reasonable principle of every 'logic of confirmation'" (Schurz, 1991, p. 394), aims at (H-D1).

In fact, (P1) is far from being "a reasonable principle of every 'logic of confirmation". Standard Bayesian accounts of confirmation, and the logic of confirmation advanced by Carnap in *The Logical Foundations of* 

*Probability*, both of which are themselves committed to a thesis very close to (H-D1), reject (P1).<sup>1</sup> For instance, suppose we take confirmation to be a matter of probabilistic favorable relevance. Thus we have the Carnapian definition

(C)  $e \text{ confirms } h \text{ iff } P(h/e) > P(h)^2.$ 

Let T be the claim 'Die A came up even', S be the claim 'Die A did not come up 5' and S\* be the statement 'Die A did not come up 5, 2 or 4'. In this case, according to (C), S confirms T, since P(T/S) = 3/5and P(T) = 1/2, and S\* clearly entails S and is consistent with T, but S\* does not confirm T, since  $P(T/S^*) = 1/3$ . So in this case, contra (P1), S confirms T, S\* logically implies S but S\* does not confirm T.

(P1), then, does not hold for Carnap's notion of confirmation as favorable relevance, and hence it is not, contra Schurz, "a reasonable principle of every logic of confirmation". Indeed, though I will not here argue the point in any detail, I believe (P1) is generally unacceptable as a constraint on theories of confirmations, since it would saddle them with the inappropriate burden of being (more or less) monotonic. Monotonicity is an appropriate requirement for non-ampliative systems, for instance deductive logics, however it is an inappropriate requirement for ampliative systems, such as inductive logics and confirmation theories.

# 3. SCHURZ'S SECOND DERIVATION AND HEMPEL'S SPECIAL CONSEQUENCE CONDITION

In Schurz's second derivation the unacceptable conclusion

(UC2) For every sentence S: If S confirms at least one theory T [by H-D1], it confirms every sentence  $T^*$  consistent with T. (Schurz 1991, p. 419)<sup>3</sup>.

is deduced from (H-D1) via the standard Hempelian special consequence condition

(P2) If S confirms T, then S confirms also every logical consequence of T. (Schurz, 1991, p. 418)

The argument from (H-D1) and (P2) to (UC2) is valid.<sup>4</sup>

*Proof*: Let S be any sentence such that for some T, S confirms T according to (H-D1) and let  $T^*$  be any sentence consistent with T.

Then,  $T \& T^*$  is consistent and, since S confirms T according to (H-D1), S is contentful, S is true [rationally acceptable], and  $T \vdash S$ , hence  $T \& T^* \vdash S$ . So by (H-D1), S confirms  $T \& T^*$ . So by (P2), S confirms  $T \& T^*$ 's logical consequence  $T^*$ .

While I share Schurz's appraisal that (P2) is "intuitively very plausible" (Schurz, 1991, p. 418), it is worth noting that it is still a fairly controversial principle. For instance, it is incompatible with the Carnapian program of identifying confirmation with favorable relevance. To see why let T be 'Die A came up even and Die B came up odd', S be 'Die A came up 2' and  $S^*$  be 'Die B came up odd'. In this case P(T/S) = 1/2 and P(T) = 1/4, so by (C), S confirm T. Now T logically implies  $S^*$ , yet  $P(S^*/S) = P(S^*) = 1/2$ , so, by (C), S does not confirm  $S^*$ . So, assuming Carnap's favorable relevance notion of confirmation, in this case S confirms T, T logically implies  $T^*$  but, contra (P2), S does not confirm  $T^{*.5}$ 

## 4. h-d reformulated and its unacceptable consequences

To avoid (UC1) and (UC2) Schurz, rather than explicitly rejecting (P1) and (P2), suggests a reformulated version of (H-D1). His reformulation rests on the notions of conclusion relevant and premise relevant deductions which apply primarily to formal languages. These we may, following Schurz, informally define as follows, counting propositional variables as Oary predicates,

Where  $\alpha \vdash \beta$ ,  $\alpha \vdash \beta$  is a conclusion relevant deduction iff no predicate in  $\beta$  is such that the replacement of some of its occurrences in  $\beta$  by any other predicate of the same arity results in a  $\beta'$  such that  $\alpha \vdash \beta'$ . (Cf. Schurz, 1991, pp. 409-411)

The following count as conclusion relevant deductions:  $p \vdash p$ ,  $(p \& q) \vdash p$ ,  $(p \& q) \vdash q$ ,  $p \& (p \rightarrow q) \vdash q$ ,  $q \vdash \sim \sim q$ ,  $(\sim p \& (p \lor q) \vdash q$ ,  $(x)(Fx \& Gx) \vdash (x)(Gx \& Fx)$ ,  $(x)Fx \vdash Fa$ . The following are not conclusion relevant deductions:  $p \vdash (p \lor q)$ ,  $p \vdash q \rightarrow p$ ,  $\sim q \vdash q \rightarrow p$ ,  $(x)Fx \vdash (Fa \lor Ga)$ .

Where  $\alpha \vdash \beta$ ,  $\alpha \vdash \beta$  is a premise relevant deduction iff (i) there is no single occurrence of a predicate in  $\alpha$  such that its replacement in  $\alpha$  by any other predicate of the same arity results in an  $\alpha'$  such that  $\alpha' \vdash \beta$  and (ii) and there are no predicate occurrences in  $\alpha$  such that they are replaceable by other predicates of the same arity resulting in an  $\alpha'$ such that  $\alpha' \dashv \alpha$ . (Cf. Schurz, 1991, p. 421-422)

The following count as premise relevant deductions:  $p \vdash p$ ,  $p & (p \rightarrow q) \vdash q$ ,  $\sim p & (p \lor q) \vdash q$ ,  $(x)Fx \vdash Fa$ . The following are not premise relevant deductions:  $(p & \sim p) \vdash q$ ,  $p \vdash (q \lor \sim q)$ ,  $(p & q) \vdash p$ ,  $(x)Fx & (x)Gx \vdash Fa$ .

Schurz (1991), p. 422, proposes the following reformulated version of (H-D1),

(H-D2) A sentence S confirms a theory T if (i) S is contentful  $(\{\emptyset\} \not\models S)$ , (ii) T is consistent  $(T \not\models (p \And \neg p), (iii) S$  is true [or 'rationally acceptable'], (iv)  $T \vdash S$ , (v) the deduction  $T \vdash S$  is a conclusion and premise relevant deduction.

The proof given above that (UC1) follows from (H-D1) and (P1) does not go through if we simply substitute (H-D2) for (H-D1). One cannot use (H-D2) to move from the premises that T is consistent, S is true [rationally acceptable] and contentful, S is consistent with T, and  $\sim S$ is consistent with  $\sim T$ , to the conclusion that, by (H-D2), ( $S \lor T$ ) confirms T. The satisfaction of these conditions does not guarantee that (H-D2)'s condition (v) is met. In particular, in this case the deduction  $T \vdash (S \lor T)$  is not conclusion relevant.

Similarly, the proof given above that (UC2) follows from (H-D1) and (P2) does not go through if we substitute (H-D2) for (H-D1). One cannot use (H-D2) to move from the premises that S confirms T and  $T^*$  is consistent with T to the conclusion that S confirms ( $T \& T^*$ ). The satisfaction of these conditions does not guarantee that (H-D2)'s condition (v) is met. In particular, in this case the deduction ( $T \& T^*$ ) + S is not premise relevant.

However the following unacceptable consequence of (UC1)

(UC1') For any S and T such that S is true [rationally acceptable, and for some R,  $(S \lor R) \& T \vdash S \lor (R \& T)$  and the deduction is conclusion and premise relevant, and S is consistent with  $(S \lor R) \& T$ , S confirms T.

is derivable from (H-D2), Schurz's (P1) and the Hempelian (P2).

*Proof*: Let S and T be any sentences such that S is true (rationally acceptable), and for some R,  $(S \lor R) \& T \vdash S \lor (R \& T)$  and the deduction is conclusion and premise relevant, and S is consistent with  $(S \lor R) \& T$ . Now since no tautology can be the conclusion of a conclusion relevant deduction  $S \lor (R \& T)$  cannot be a tautology. Since

no contradiction can be the premise of a premise relevant deduction,  $(S \lor R) \& T$  cannot be a contradiction. Since S is true [rationally acceptable],  $S \lor (R \& T)$  is true [rationally acceptable]. So by (H-D2),  $S \lor (R \& T)$  confirms  $(S \lor R) \& T$ . Now  $S \lor (R \& T)$  is a logical consequence of S, and ex hypothesi S is consistent with  $(S \lor R) \& T$ , so by (P1), S confirms  $(S \lor R) \& T$ . So, by (S2), S confirms  $(S \lor R) \& T$ 's consequence T.

This, for instance, again yields the unacceptable result that 'Bs', if true [rationally acceptable], confirms 'Bp', since  $(Bs \lor Os) \& Bp \vdash Bs \lor (Os \& Bp)$  and the deduction is conclusion and premise relevant and 'Bs' is consistent with ' $(Bs \lor Os) \& Bp'$ . More generally (UC1'), like (UC1), applied to standard first order quantification languages, has the totally unacceptable consequence (UCC).

The following unacceptable consequence of (UC2) is derivable from the combination of (H-D2) and (P2) without recourse to the suspect principle (P1).

(UC2') For every sentence S: If S confirms at least one sentence T (by (H-D2)), it confirms every sentence  $T^*$  such that  $T^* \& (T^* \to T) \vdash S$  and the deduction is premise and conclusion relevant.

*Proof:* Let S be any sentence such that for some T, S confirms T according to (H-D2), and  $T^* \& (T^* \to T) \vdash S$  and the deduction is premise and conclusion relevant. Then, since no premise of a premise relevant deduction can be a contradiction,  $T^* \& (T^* \to S)$  is not a contradiction. Since no conclusion of a conclusion relevant deduction can be a tautology, S is not a tautology. Since S confirms T, S is true [rationally acceptable]. So by (H-D2), S confirms  $T^* \& (T^* \to T)$ . So by (P2), S confirms  $T^* \& (T^* \to T)$ 's logical consequence  $T^*$ .

(UC2') yields the result that 'Bs', if true [rationally acceptable], confirms 'Bp', since 'Bs', if true [rationally acceptable], confirms 'Bs' and Bp &  $(Bp \rightarrow Bs) \vdash Bs$  and the deduction is premise and conclusion relevant. More generally (UC2'), like (UC2), applied to standard first order quantification languages, has the totally unacceptable consequence (UCC).

### 5. PREMISE AND CONCLUSION RELEVANCE, THE TACKING PROBLEMS, AND SOME UNACCEPTABLE CONSEQUENCES OF (H-D2)

While Schurz has tried to expose the problems of canonical formulations of hypothetico-deductivism such as (H-D1) by appeal to alleged principles of confirmation such as (P1) and (P2), other authors have attacked related versions of hypothetico-deductivism more directly. Glymour (1980) and (1980a) faults such simple versions of hypotheticodeductivism as

by adducing the following type of consequence of (H-D3)

(UC3) If E confirms T then E confirms (T & H) for any H consistent with T.

For example, according to (H-D3), 'Bs' confirms 'Bs & Bp', which under an obvious interpretation may be read as 'Sydney has a harbor bridge' confirms 'Sydney has a harbor bridge and Paris has a harbor bridge'. Similarly, (H-D3) has the consequence that 'Fa' confirms '(x)Fx & (x)Gx'. This in Gemes (1993) is called the problem of tacking by conjunction, since it stems from the fact that where T logically implies E, one can tack on any arbitrary H to T by conjunction to form (T & H) which also logically implies E.

Gemes (1990), (1993), and Grimes (1990) criticize (H-D3) for having the consequence that

(UC4) If S confirms T then so does  $(S \lor S')$ , provided  $(S \lor S')$  is not a tautology.

For example, according to (H-D3), one can confirm Newton's second law, f = ma, by observing that its consequence 'f = ma or Sydney has a harbor bridge' is true. This in Gemes (1993) is called the problem of tacking by addition since its stems from the fact that where T logically implies S then one can tack on any arbitrary S' to S by addition to form  $(S \vee S')$  which is also a consequence of T.

Appealing to the notion of deductions which are conclusion relevant and premise relevant goes some way to addressing these tacking problems. If we demand that the deduction of S from T in (H-3) be con-

<sup>(</sup>H-D3) If (non-contradictory) T logically implies (non-tautologous) E then E confirms T

clusion relevant we avoid the tacking by disjunction problem. However adding the requirement that the deduction be premise relevant does not solve the tacking by conjunction problem save that one gives up the canonical equivalence principle

# (P3) If S confirms T and T is logically equivalent to $T^*$ , S confirms $T^*$ .

We shall now see that (H-D2) combined with the canonical equivalence principle (P3) engenders problematic tacking consequences.

Recall, (H-D3), besides having such plausible consequences as that 'Fa' confirms '(x)Fx', also has implausible consequences such as that 'Fa' confirms '(x)Fx & (x)Gx'. Now (H-D2) by itself does not have the consequence that 'Fa', if true [rationally acceptable], confirms '(x)Fx & (x)Gx' since the deduction (x)Fx & (x)Gx + Fa is not premise relevant. However (H-D2) does have the consequence that 'Fa', if true [rationally acceptable], confirms '(x)Fx & (x)Gx & (x)Gx' since the deduction (x)Fx & (x)Gx + Fa is not premise relevant. However (H-D2) does have the consequence that 'Fa', if true [rationally acceptable], confirms '(x)Fx & (x)Gx's logical equivalent '(x)Gx & (x)(Gx \to Fx)'. Note, the inference (x)Gx & (x)(Gx \to Fx) + Fa is premise relevant. So (H-D2) combined with the canonical equivalence principle (P3) has the consequence that 'Fa' confirms '(x)Fx & (x)Gx'.

So far we have seen that (H-D2) combined with (P3) is subject to Glymour's tacking by conjunction problem. Previously we noted (i) that (H-D2) combined with Schurz's favored principle (P1) and the Hempelian consequence condition (P2) has unacceptable consequences, including, for instance, that 'Bs' confirms 'Bp' and (ii) that (H-D2) combined simply with (P2) has unacceptable consequences. Before concluding it is worth noting that (H-D2) by itself has highly unacceptable consequences.

Let 'Bs' stand for 'Sydney has a harbor bridge', 'Os' stand for 'Sydney has an opera house' and 'Bp' stand for 'Paris has a harbor bridge'. Now 'Bs  $\lor$  (Os & Bp)' is contentful and true [rationally acceptable], '(Bs  $\lor$  Os) & Bp' is consistent and (Bs  $\lor$  Os) & Bp  $\vdash$  Bs  $\lor$  (Os & Bp) and the deduction is premise and conclusion relevant, so according to (H-D2), 'Bs  $\lor$  (Os & Bp)' confirms '(Bs  $\lor$  Os) & Bp'. That is to say, that the claim 'Sydney has a harbor bridge, or Sydney has on Opera house and Paris has a harbor bridge' confirms 'Sydney has a harbor bridge or an Opera house, and Paris has a harbor bridge'. If this were true then confirming the claim 'Sydney has a harbor bridge or an Opera

house, and Paris has a harbor bridge' would take no more than a simple visit to Sydney harbor!

The root problem here is that Schurz's notion of relevant consequence allows 'Bs  $\lor$  (Os & Bp)' to count as a relevant consequence of  $(B_s \vee O_s) \& B_p$ . Another problematic results stems from that fact that Schurz's notion of relevant consequence allows, for instance,  $(Bs \lor \sim Bp)$  to count as a relevant consequence of  $(Bs \& Bp) \lor$  $(\sim Bs \& \sim Bp)$ '. To see how unintuitive this result is let 'Bs' be 'Sydney has a harbor bridge' and 'Bp' be 'Paris has a harbor bridge'. Then Svdnev and observing that it has harbor going to а bridge and hence that  $(Bs \lor \sim Bp)$  is true confirms  $(Bs \& Bp) \lor$  $(\sim Bs \& \sim Bp)$ '. Now note, ' $(Bs \& Bp) \lor (\sim Bs \& \sim Bp)$ ' is equivalent to  $(Bs \equiv Bp)$ '. But surely a trip to Sydney is not sufficient to confirm Sydney has a harbor bridge if and only Paris has one. Perhaps worse still note that a traveler to Sydney can confirm  $(Bs \& Bp) \lor$  $(\sim Bs \& \sim Bp)$ ' and hence its logical equivalent ' $(Bs \equiv Bp)$ ' irrespective of what he sees in Sydney. For if he see a bridge he knows  $(Bs \lor \sim Bp)$  is true and this by (H-D2) confirms  $(Bs \& Bp) \lor$  $(\sim Bs \& \sim Bp)$ '. If he does not see any bridge he knows  $(\sim Bs \lor Bp)$ ' is true and this by (H-D2\*) confirms ' $(Bs \& Bp) \lor (\sim Bs \& \sim Bp)$ '. Indeed, one who used (H-D2) liberally can confirm just about anything by observing some totally unrelated matter. For instance, to confirm atomic  $\alpha$  it will suffice to observe that unrelated atomic  $\beta$  is true. From  $\beta$  he can draw the consequence that  $\lceil (\sim \alpha \lor \beta) \rceil$  is true. From this by (H-D2\*) he has confirmation of  $\lceil (\alpha \& \beta) \lor (\neg \alpha \& \neg \beta) \rceil$  which combined with the observed truth  $\beta$  entails  $\alpha$ .

The notion of relevant consequence is helpful in the task of formulating a precise version of hypothetico-deductivism in that it allows us to discount inferences such as the inference from T to  $(T \vee S)$ . However, since it allows such inferences as the inference from  $(Bs \lor Os) \& Bp'$  $Bs \vee (Os \& Bp)',$ to and the inference from  $(Bs \& Bp) \lor$  $(\sim Bs \& \sim Bp)$ ' to ' $(Bs \land - Bp)$ ' it, without further supplementation, is ultimately inadequate to the task. Perhaps more helpful is the notion of content briefly mentioned in Gemes (1990) as a possible means for succor in the task of formulating hypothetico-deductivism. This notion of content is more fully developed in Gemes (1993). A simplified version is as follows:

 $\alpha$  is a content part of  $\beta$  iff  $\beta \vdash \alpha$  and there is no stronger consequence of  $\beta$ , constructible in the atomic wffs of  $\alpha$ .<sup>6</sup>

Note,  $(T \lor S)$  does not count as a content part of T since T itself is a consequence of T, is stronger than  $(T \lor S)$  and contains only atomic wffs occurring in  $(T \lor S)$ . Furthermore, 'Bs  $\lor (Os \& Bp)$ ' is not a content part of ' $(Bs \lor Os)$  & Bp' since ' $(Bs \lor Os)$  & Bp' is a consequence of  $(Bs \lor Os) \& Bp'$  that is stronger than  $(Bs \lor (Os \& Bp))'$  and only contains only atomic wffs occurring in 'Bs  $\vee$  (Os & Bp)'. Similarly,  $(Bs \lor \sim Bp)$  is not a content part of  $(Bs \& Bp) \lor (\sim Bs \& \sim Bp)$  since  $(Bs \& Bp) \lor (\sim Bs \& \sim Bp)'$ consequence is itself а of  $(Bs \& Bp) \lor (\sim Bs \& \sim Bp)$  that is stronger than  $(Bs \lor \sim Bp)$  and contains only atomic wffs that occur in '(Bs & Bp)  $\vee$  (~Bs & ~Bp)'. I suggest then that we do better then to recast hypothetico-deductivism in terms of demanding that where S confirms T, S be a content part of T. But that is a story I have pursued elsewhere (Cf. Gemes, 1993).

#### NOTES

<sup>1</sup> Carnap and Bayesians are committed to the claim that where  $T \vdash S$ , S confirms T, provided 0 < P(T) and P(S) < 1. This is very close to such standard versions of hypothetico deductivism as (H-D3) below.

<sup>2</sup> Cf. Carnap (1962), preface to the second edition, pp. xvi-xx and §86, pp. 462-468.

<sup>3</sup> The condition that the confirmation of T by S occurs by (H-D1) is omitted in Schurz's statement of (UC2) – proposition (30) on p. 419 of Schurz (1991) – though it is later assumed in the derivation of (30) from (H-D1) and (P2).

 $^4$  The previous unacceptable conclusion (UC1) is also deducible from the combination of (H-D1) and (P2).

*Proof:* Let T by any consistent theory and S be any true [rationally acceptable] contentful sentence such that S is consistent with T and  $\sim S$  with  $\sim T$ . Under these conditions, S is true [rationally acceptable] and contentful and (T & S) is consistent and  $(T \& S) \vdash S$ , so, by (H-D1) S confirms (T & S). Also  $(T \& S) \vdash T$ , so by (P2), S confirms T.

<sup>5</sup> The incompatibility of the favorable relevance notion of confirmation and Hempel's special consequence condition was pointed out in Carnap (1962), pp. 474–475.

<sup>6</sup> More formally, where  $\lceil At(X) \rceil$  designates the set of all atomic wffs occurring in X,

 $\alpha$  is a content part of  $\beta$  iff  $\beta \vdash \alpha$  and there is no  $\sigma$  such that  $\beta \vdash \sigma \vdash \alpha$ ,  $\alpha \not\vdash \sigma$ , and  $At(\sigma) \subseteq At(\alpha)$ .

### References

Carnap, R.: 1962, Logical Foundation of Probability Theory, The University of Chicago Press, Chicago.

Gemes, K.: 1990, 'Horwich, Hempel, and Hypothetico-Deductivism', *Philosophy of Science* 57, 690-702.

- Gemes, K.: 1993, 'Hypothetico-Deductivism, Content, and & The Natural Axiomatization of Theories', *Philosophy of Science* 60, 477-487.
- Gemes, K.: 1994, 'A New Theory of Content I: Basic Content', forthcoming in the Journal of Philosophical Logic 23.
- Grimes, T.: 1990, 'Truth, Content and the Hypothetico-deductive Method', *Philosophy* of Science 57, 514-522.
- Glymour, C.: 1980, *Theory and Evidence*, Princeton University Press, Princeton, New Jersey.
- Glymour, C.: 1980a, 'Discussion: Hypothetico-Deductivism is Hopeless', *Philosophy of Science* 47, 322-325.

Schurz, G.: 1991, 'Relevant Deduction', Erkenntnis 35, 391-437.

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Department of Philosophy Yale University Box 208306 Yale Station New Haven, Connecticut 06520-8306 U.S.A.