# NONINERTIAL NONLINEAR ELEMENTS AND THEIR USE TO CONTROL THE LASING REGIME

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Various noninertial nonlinear optical effects of electronic character that can be used as the basis of noninertial nonlinear elements capable of laser control, e.g., mode locking, separation of a single pulse, shaping a flat-top pulse, chopping a pulse, or stabilizing the emission, are condidered. The transmission of all such nonlinear filters is described in like fashion on the basis of a piecewise-linear approximation. The results are compared in a table. Various laser regimes with a similar noninertial elements inside a ring cavity are investigated theoretically.

# INTRODUCTION

To control the emission regime of a laser, extensive use is made of nonlinear elements such as filters, mirrors, etc. whose optical properties are changed by the radiation [1]. The transmission and reflection of such an element can be expressed in the form

$$R (J) = \gamma_0 + \gamma (J),$$
  

$$\gamma(0) = 0,$$
(1)

where J is the intensity of the radiation passing through the element. Such elements can vary the cavity Q as a function of the emission intensity. Thus, the use of nonlinear saturable dye filters, in which the loss decreases with increase of the intensity ( $\gamma > 0$ ), make it possible to lock effectively the laser modes and obtain high-power and short pulses.

Elements in which  $\gamma < 0$ , and accordingly the loss increases with intensity, can be used to stabilize the lasing and to suppress small spikes.

The inertia of the employed saturable filters (the filter reaches the initial state after a time  $\tau_{f}$ , the relaxation time of the dye molecules) makes it difficult to obtain pulses with durations much shorter than  $\tau_{f}$ , since such a filter poorly "cuts off" the trailing edge of the powerful pulse that makes the filter transmitting, inasmuch as the filter does not break down during the time  $\tau_{f}$ . Similarly, a stabilizing inertial device cannot suppress effectively the small-scale ripples.

No such difficulties arise in nonlinear elements with noninertial nonlinearity of "electronic" nature. In the case of mode locking, in particular, no restrictions will be imposed on the pulse duration by the relaxation time.

The various nonlinear elements considered below can be compared with the following parameters: initial transmission  $\gamma_0$ , initial rate of change of the transmission

$$\gamma_1 = \frac{\partial \mathbf{R} (\mathbf{J})}{\partial \mathbf{J}} \Big|_{\mathbf{J}=\mathbf{0},}$$
(2)

and also the saturation intensity  $J_{sat}$ , which we define, recognizing that for  $\gamma_0 > 0$  we have

$$\gamma_0 \leq \mathrm{R}(\mathrm{J}) \leq 1,$$

as that intensity J<sub>s</sub> for which

$$R\left(\frac{J_{s}}{2}\right) = \frac{1+\gamma_{0}}{2}$$
(3)

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Fig. 1. Plot of R(p) (dashed) and its piecewise equivalent (solid).



Fig. 2. Reflection under conditions close to TIR. I) prism of nonlinear material; II) medium of linear material  $(n_{20} < n_{10}), \varphi_0$  total internal reflection angle (sin  $\varphi_0 = n_{20}/n_{10}$ );  $\varphi$ ) incidence angle,  $\varphi = \varphi_0 - \Delta$ ; N - normal; 1, 2) incident and reflected rays.

Introducing the relative flux density

$$p = \frac{J}{J_s}$$
(4)

we can replace the real transmission by a standard piecewise-linear one having the same form in dimensional variables for all the elements considered above. This method, called the piecewise-linear approximation of the characteristics of nonlinear elements, is extensively used in the theory of nonlinear electric circuits [3]. The equivalent piecewise-linear transmission will be

$$\mathbf{R}(\mathbf{p}) = \begin{cases} \gamma_0 + (1 - \gamma_0) \mathbf{p} & (\mathbf{p} \le 1) \\ 1 & (\mathbf{p} \ge 1) \end{cases}$$
(5)

Figure 1 shows the real transmission of a certain element and its piecewise linear equivalent. In form, the piecewise-linear equivalents R(p) for different elements differ only by  $\gamma_0$ . The reason for the latter is we are using the dimensionless quantity  $p = J/J_s$ , for which  $p_s = 1$ .



Fig. 3. Refraction, in a prism, near the total internal reflection angle. I) prism of linear material; 2) prism of nonlinear material; 3) normal to the interface of the media;  $n_{10} \ge n_{20}; \varphi_0$ ) total internal reflection angle (sin  $\varphi_0 = n_{20}/n_{10}$ );  $\varphi$ ) incidence angle;  $\psi$ ) refraction angle; 1, 2, 3) incident, reflected, and refracted beams, respectively.



Fig. 4. Transmission in refraction close to the TIR angle on the interface of a linear and nonlinear medium;  $n_{10} = 1.3003$ ,  $n_{20} = 1.30000$ ,  $\kappa = 2 \cdot 10^{-11}$ ; TIR angle  $\varphi_0 = 88.77^\circ$ ; curve 1 corresponds to  $\Delta = 0$  and curve 2 to  $\Delta = 0.1^\circ$ .

We consider next real physical noninertial nonlinear elements of various physical types, calculate the corresponding R(J), and also consider how such elements operate in lasers.

# CALULATION OF THE TRANSMISSION OF A NONLINEAR ELEMENT

We consider first a flat mirror of nonlinear material, with a refractive index that depends on the intensity [4, 5]

$$n = n_0 + \delta n_2 = n_0 + \frac{8\pi\kappa}{cn_0} J$$
 (6)



Fig. 5. Nonlinear filter based on self-induced rotation of the polarization plane: 1, 2) polarizers crossed at an angle  $\varphi_0$ ; 3) "rotating" sample; 4) light beam.



Fig. 6. Nonlinear element based on self-rotation of polarization ellipse: 1, 5) polarizers crossed at an angle 90°; 2, 4) crystalline phase plates; 3) "rotating" sample; 6) beam.

where  $n_0$  is the refractive index in a weak field, and  $\kappa$  is the nonlinearity coefficient, which we assume in all estimates that follow to be  $2 \cdot 10^{-11}$  CGSE. This value of  $\kappa$  is reached, for example, in CS<sub>2</sub> [4] and receives the main contribution by an "orientational" mechanism whose relaxation time is  $10^{-12}$  sec. The noninertial purely "electronic" mechanism in glass yields  $\kappa = 2 \cdot 10^{-13}$  CGSE [6], but in certain semiconductor materials the "electronic" nonlinearity is considerably higher and  $\kappa$  can reach  $10^{-10}$ - $10^{-9}$  CGSE.

Let the radiation be incident at an angle  $\varphi$  on the interface of two media with respective refractive indices  $n_{10}$  and  $n_{20}$ . The amplitude reflection coefficients for waves with s- and p-polarization will be respectively [4]:

$$\mathbf{r}_{\mathbf{S}} = \frac{\sqrt{1 - n^2 \sin^2 \varphi} - \tilde{n} \cos \varphi}{\sqrt{1 - \tilde{n}^2 \sin^2 \varphi} + \tilde{n} \cos \varphi} , \qquad (7)$$

$$\mathbf{r}_{\mathbf{p}} = -\frac{\cos\varphi - \widetilde{n}\sqrt{1 - \widetilde{n}^{2}\sin^{2}\varphi}}{\cos\varphi + \widetilde{n}\sqrt{1 - \widetilde{n}^{2}\sin^{2}\varphi}}$$
(8)

where  $\tilde{n}$  is the relative refractive index

$$\widetilde{\mathbf{n}} = \frac{\mathbf{n}_{10}}{\mathbf{n}_{20}} \tag{9}$$



Fig. 7. Nonlinear element with distributed nonlinear mirror with external feedback: 1) fundamental harmonic beam; 2) second-harmonic beam; 3) splitting plate; 4) frequency doubler; 5, 6) mirrors for second harmonic.

The energy reflection coefficient is correspondingly

$$\mathbf{R}_{\mathbf{s},\mathbf{p}} = \mathbf{r}_{\mathbf{s},\mathbf{p}}^2 \tag{10}$$

To calculate the reflection R(J) of a nonlinear medium in the nonlinear case, when the index i of the medium changes

$$\mathbf{n}_{i} = \mathbf{n}_{i0} + \delta \mathbf{n}_{2i} \tag{11}$$

it is necessary to substitute (11) in (9) and correspondingly (7), (8) and (10). Consider the case when the second medium is nonlinear; then

$$R_{s}(J) = \left( \frac{\sqrt{\left(\frac{n_{20} + \delta n_{2}}{n_{10}}\right)^{2} - \sin^{2} \varphi - \cos \varphi}}{\sqrt{\left(\frac{n_{20} + \delta n_{2}}{n_{10}}\right)^{2} - \sin^{2} \varphi + \cos \varphi}} \right)^{2}$$
(12)

$$R_{p}(J) = \left(\frac{\cos\varphi - \frac{n_{10}}{n_{20} + \delta n_{2}} \sqrt{1 - (\frac{n_{10}}{n_{20} + \delta n_{2}})^{2} \sin^{2}\varphi}}{\cos\varphi + \frac{n_{10}}{n_{20} + \delta n_{2}} \sqrt{1 - (\frac{n_{10}}{n_{20} + \delta n_{2}})^{2} \sin^{2}\varphi}}\right)^{2},$$
(13)

$$\delta \mathbf{n}_2 = \frac{8\pi\kappa}{c\mathbf{n}_{20}} \mathbf{J},\tag{14}$$

We have accordingly (only for s polarization)

$$\gamma_{\mathbf{0S}} = \mathbf{r}_{\mathbf{S}}^2 , \qquad (15)$$

$$\gamma_{1s} = \frac{32\pi n_{10} \cos \varphi \kappa \sqrt{\gamma_{0s}}}{\operatorname{cn}_{20}^3 \left(\sqrt{1-\widetilde{n}^2 \sin^2 \varphi} + \widetilde{n} \cos \varphi\right)^2 \sqrt{1-\widetilde{n}^2 \sin^2 \varphi}}$$
(16)

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Fig. 8. Diagram of traveling-wave laser in a ring cavity with nonlinear element: 1) active element; 2) nonlinear element; 3, 4, 5) mirrors; 6) light beam.

From (12), with allowance for the definition (3), we obtain the saturation intensity

$$J_{SH} = \frac{cn_{20}}{4_{sat}} (n_{10}\sqrt{\sin^2\varphi + \cos^4\varphi} (\frac{1\pm\sqrt{\frac{1+\gamma_{0S}}{2}}}{1\pm\sqrt{\frac{1+\gamma_{0S}}{2}}})^2 - n_{20}),$$
(17)

where  $\gamma_{os}$  is defined by (15), and the upper sign in front of the square root is used if

$$\sqrt{1-\widetilde{n}^2\sin^2\varphi} \geq \widetilde{n}\,\cos\varphi,$$

and the lower sign is used in the opposite case.

For normal incidence, Eqs. (15), (16), and (17) take, with allowance for (7), the form

$$\gamma_0 = \left(\frac{1-\widetilde{n}}{1+\widetilde{n}}\right)^2 = \left(\frac{n_{20} - n_{10}}{n_{20} + n_{10}}\right)^2,\tag{18}$$

$$\gamma_1 = \frac{32\pi\kappa \ n_{10} \ (n_{20} - n_{10})}{c \ n_{20} \ (n_{20} + n_{10})^3}, \qquad (19)$$

$$J_{\rm H} = \frac{cn_{20}}{4\pi\kappa} \left( n_{10} \frac{1 + \sqrt{\frac{1+\gamma_0}{2}}}{1 - \sqrt{\frac{1+\gamma_0}{2}}} - n_{20} \right). \tag{20}$$

In the last equation we assume that  $n_{10} \le n_{20}$ .

Example. In the case of normal incidence ( $\varphi = 0$ ) at  $\kappa = 2 \cdot 10^{-11}$  CGSE  $n_{10} = 1$  (air), and  $n_{20} = 1.52$ , we find from (18)-(20) that  $\gamma_0 = 0.043$ ,  $\gamma_1 = 1.43 \cdot 10^{-5}$  cm<sup>2</sup>/GW and  $J_s = 8.4 \cdot 10^4$  GW/cm<sup>2</sup>.

We consider now the case of reflection of an s wave from a nonlinear medium at a large angle.

**Example.** In the case  $\varphi = 85^\circ$ ,  $\kappa = 2 \cdot 10^{-11}$  CGSE, and  $n_{10} = 1.52$ , we obtain from (15), (16), (7), and (17)  $\gamma_0 = 0.974$ ,  $\gamma_1 = 3.76 \cdot 10^{-5}$  cm<sup>2</sup>/GW, and recognizing that in (17) it is necessary to take the upper sign,  $J_s = 1.8 \cdot 10^4$  GW/cm<sup>2</sup>. The foregoing examples show that, depending on  $\varphi$ , the initial reflection  $\gamma_0$  from a nonlinear medium can be arbitrary, the saturation intensity  $J_s$  is high and exceeds apparently the ultimate strength of the material.

![](_page_6_Figure_0.jpeg)

Fig. 9. Mode locking. Steepening of pulse from the noise pattern. Values of parameters in CGSE:  $\gamma_0 = 0.12, \chi = 1.7 \cdot 10^{-4}, \delta = 0.18, \mathcal{H}_0 = 10, \tau = 3 \cdot 10^3$  psec;  $p_0 = 0.1, r = 10, \hbar\omega = 2.8 \cdot 10^{-12}, \sigma = 2.5 \cdot 10^{-20}$ . a) dashed — initial profile, M1:0.1;  $\overline{p} = 0.01, \tau = 10$  psec, r = 10; b) dashed — third pass, M1:1,  $\overline{p} = 7 \cdot 10^{-3}, \tau = 5$  psec; c) solid — 7th pass, M1:100,  $\overline{p} = 8.7 \cdot 10^{-5}, r = 2 \cdot 10^6, \tau = 4.3$  psec. d, e, f) conditional representation of noise in the 0th, 3rd, and 7th passes.

### Nonlinear Element Based on Total Internal Reflection (TIR)

We consider now a nonlinear element (see Fig. 2) in which the light is incident at an angle  $\varphi$  close to the angle of the total internal reflection angle  $\varphi_0$  (now  $n_{10} > n_{20}$ ):

$$\sin\varphi_0 = \frac{n_{20}}{n_{10}} , \qquad (21)$$

$$\varphi_0 - \varphi = \Delta, \tag{22}$$

where  $\Delta$  is small but larger than the diffraction divergence  $\delta \varphi_g$ , which amounts at a wavelength  $\lambda = 7 \cdot 10^{-5}$  cm and a beam width a = 1 cm to  $\delta \varphi_g = 7 \cdot 10^{-5}$  (0.24').

Let the nonlinear medium be the first, and correspondingly from (7), (9), (10), and (11) we obtain for an s-polarization wave (we consider only this case) a reflection coefficient

$$R(J) = \begin{pmatrix} (1 + \frac{\delta n_2}{n_{10}}) \cos(\varphi_0 - \Delta) - \sqrt{\sin^2 \varphi_0 - (1 + \frac{\delta n_2}{n_{10}})^2 \sin^2(\varphi_0 - \Delta)} \\ (1 + \frac{\delta n_2}{n_{10}}) \cos(\varphi_0 - \Delta) + \sqrt{\sin^2 \varphi_0 - (1 + \frac{\delta n_2}{n_{10}})^2 \sin^2(\varphi_0 - \Delta)} \\ 1; \quad J > J_{so}, \end{cases}$$
(23)

![](_page_7_Figure_0.jpeg)

Fig. 10. Mode locking under conditions of "hard" start: a) dashed, initial profile, for left-hand spike  $\tau = 10$  psec,  $p_0 = 0.12$ , M1:0.1; b) dashed, 5th pass,  $\tau = 5$  psec, M1:3; c) solid, 8th pass,  $\tau = 2.5$  psec, M1:600

where

$$\delta n_2 = \frac{8\pi\kappa}{cn_{10}} J,$$

 $J_{s0}$  is determined from the equality of the radicand in (23) to zero and corresponds to the intensity at which the TIR sets in:

$$J_{so} = \frac{n_{10}^2 c}{8\pi\kappa} \left( \frac{\sin\varphi_0}{\sin(\varphi_0 - \Delta)} - 1 \right) \approx \frac{n_{10}^2 c}{8\pi\kappa} \operatorname{ctg} \varphi_0 \cdot \Delta, \tag{24}$$

The latter was obtained in the limit as  $\Delta \rightarrow 0$ . It can be seen from (24) that as the TIR is approached ( $\Delta \rightarrow 0$ )  $J_{s0}$  tends to zero. From (23) we get for J = 0:

$$\operatorname{OH}(23) \text{ we get for } 3 = 0.$$

$$\gamma_0 = \left(\frac{\cos\left(\varphi_0 - \Delta\right) - \sqrt{\sin^2\varphi_0 - \sin^2\left(\varphi_0 - \Delta\right)}}{\cos\left(\varphi_0 - \Delta\right) + \sqrt{\sin^2\varphi_0 - \sin^2\left(\varphi_0 - \Delta\right)}}\right)^2 \tag{25}$$

$$\gamma_{1} = \frac{32\pi\kappa n_{20}^{2} \cos(\varphi_{0} - \Delta)\sqrt{\gamma_{0}}}{\operatorname{cn}_{10}^{4} \sqrt{\sin^{2}\varphi_{0} - \sin^{2}(\varphi_{0} - \Delta)} (\cos(\varphi_{0} - \Delta) + \sqrt{\sin^{2}\varphi_{0} - \sin^{2}(\varphi_{0} - \Delta)})} \approx$$

$$\approx \frac{32\pi\kappa n_{20} \sqrt{\gamma_{0}}}{\operatorname{cn}_{10}^{3} \cos^{2}\varphi_{0} \sqrt{2} \operatorname{ctg} \varphi_{0} \Delta}$$
(26)

the latter obtained as  $\Delta \rightarrow 0$ . It is seen from (26) that  $\gamma_1 \rightarrow \infty$  as  $\Delta \rightarrow 0$ . In accordance with the definition 3, we obtain from (23) the saturation intensity  $J_s$ 

$$J_{s} = \frac{cn_{10}^{2}}{4\pi\kappa} \left( \frac{\frac{\sin\varphi_{0}}{\sqrt{1+\gamma_{0}}^{2}} - 1}{\sqrt{\sin^{2}(\varphi_{0} - \Delta) + \frac{1 - \sqrt{1+\gamma_{0}}^{2}}{1 + \sqrt{\frac{1+\gamma_{0}}{2}}}} \cos^{2}(\varphi_{0} - \Delta)} \right)$$
(27)

Inasmuch as  $\Delta \rightarrow 0$  we have from (25)  $\gamma_0 \rightarrow 0$ , we see from (27) that  $J_s \rightarrow 0$  as  $\Delta \rightarrow 0$ . Note also that the saturation intensity  $J_s$  is indicative of the equivalent transmission or reflection (5) and need not necessarily coincide with  $J_{s0}$ .

Nonlinear element	Number of equations	$\gamma_0$	$\gamma_1 \text{ cm}^2/\text{GW}$	J <sub>S</sub> G₩/cm²
Normal reflection from nonlinear medium	18-20	0,043	I,43 I0 <sup>-5</sup>	8,4 I0 <sup>4</sup>
Reflection of wave at an angle 85°	15–17	0,976	3,76 IO <sup>-5</sup>	I,8 I0 <sup>4</sup>
Reflection under TIR conditions. The refractive indices of the media differ	25–27	0,85	4,7 I0 <sup>-3</sup>	24,8
Reflection under TIR conditions, Refrac- tive indices of the media are close	25–27	0	0	5,5
Transmission under TIR conditions	2 <b>9-</b> 3I	0	80	0,10
Self-induced rotation of the polarization	35–38	0,5	4	0,13
Self rotation of the polarization ellipse	40-42	0,5	0	16,8
Nonlinear distri- buted mirror	43,44	0,04	0	3,3
Dense dye-solution filter	4648	0,135	I9 <b>,</b> 3	0,071
Weak dye-solution filter	50-52	0,8	I,93	0,028

TABLE 1

**Example.** For  $\mathbf{n}_{10} = 1.52$ ,  $\mathbf{n}_{20} = 1$ ,  $\kappa = 2 \cdot 10^{-11}$  CGSE,  $\varphi_0 = 41^{\circ}3'$  we choose  $\varphi = 41^{\circ}$  ( $\Delta = 3'$ ). From (25)-(27) we find  $\gamma_0 = 0.85$ ,  $\gamma_1 = 4.7 \cdot 10^{-3}$  cm<sup>2</sup>/GW, and  $\mathbf{J}_s = 24.8$  GW/cm<sup>2</sup>. Thus,  $\mathbf{J}_s$  is much smaller, and  $\gamma_1$  larger than in the preceding case, while  $\gamma_1 \rightarrow \infty$  and  $\mathbf{J}_s \rightarrow 0$  as  $\Delta \rightarrow 0$ . However, the nonlinear element has in the last case a high initial reflectivity. In such an element a small initial reflectivity can be obtained if both media are chosen to have close refractive indices. For example, if  $\mathbf{n}_{10} = \mathbf{n}_{20}$  and  $\varphi = 90^{\circ}$  and there is no reflection in a weak field, when the optical field increases the reflectivity will also increase, all the way to TIR.

Example.  $n_{10} = n_{20} = 1.52$ ,  $\kappa = 2 \cdot 10^{-11}$  CGSE,  $\varphi_0$ , 90°;  $\varphi = 89°$ ; we obtain then from (25)-(27) that  $\gamma_0 = 0$ ,  $\gamma_1 = 0$ ,  $J_s = 5.5$  GW/cm<sup>2</sup>.

#### Nonlinear Filter under Conditions Close to TIR

For another variant of an element that uses TIR see Fig. 3. In this variant we have again  $n_{10} \ge n_{20}$ , but it is the second medium which is nonlinear, and the element operates in transmission: with an increase in J the refractive index of the second medium increases and the total internal reflection is violated all the more. The transmission in this case can be obtained by subtracting from (1) the reflection R(J) given by Eq. (12) for s polarization, for example, and the transmission of the element is given by

$$R(J) = 1 - \left(\frac{\cos(\varphi_0 - \Delta) - \sqrt{\sin^2 \varphi_0 (1 + \frac{\delta n_2}{n_{10}})^2 - \sin^2(\varphi_0 - \Delta)}}{\cos(\varphi_0 - \Delta) + \sqrt{\sin^2 \varphi_0 (1 + \frac{\delta n_2}{n_{10}})^2 - \sin^2(\varphi_0 - \Delta)}}\right)^2$$
(28)

![](_page_9_Figure_0.jpeg)

Fig. 11. Generation of square pulses with the aid of an element with self-induced rotation of the polarization plane: a) initial pulse; b) pulse in 4th pass; d) pulse in 6th pass.

The transmission (28) (see Fig. 4) differs from the preceding case (compare Figs. 1 and 4) and has a maximum  $R_{max} = 1$  and  $J = J_m$ , at which the refractive indices of both media become equal and the numerator in (28) is zero. In this case

$$J_{m} = \frac{cn_{10} (n_{10} - n_{20})}{8\pi\kappa} , \qquad (29)$$

As  $J \rightarrow \infty$  it can be seen from (28) that  $R(\kappa) \rightarrow 0$  (see Fig. 4).

**Example.**  $\kappa = 2 \cdot 10^{-11}$  CGSE,  $n_{10} = 1.3003$ ,  $n_{20} = 1.3000$ ,  $\varphi_0 = 89^\circ$ ,  $J_m = 2.3$  GW/cm<sup>2</sup>. It is seen from (29) that  $n_{10} - n_{20} \rightarrow 0$  as  $J_m \rightarrow 0$ , i.e., the saturation intensity decreases. From (28) we obtain  $\gamma_0 \gamma_1$ :

$$\gamma_{0} = 1 - \left(\frac{\cos(\varphi_{0} - \Delta) - \sqrt{\sin^{2}\varphi_{0} - \sin^{2}(\varphi_{0} - \Delta)}}{\cos(\varphi_{0} - \Delta) + \sqrt{\sin^{2}\varphi_{0} - \sin^{2}(\varphi_{0} - \Delta)}}\right)^{2}$$
(30)

$$\gamma_{1} = \frac{32\pi\kappa n_{20} \sqrt{1 - \gamma_{0} \cos(\varphi_{0} - \Delta)}}{cn_{10}^{3} \sqrt{\sin^{2}\varphi_{0} - \sin^{2}(\varphi_{0} - \Delta)} (\cos(\varphi_{0} - \Delta) + \sqrt{\sin^{2}\varphi_{0} - \sin^{2}(\varphi_{0} - \Delta)})^{2}}$$
(31)

From (28) and the definition (3) we obtain

$$J_{s} = \frac{cn_{10}n_{20}}{4\pi\kappa} \left( \frac{\sqrt{\sin^{2}(\varphi_{0} - \Delta) + \cos^{2}(\varphi_{0} - \Delta)} + \frac{1 - \sqrt{(1 - \gamma_{0})/2}}{1 + \sqrt{(1 - \gamma_{0})/2}}}{\sin\varphi_{0}} \right) - 1$$
(32)

**Example.** For  $\kappa = 2 \cdot 10^{-11}$  CGSE,  $n_{10} = 1.3003$ ,  $n_{20} = 1.3000$ ,  $\varphi_0 = 89^\circ$ ,  $\Delta = 0$ , we find from (30), (31), and (32) that  $\gamma_0 = 0$ ,  $\gamma_1 = \infty$ ,  $J_s = 0.1$  GW/cm<sup>2</sup>. Thus,  $\gamma_1$  is large and  $J_s$  is relatively small. By varying  $\Delta$ , we can obtain various initial transmissions  $\gamma_0$ . A shortcoming of the considered element may be that the refraction angle  $\psi$  varies with intensity.

To produce a noninertial filter we can use also the effect of self-induced rotation of the polarization plane (SIRPP), which arises in certain nonlinear substances [7], with the angle  $\psi$  of rotation of the polarization plane proportional to the intensity J and to the sample length *l*:

$$\psi = \beta \mathbf{J} \mathbf{I} \tag{33}$$

where the coefficient  $\beta$  reaches 1-100 (rad/cm)(GW/cm<sup>2</sup>) [7]. We consider the diagram of the corresponding device (see Fig. 5), which consists of a self-rotating sample placed between polarizers crossed at an a angle  $\varphi_0$ . Let light of intensity J with a polarization parallel to the axis of the first polarizer pass through our device. Then, according to (33), the light passing through the second polarizer has an intensity

$$\mathbf{\tilde{J}} = \mathbf{J}\cos^2\left(\varphi_0 - \beta l\mathbf{J}\right) \tag{34}$$

where  $0 \le \varphi_0 \le \pi/2$ , and we have chosen the sign of  $\varphi_0$  to be the same as the sign of the rotation angle  $\psi$ . The transmission of such a filter is then

$$\mathbf{R}(\mathbf{J}) = \cos^2\left(\varphi_0 - \beta l \mathbf{J}\right) \tag{35}$$

and

$$\gamma_0 = \cos^2(\varphi_0), \tag{36}$$

$$\gamma_1 = \sin 2\varphi_0 \beta l, \tag{37}$$

and from (34) and from the definition (3) of  $J_s$  we get

$$J_{s} = \frac{2\varphi_{0} - 2 \arccos \sqrt{\frac{1 + \cos^{2}\varphi_{0}}{2}}}{\beta l}$$
(38)

Note that R(J) in (35) is a periodic function of J, out of the set  $J_s$  we have selected the smallest value, and the equivalent piecewise linear transmission (5) is valid in fact only up to intensities of order  $\varphi_0/\beta l$ . Note that by choosing  $\varphi_0$  and l we can obtain any  $\gamma_0$  and independently any  $J_s$ , i.e., realize any equivalent characteristic (5).

Example.  $\beta = 4 \text{ (rad/cm)/(GW/cm^2)}$  in  $Z_n P_2$  [7], choosing  $\varphi_0 = \pi/4$ , l = 1 cm, we obtain from (35)-(37),  $\gamma_0 = 0.5$ ,  $\gamma_1 = 4 \text{ cm}^3/\text{GW}$  and  $J_s = 0.13 \text{ GW/cm}^2$ .

### Element Based in Self-Rotation of the Polarization Ellipse (SRPE)

A noninertial nonlinear filter can be produced on the basis of the effect of self-rotation of the polarization ellipse (SRPE) [14], in which the rotation angle  $\psi$  of the polarization ellipse in a nonlinear medium is proportional to the intensity J and to the length of the nonlinear sample. A filter based of this effect was considered in [15, 16, 17], where the relaxation time of the nonlinearity in the medium was determined by an "orientational" mechanism and amounted to  $10^{-12}$  sec. The results of these papers demonstrated experimentally the effectiveness of such a filter for passive mode locking, noise discrimination, and pulse-duration shortening. A similar effect should be expected in media with nonlinearity of electronic character. Let us consider the scheme of a corresponding device (see Fig. 6). Let the light pass through a polarizer (1), whose axis makes an angle  $\theta$  with the axis at the phase plate (2) that shifts the phase of the y component by  $\varphi$ . After passing through plate 2 the light acquires an elliptic polarization and is incident on a nonlinear self-rotating sample (3) of length *l*, which rotates the polarization ellipse through an angle  $\psi$ :

$$\psi = -\frac{4\pi^2 \omega}{\mathrm{cn}^2} \,\mathrm{B} I \sin 2\theta \sin \varphi \mathrm{J},\tag{39}$$

where B is a coefficient whose value, e.g., for CS<sub>2</sub> is  $2.5 \cdot 10^{-13}$  CGSE [14]. The light passes next through a similar phase plate (4) and a polarizer (5) crossed at an angle 90° with the first polarizer (1). The transmission coefficient of such a system at  $\theta = \pi/4$  is

$$\mathbf{R} (\mathbf{J}) = \sin^2 \psi + \cos^2 \psi \, \sin^2 \varphi \, . \tag{40}$$

From this we find that

$$\gamma_0 = \sin^2 \varphi \,, \, \gamma_1 = 0, \tag{41}$$

and in accordance with the definition (3):

$$J_{s} = \frac{c^{2}n^{2}}{8\pi\omega Blsin\varphi}$$
(42)

Example. At the following parameters (in CGSE)  $c = 3 \cdot 10^{10}$ , n = 1.5,  $\omega = 2.7 \cdot 10^{15}$ ,  $B = 2.5 \cdot 10^{13}$  (a value corresponding to  $\kappa = 2 \cdot 10^{-11}$ ) we find from (39)-(40) for  $\varphi = \pi/4$  that  $\gamma_0 = 1/2$ ,  $\gamma_1 = 0$ ,  $J_s = 1.7 \cdot 10^{17}$  GW/cm<sup>2</sup>).

Just as in the preceding case, the transmission is periodic in the intensity.

# Element with Distributed Mirror and "External" Feedback

To produce a noninertial filter one can use a distributed mirror. An example of such an element is the device shown in Fig.7, in which a definite fraction  $\xi J$  of the principal radiation, with frequency  $\omega$ , is extracted first to the outside, is transformed with efficiency  $\eta$  into second-harmonic ( $2\omega$ ) radiation, passes through a nonlinear medium having a noninertial nonlinearity  $\kappa$ , and is totally reflected from the mirror (mirror at frequency  $2\omega$ ) at a glancing angle  $\alpha$  (the radiation polarization is perpendicular to the incidence plane). In the nonlinear medium is produced a grating of a nonlinear addition to the refractive index [8]. The fundamental-frequency radiation, which is not reflected from the mirror 8) undergoes then Bragg reflection of first order from the induced grating, at a glancing angle  $\varphi$ :

$$\sin\varphi = \frac{m\sin a\,\lambda\,(\omega)}{\lambda\,(2\omega)}$$

where  $\lambda(\omega)$ ,  $\lambda(2\omega)$  are the wavelengths in the medium at the frequencies  $\omega$  and  $2\omega$ , respectively, and m is the order of the reflection. In our case m = 1. We use a grating of frequency  $2\omega$ , since, if it were made up by radiation of frequency  $\omega$ , at m = 1 the angle is  $\varphi$  = a and there would be no additional self-Q-switching in a medium with a real nonlinearity [9].

The energy reflection coefficient of a distributed mirror of length l at a frequency  $\omega$  will be [8]:

$$\mathbf{R}(\mathbf{J}) = \gamma_0 + (1 - \gamma_0) (1 - e^{-\alpha t})^2$$
(43)

where a(J) is the "distributed" reflection coefficient

$$a(J) = \frac{8\pi}{3} \quad \frac{\omega\kappa}{c^2} \quad J(2\omega) = \frac{16\pi^2\kappa(\xi\eta)}{3\lambda c} \quad J(\omega).$$

We have taken into account here the additional Fresnel reflection  $\gamma_0$  from the surface of the nonlinear element. It follows from (43) that  $\gamma_1 = 0$ . From (43) and (3) we obtain  $J_s$ :

$$J_{s} = \frac{\lambda c}{l\kappa \xi \eta} \frac{3}{8\pi^{2}} \ln \frac{\sqrt{2}}{\sqrt{2} - 1} = \frac{\lambda c}{l\xi \eta \kappa} 4,7 \cdot 10^{-2}.$$
 (44)

Example.  $\gamma_0 = 0.04$ , l = 1 cm,  $\kappa = 2 \cdot 10^{-11}$ ,  $\lambda = 7 \cdot 10^{-5}$  cm<sup>-1</sup>,  $\xi = 0.3$ ,  $\eta = 0.5$ ; we have then from (44) J<sub>s</sub> = 3.3 GW/cm<sup>2</sup>. The radiation losses to the "external" feedback can be included in the total radiation loss in the cavity.

#### Saturable Nonlinear Dye-Solution Filter

We present for comparison the parameters of a saturable filter using a high-density dye solution [10. 18]. The filter transmission will be

$$\mathbf{R}(\mathbf{J}) = \mathbf{e}^{-\sigma_{\mathbf{f}} l_{\mathbf{f}} \mathbf{n}_{\mathbf{I}}}$$
(45)

where  $\sigma_f$  is the dye-molecule absorption cross section,  $l_f$  is the filter length, and  $n_1$  is the number of molecules per unit volume in the absorbing state. Neglecting the inertia of the filter, we get

$$n_{1} = \frac{n_{0}}{1 - \sigma_{f} J \tau_{f}^{h} \omega}$$
(46)

![](_page_12_Figure_0.jpeg)

Fig. 12. Breakup of pulse by a nonlinear element based on SIRPP; a) initial pulse, M1:1; b) pulse in second pass, M1:50.

where  $n_0$  is the density of the dye molecules,  $\omega$  is the cyclic frequency of the light, and  $\tau_f$  is the filter relaxation time. We obtain then from (45) and (46):

$$\gamma_0 = e^{-\sigma_f l_f n_0}, \tag{47}$$

$$\gamma_{1} = \sigma_{f}^{2} l_{f} e^{-\sigma_{f} l_{f} n_{0}} \frac{\tau_{f}}{\hbar\omega}$$
(48)

from (3), (45) and (46) we obtain  $J_s$ :

$$J_{H} = \frac{2\hbar\omega}{\sigma_{f}^{\tau}f_{f}} \left( \frac{\sigma_{f}^{l}f_{f}^{n}o}{\ln\left(\frac{2}{1+\exp(-\sigma_{f}l_{f}^{n}o)} - 1\right)} \right).$$
(49)

**Example.** (All the parameters are in CGSE )  $\sigma_{\rm f} = 5 \cdot 10^{-16}$ ,  $l_{\rm f} = 0.4$ ,  $\tau_{\rm f} = 4 \cdot 10^{-11}$ ,  $\hbar\omega = 2.8 \cdot 10^{-12}$ ,  $n_0 = 10^{16}$ . We obtain then from (47)-(49),  $\gamma_0 = 0.135$ ,  $\gamma_1 = 1.93 \cdot 10^{-15}$ , 19.3 cm<sup>2</sup>/GW,  $J_{\rm s} = 7.1 \cdot 10^{14}$ , 7.1  $\cdot 10^{-2}$  GW/cm<sup>2</sup>. From (45)-(49) it is easy to obtain the parameters of a filter of low density  $\sigma_{\rm f} l_{\rm f} n_0 << 1$ :

$$\mathbf{R}(\mathbf{J}) = 1 - \frac{\sigma_{\mathbf{f}}^{t} \mathbf{f}^{\mathsf{n}}_{\mathsf{0}}}{1 + \sigma_{\mathbf{f}} \mathbf{J} \tau_{\mathbf{f}} / \hbar \omega}, \qquad (50)$$

$$\gamma_1 = \frac{\sigma_f' f}{\hbar \omega} (\sigma_f l_f n_0), \qquad (51)$$

$$\gamma_0 = 1 - \sigma_f l_f n_0, \tag{52}$$

$$J_{s} = \frac{2\hbar\omega}{\sigma_{f}\tau_{f}} , \qquad (53)$$

Example. We decrease by an order of magnitude the density in the preceding example, and leave the remaining parameters unchanged:  $n_0 = 10^{15} \text{ cm}^{-3}$ , so that  $\sigma_f n_0 l_f = 0.2$ . We have then from (51)-(53)  $\gamma_0 = 0.8$ ,  $\gamma_1 = 1.93 \text{ cm}^2/\text{GW}$  and  $J_s = 2.8.10^{-2} \text{ GW/cm}^2$ .

The values of  $\gamma_0$ ,  $\gamma_1$ ,  $J_s$  for the different elements are listed in Table 1.

For lack of space we have not considered some other variants of nonlinear transparentizing elements. For example, an element based on varying the Brewster angle in a strong field, an element with "external" feedback using a shutter based on the high-frequency Kerr effect [11], etc.

We consider now "obscuring" noninertial elements, in which the losses increase with increase of intensity ( $\gamma < 0$ ). It is easy to note that all the elements with  $\gamma > 0$ , considered above, can be "reversed" and made into elements with  $\gamma < 0$ . For example, an element working in reflection can be used in transmission. The transmission of an element based on SIRPP, contained in parallel polarizers, will be

$$\mathbf{R}(\mathbf{J}) = \cos^2(\beta l \mathbf{J}) \tag{54}$$

and decreases with increase of intensity at  $\beta IJ \leq \pi/2$ . An equivalent piecewise-linear transmission of such elements in dimensionless variables (4) will be:

$$R(\mathbf{p}) = \begin{cases} \widetilde{\gamma}_0 - \widetilde{\gamma}_1 \mathbf{p}, \quad \mathbf{p} \le 1\\ \widetilde{\gamma}_0 - \widetilde{\gamma}_1, \quad \mathbf{p} > 1, \end{cases}$$
(55)

where  $\tilde{\gamma}_0 \leq 1$ , and  $\tilde{\gamma} - \tilde{\gamma}_1 \geq 0$ , and cases are possible when  $\tilde{\gamma}_0 - \tilde{\gamma}_1 = 0$ , i.e., the element is completely shut off. In (54), for example, takes place at  $\beta IJ = \pi/2$ . A similar effect will be produced in an inverted element with TIR, etc.

## 2. EQUATION FOR TRAVELING-WAVE GENERATOR IN A RING LASER WITH NONLINEAR OPTICAL ELEMENT

We consider for simplicity a traveling-wave laser in a ring cavity (Fig. 8), into which we introduce a nonlinear element. The latter is of no principal importance but is convenient from structural considerations, and also simplifies the calculation. Without assuming a small change after one pass through the cavity, and introducing the dimensionless flux (4), we write a difference equation for  $p_k(\tau)$ , where k is the number of the pass and

$$0 \leq \tau \leq T$$
, (56)

where T is the time of passage of the light pulse through the cavity

$$\mathbf{T} = \frac{\mathbf{L}}{\mathbf{c}}$$
(57)

here L is the cavity length.

The difference equation for  $p_k(t)$  is

$$\mathbf{p}_{\mathbf{k+1}}(\tau) = \mathbf{R} \left( \mathcal{H}_{\mathbf{k}}(\mathbf{p}_{\mathbf{k}}(\tau)) \cdot \mathbf{p}_{\mathbf{k}}(\tau) \right) \mathcal{H}_{\mathbf{k}}(\mathbf{p}_{\mathbf{k}}(\tau)) \mathbf{p}_{\mathbf{k}}(\tau), \tag{58}$$

where R(p) is a real or equivalent piecewise-linear transmission of element 5 or 55.  $\mathcal{H}_{k}(p_{k}(\tau))$  is the gain in the k-th pass

$$\mathcal{H}_{\mathbf{k}}(\mathbf{p}_{\mathbf{k}}(\tau)) = \mathbf{e}^{\sigma \mathbf{N}_{\mathbf{k}}(\tau)^{\prime}} \delta$$
(59)

here  $\sigma$  is the cross section for stimulated emission of the active rod, N<sub>k</sub> is the inverted population on the k-th pass in the active cavity rod (assumed to be uniform over the sample), *l* is the length of the active rod,  $\gamma$  accounts for the mirror loss, for the distributed losses, etc., with  $0 \le \gamma \le 1$ .

.. . . .

The equations for the inversion of an active (3-level) medium will be

$$\frac{\partial N_{\mathbf{k}}(\tau)}{\partial \tau} = -2 \frac{J_{\mathrm{s}}}{\hbar \omega} \frac{e^{\sigma N_{\mathbf{k}}(\tau) l} - 1}{l} p_{\mathbf{k}}(\tau) - \frac{N_{\mathbf{k}}(t)}{T_{1}} + \frac{N_{0}}{T_{1}}$$
(60)

here  $T_1$  is the time of spontaneous decay of the inversion, and  $N_0/T_1$  describes the pump.

If we neglect the spontaneous decay of the inversion and the pump, and are interested precisely in those lasing stages in which these terms are not significant ( $T_1$  is long and is of the order of  $10^{-3}$  sec), then we get from (59) and (60) for the gain in the k-th pass at the instant  $\tau$ :

$$\mathcal{H}_{\mathbf{k}}(\tau) = \frac{\delta \mathcal{H}_{0}}{\mathcal{H}_{0} - (\mathcal{H}_{0} - \delta) \exp\left(-\chi(\int_{0}^{\tau} p_{\mathbf{k}}(\tau) d\mathbf{i} + W_{\mathbf{k}})\right)} , \qquad (61)$$

here  $\mathcal{H}_0$  is the initial gain, and the dimensionless quantity

$$\chi = 2\sigma \frac{J_s}{\hbar \omega} \Delta t, \qquad (62)$$

where  $\Delta t$  is the time-measurement unit;  $W_k$  is the density of the energy passing in the k-th pass through the active element. Note that when the equivalent transmission (5) or (55) is used, Eq. (58) depends only on  $\gamma_0(\tilde{\gamma}_0)$  of the element, and the dependence on J<sub>s</sub> is contained in  $\chi$  [Eq.(62)].

The difference equations (58) and (61) are easily solved with a computer given the initial intensity profile  $p_0(\tau)$ . It follows from (57) directly that the condition of "soft" self excitation of the lasing is:

$$\mathcal{H}\gamma_{0}-1 \geq 0, \tag{63}$$

or, with account taken of (58),

$$e^{\sigma N_0 l} \delta \gamma_0 - 1 > 0. \tag{64}$$

A situation can occur, however, when the conditions (63) and (64) are not met, but a "hard" start is possible under the condition

$$p > p_1 = \frac{1}{\mathcal{H}_0} \frac{(1 - \mathcal{H}_0 \gamma_0)}{(1 - \gamma_0)} ,$$
 (65)

i.e., an individual spike satisfying (65) bleaches the filter and is subsequently amplified.

Let us find now the conditions for mode locking when the laser contains a bleaching noninertial element. Assume that the initial noise pattern, with average intensity  $\overline{p}$  contains a spike with maximum intensity  $p_{max}$  and duration  $r_{b}$  such that

$$p_{\max} \tau_b < \overline{p} T, p_{\max} = r \overline{p},$$
 (66)

where r > 1.

Without loss of generality, it can be assumed that the maximum spike occurs at  $\tau = T$ . If the self-excitation condition (63) is met, the condition for further selection of the maximum spike, i.e., the condition for mode locking, will be

$$\frac{\partial}{\partial p} \left( \left| \mathcal{K}_{i}(T) R(\mathcal{K}_{i}(T) p) \right| \right|_{p = p_{max}} > 0.$$
(67)

whence, taking into account (66) and the fact that

$$\int_{0}^{T} p_{\mathbf{k}}(\tau) d\tau = \overline{p} T = \frac{1}{r} p_{\max} T, \qquad (68)$$

we obtain the mode-locking condition

$$\frac{\mathcal{K}(0) (1-\gamma_0) r h \omega}{(\frac{\mathcal{H}(0)}{\delta} - 1) T 2 \sigma J_{s}} > \gamma_0, \tag{69}$$

which corresponds at  $\sigma N \ll 1$  to the so-called "second threshold" [2]. The main difference in our case is the appearance in the right-hand side of (69) of the coefficient  $\gamma_0$  in place of unity.

In the case  $\gamma < 0$  a transmitting "obscuring" noninertial element is capable of effectively stabilizing the emission of the laser. Thus, at  $\tilde{\gamma}_0 = \tilde{\gamma}_1$ , when the self-excitation condition (63) is met, there exists a stationary intensity

$$p_{st} = 1 - \frac{1}{\mathcal{H}_0 \gamma_0} > 0, \qquad (70)$$

whose stability can be shown to be of the stable saddle type at  $1 < \mathcal{H}_0\gamma_0 < 2$  and of the stable focus type at  $2 < \mathcal{H}_0\gamma_0 < 3$ . (We assume that the fields are weak and the saturation of the active element is insubstantial, i.e.,  $\mathcal{H} = \mathcal{H}_0$ ). We proceed now to numerical calculations.

# 3. NUMERICAL SIMULATION OF LASING REGIME OF A LASER WITH NONINERTIAL NONLINEAR ELEMENT

We consider first the case of mode locking with the aid of noninertial nonlinear element with equivalent transmission (5) in accordance with Eqs. (58) and (61), and with transmission parameters  $\gamma_0 = 0.12$  and  $J_s = 0.97$  GW/cm<sup>2</sup>. Such parameters can be possessed, for example, by an element with SIPPR or by an element with a distributed nonlinear medium (see Table 1). Choosing the time unit  $\Delta t$  to be 1 psec, we obtain from (62)  $\chi = 1.7 \cdot 10^{-4}$  and the other parameters (in the CGSE) will be  $\delta =$ 0.18,  $H_0 = 10$ . The latter is realized at  $\sigma = 2.5 \cdot 10^{-20}$ ,  $N_0 = 1.6 \cdot 10^{19}$ , l = 10,  $\hbar\omega = 2.8 \cdot 10^{-12}$ ,  $T = 3 \cdot 10^3$  psec. Let the initial intensity (K = 0) consist of noise with average intensity  $\overline{p}_0 = 0.01$  and with maximum intensity  $p_0 = 0.1$  (see Fig. 9). In this case  $r = p_0/\overline{p_0} = 10$ . The conditions for self-excitation (64) and mode locking (65) are met. Figure 9 shows, for the 3rd and 7th passes respectively, the decrease of the noise level and the selection of the spike when its duration is decreased from 10 to 4.3 psec, which increases in this case to 2.106. In the seventh pass, the active medium saturates (*H* decreases to unity), the maximum is shifted to the left on account of the predominant amplification of the leading front under the saturation conditions. We consider now the mode locking under the conditions of a "rigid" triggering, when the condition (64) is not met,  $\gamma_0 = 0.03$ , the remaining parameters are the same as in the preceding case, and the mode-locking conditions (69) are met. In this case the intensities satisfying the condition of "hard" triggering (65) will become stronger, and activities that do not satisfy (65) will be attenuated. Figure 10 shows the initial intensity profile with maximum  $p_0 = 0.12$  and left-peak duration  $\tau = 10$  psec. A numerical computation of (58) and (61) shows a steepening and enhancement of the left-hand peak and a suppression of the right-hand one (see Fig. 10). In the 8th pass the active medium becomes saturated, the duration of the remaining pulse is shortened to  $\tau = 2.5$  psec, and its maximum shifts to the left just as in the preceding case. In our opinion, the considered case of "hard" triggering can find practical applications for generation of particularly short and high-power pulses. A laser with a nonlinear element with very small  $\gamma_0$  stores the pump energy, the gain reaches a high value without self-excitation. after which a sufficiently strong pulse passes from the outside into the laser from another laser and everything takes place as described above: the wings of the pulse are cut off, only the most intense part is amplified, and the duration is decreased.

Among the considered nonlinear elements, a special position is occupied by a filter using SIRPP, owing to the periodicity of the transmission and to the dependence (35) on the intensity: the filter will be alternately bleached or darkened. In particular, at intensities  $J \approx \varphi_0/\beta l$  the first transmission intensity sets in, and further growth of the intensity will "darken" the filter. If the gain is not large and saturates when  $J \sim \varphi_0/\beta l$  is reached, pulse intensity limitation is possible, i.e., square pulses can be generated. In fact, at  $\beta = 1$  rad  $\cdot$  cm/GW, l = 1 cm,  $\varphi_0 = \pi/3$ ,  $\chi = 5.4 \cdot 10^{-4}$ ,  $\mathcal{I}_0 = 1.2$ ,  $\delta = 0.18$ . Substituting in (58) the transmission

$$\mathbf{R}(\mathbf{J}) = \cos^2(\varphi_0 - \beta I \mathbf{J}_s \mathbf{p})$$

where  $J_s$  is defined by (38). Choosing the initial profile (Fig. 11), we obtain after several passes a square pulse. Similarly, a square pulse can be obtained in the case of a nonlinear element based on refraction under TIR conditions (29), since this filter will initially become bleached with increase of the intensity, and then darken (see Fig. 4). If the gain  $\mathcal{H}$  in a laser with a filter based on SIRPP is large and is not saturated at  $J \sim \varphi_0/\beta I$  the intensity will increase further, we then get  $\beta I J >> \pi/2$ , and at certain intensities the filter becomes completely "darkened" and as a result, the braking up of the pulse into irregular pulses takes place. Indeed, at the parameter values  $\mathcal{H} = 10$ ,  $\chi = 1.7 \cdot 10^{-4}$  (the remaining are the same as in the preceding case) we obtain such a breakup (see Fig. 12). Note that all the nonlinear "darkening" elements having a transmission of type (55), should, at sufficiently high intensities p > 1, cause sufficient "chipping" of the pulse, i.e., "eat away" the most important part. We shall show now the possibility of noninertial stabilization of lasing with the aid of a "darkening" element of type (55). Let the radiation intensity be low, and let  $\chi$  be small enough, so that the gain does not saturates and  $\mathcal{H}_0$  is constant. Let  $\tilde{\gamma}_1 = \tilde{\gamma}_0$  and  $\mathcal{H}_0\gamma_0 = 1.9$ , so that the selfexcitation condition is satisfied and the stationary value of lasing from (70) will be  $p_{st} = 0.474$  and the initial emission (see Fig. 13) with sinusoidal "ripples"  $p_0 = 0.2 + 0.4\sin (2\pi t/T)$ , where T = 12 psec, becomes stabilized after several passes (see Fig. 12).

### 4. INSTABILITY IN LASERS WITH NONINERTIAL NONLINEAR ELEMENTS

The nonlinear optical elements considered above use, besides an element with SIRPP, also the dependence (6) of the refractive index on the light intensity. It is desirable in this case that the nonlinearity coefficient  $\kappa$  not be small. At the same time, an increase of  $\kappa$  can contribute to the onset of instabilities of the self-focusing and mode-locking type.

![](_page_16_Figure_0.jpeg)

Fig. 13. Stabilization of "darkening" noninertial element  $\mathcal{K}_{0\gamma_{0}} = 1.9$ , k) number of pass.

Consider first self-focusing, which requires for its development a definite length [4]:

$$l_{\rm sf} = \frac{a}{2} \left(\frac{n_0}{\frac{8\pi}{\kappa} J}\right)^{1/2}$$
(71)

where a is the initial beam diameter. The contribution of a nonlinear element of length  $l_f$  and with nonlinearity  $\kappa$  to self-focusing is insignificant compared with the contribution of an active medium of length l and with weak nonlinearity  $\kappa_1$ , if

$$l_{\rm f} < l \left(\frac{\kappa_1}{\kappa}\right)^{1/2} \tag{72}$$

Example. Let  $\kappa \sim 2 \cdot 10^{-11}$  CGSE and  $\kappa_1 \sim 2 \cdot 10^{-13}$  CGSE (nonlinearity of electronic origin in glass [6]). We have then from (72)  $l_f < 0.1l$  and if l = 10 cm then  $l_f < 1$  cm.

We consider now mode self-locking. A measure of mode locking is the broadening of the spectrum [4]:

$$\delta\omega = \frac{8\pi}{c} \frac{\omega \kappa I J}{\tau_{\rm p}} , \qquad (73)$$

Here *l* is the length of the nonlinear medium, and  $\tau_{\rm p}$  is the pulse time.

It can be seen from (73) that mode locking of a nonlinear element with length  $l_f$  and nonlinearity  $\kappa$  does not exceed the mode locking of an active medium of length l and nonlinearity  $\kappa_1$  if

$$l_{f} < l\left(\frac{\kappa_{1}}{\kappa}\right) \tag{74}$$

Example. At  $\kappa_1 = 2 \cdot 10^{-13}$  CGSE (glass in [11]),  $\kappa = 2 \cdot 10^{-11}$  CGSE [from (73)],  $l_f < 0.01l$ , l = 10 cm,  $l_f < 0.1$  cm. This imposes a more stringent condition on the length of the nonlinear element. Note that when light passes through saturable dye filters, the refractive index changes by virtue of the Kramers–Kronig relation, and  $\delta n_2$  and  $\kappa$  can in this case larger by 1-2 orders than the corresponding  $\delta n_2$  and  $\kappa$  used by us in the calculations. Accordingly, the difficulties with generation of ultrashort pulses, connected with the mode locking are encountered to and equal for in the nonlinear elements considered by us and in already operating saturable filters, in which, as is well known, locking is successfully realized. We note also that in nonlinear elements, owing to the nonlinearity of the nonuniform transmission across a beam, the cross section will decrease and accordingly the diffraction losses will increase. This should be equally manifested in the already operating saturable dye filters. It can thus be stated [(see (72) and (74)] that the aforementioned instabilities need not necessarily be due to the nonlinear element, and if they do occur, they do not differ from those in a saturable dye filter.

# 5. DISCUSSION OF RESULTS

We have considered a number of nonlinear noninertial elements that can be conveniently compared, with respect to piecewise-linear transmission (5), the latter characterized by an initial transmission  $\gamma_0$  and a saturation intensity and  $J_s$  (see Table 1). Each of them has its own shortcomings and advantages. Any initial transmission can exist in the considered elements. For example, in an SIRPP element the initial transmission is determined by the angle  $\varphi_0$  between the polarizers [see (35)]. The most promising are elements having the smallest  $J_s$ , since this means effective operation of the element in relatively weak fields. From this point of view the most suitable element is one based on SIRPP or SRPE, a filter under TIR conditions, and also based on a distributed mirror. They can be used for mode locking and to obtain, since they have no inertia, ultrashort radiation pulses (see Figs. 9 and 10). To be sure, an element with SIRPP or SRPE may not be suitable for mode locking at high intensities (see Fig. 12), owing to the breakup of the pulses.

We have shown that such elements can form square pulses of light (Fig. 11), stabilize (see Fig. 13) and break up light pulses (Fig. 12), etc., i.e., have exert the same action as nonlinear elements in radio engineering [13]. Investigations of saturable dye filters [12] have shown that the nonlinear change of the refractive index in such filters is quite large ( $\kappa \sim 10^{-9}$  CGSE), and our estimated ones  $\kappa \sim 10^{-11}$  CGSE are smaller.

Thus, the difficulty with mode locking is inherent in dye filters to an even grater degree than in the nonlinear elements considered here. To decrease the mode locking it is necessary, in accord with (73) and (74), to choose elements with short lengths  $l_{\rm f}$ . The best from the standpoint would be elements based on reflection from a nonlinear medium  $(l_{\rm f} \sim \lambda)$ , but they are not very effective, since they have a large  $J_{\rm s}$  (see Table 1). In the case when the inertia in not significant, the foregoing elements can be made also from materials with inertial nonlinearity, considerably expanding the range of materials suitable for the production of such elements.

Successful experiments with filters based on SREP [14-17] show that the development of many of the considered noninertial elements and their use in laser technology is quite possible.

In conclusion, I wish to take this opportunity to thank M. V. Fok, Doctor of Physico-Mathematical Sciences, for discussions and very useful advice.

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