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Failure of fibre-reinforced composites by pull-out fracture

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A simple model is proposed to predict the ultimate tensile strength of fibre-reinforced composites when the failure is governed by fibre debonding. The theoretical analysis is based on the concept of fracture mechanics where the debonded zone is considered as an interfacial crack. The analysis is first applied to the classical pull-out test in order to determine the specific work of interfacial cracking. Using this value, the uniaxial tensile strength of the composites can be predicted from an approximate, closed-form equation proposed here. The theoretically predicted results seem to compare favourably with experimental values for fibre-reinforced cement based composite.

1. Introduction

In many fibre-reinforced composites, the failure (defined as the maximum load-carrying capacity) is dominated by the debonding of fibres which occurs after transverse fracture in the matrix. Failure of cord-rubber composites by pull-out fracture has been discussed by Gent *et al.* [1]. For failure of Portland cement-based composites which is the subject of this paper, the fibre–matrix interface is comparatively weak, and as a result fibre debonding plays a critical role.

Experimentally observed average tensile stress–strain responses of Portland cement composites reinforced with randomly distributed, short, steel fibres [2, 3], glass fibres [3] and polyproplene fibres [4], are shown in Fig. 1. The response of these composites subjected to monotonically increasing uniaxial tensile loading is linear until the stress aproximately equals the tensile strength of the unreinforced matrix. The linear behaviour is followed by: (i) non-linear behaviour, (ii) the peak response (termed failure), and (iii) the post-peak response, as seen in Fig. 1. One or more transverse cracks are often observed before the peak of the stresss–strain curve [2, 4]. The post-peak response is generally dominated by widening of a single major crack [2]. As a result of these localized deformations, the strains are non-homogeneously distributed and the definition of the stress–strain curve in the post-peak region is no longer unique [5, 6].

Based on a larger number of experimental results, empirical relations have been proposed to predict the failure stress of fibre-reinforced cement-based composites (FRC). Such equations relate the failure stress to the volume fraction of fibres, aspect ratio of fibres and the bond strength of the fibre–matrix interface [7–9]. Such empirical observations indicate the importance of debonding and pull-out resistance of fibres in determining the failure of stress of the composites.

2. Some previous theoretical models

Theoretical models relating the properties of fibre, matrix and their interface to the failure of the FRC composite have been suggested. For example, a model to predict the occurrence of multiple transverse cracking has been proposed by Aveston *et al.* [10]. Their model assumes a constant shear stress distribution at the fibre–matrix interface. A strain relief model developed by Irwin for unreinforced matrix has been proposed for FRC composites [11, 12]. It is assumed that the strain energy released by formation of a transverse Griffith crack is contributed only by an elliptical region surrounding the crack. For a crack of a given length, energy contributed by this elliptical zone is compared with the energy required to form new cracked surfaces. The elastic strain energy in the matrix and the fibres, and the energy absorbed in debonding (assuming a constant shear stress at the interface and friction type of bond), are included in the analysis with some simplfying assumptions regarding the strain distribution away from the crack.

It is generally difficult to determine, theoretically as well as experimentally, the shear stress (bond stress) distribution at the interface and the strain field in the matrix surrounding the fibre during the debonding process [13–22]. However, some approximate theoretical solutions which provide a basic understanding of the problem have been obtained. For example, Sternberg and Muki [13–15] have proposed an analytical solution of the classical pull-out problem (a single fibre being pulled out from a semi-infinite matrix). They equate the presence of the fibre with a distribution of disc loads in the matrix, and they formulate and solve the corresponding integral equation. Phan Thien and co-workers [16–19] equate the presence of a slender fibre with a distribution of Mindlin forces in the matrix using slender-body theory.

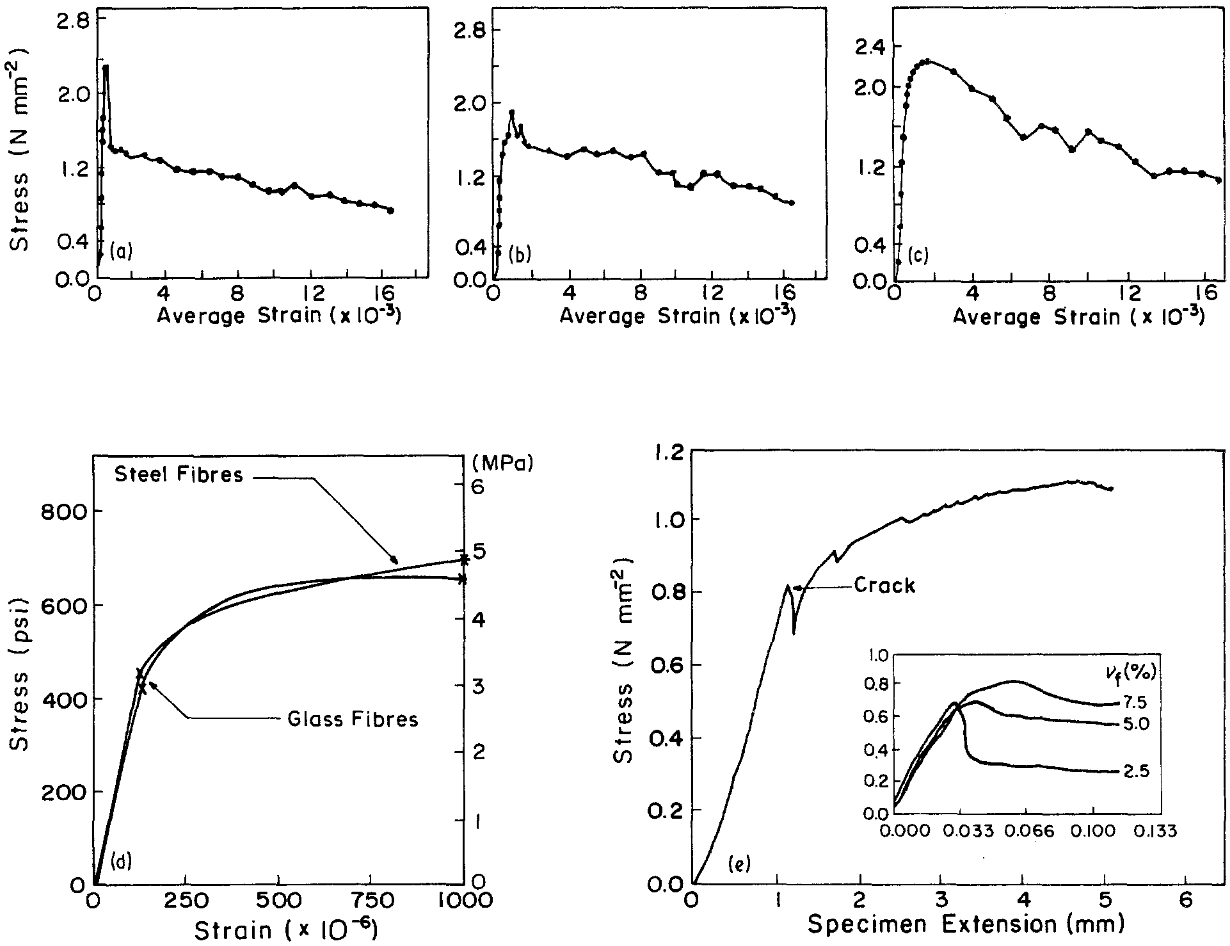

Figure 1 Experimentally observed tensile behaviour of fibre-reinforced concrete composites made using (a, b, c) steel fibres of different types (from Shah, Stroeven, Dalhuisen and van Stekelenburg [2]); (d) smooth steel and glass fibres (from Shah and Naaman [3]); (e) monofilament polypropylene fibres with inset showing response of fibrillated polyproplene fibres (from Baggot [4]). (a) Smooth, straight steel fibres with $V_f = 1.23\%$, $l = 25$ mm, $2a = 0.38$ mm. (b) Hooked ends, $V_f = 0.90\%$, $l = 30$ mm, $2a = 0.40$ mm. (c) Enlarged ends, $V_f = 1.40\%$, $l = 50$ mm, $2a = 0.75$ mm (d) Smooth, straight steel fibres, $V_f = 2\%$, $l = 25$ mm, $2a = 0.43$ mm; glass-fibres (bundles) $V_f = 2\%$, $l = 13$ mm; (×) first crack, (*) maximum load. (e) Polypropylene monofilaments, $V_f = 5\%$, $l = 26$ mm, $d = 130\,\mu$m; inset, fibrillated fibres, $l = 6$ mm, average thickness $72\,\mu$m.

To evaluate the results of single-fibre pull-out tests and to determine the failure load on composites an approach based on fracture mechanics is proposed here. No assumption regarding the distribution of shear stresses at the interface is explicitly made. In the analytical model, the zone of debonding is treated as an interfacial crack. Failure modes such as fibre yielding, fibre fracture and further transverse matrix cracking are disregarded. It is assumed that a transverse crack has already been formed and that the debonding starts at the transversely crack matrix surfaces.

3. Fracture mechanics approach

Consider a linear elastic body with a crack (Fig. 2). The surface of the crack Ω is determined by a single parameter, δ, such that

$$\Omega = \Omega(\delta) \qquad (1)$$

The displacements are described on a part of the boundary of the body and the body is loaded with a single force P. The displacement of the boundary at the force P in the same direction as the force is called u and is governed by

$$u = CP \qquad (2)$$

where C is the compliance of the body. The compliance is, among other parameters, a function of the crack size:

$$C = C(\delta, \ldots) \qquad (3)$$

It can be shown that the total energy released (work done by the force minus the change in strain energy of the body) when the crack grows an infinitesimal amount $\partial\delta$ is given by

$$\frac{1}{2}\frac{\partial C}{\partial \delta} P^2 \, d\delta = \text{energy released} \qquad (4)$$

A simple Griffith-type criterion for the crack-growth load, P_{cr}, can then be written as

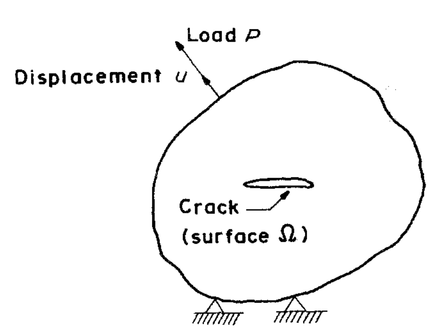

Figure 2 Definition of some parameters used in the problem.

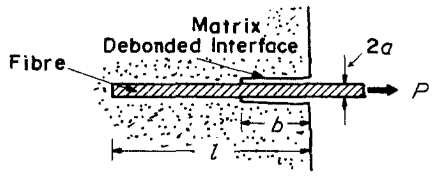

Figure 3 The classical fibre pull-out problem.

$$\frac{1}{2}\frac{\partial C}{\partial \delta}(P_{cr})^2 = \frac{d\Omega}{d\delta}\gamma_i \qquad (5)$$

where γ_i is the specific work of fracture. This criterion will be used to determine γ_i from the single-fibre pull-out test and to determine the failure load of FRC composites.

4. Single-fibre pull-out test

The classical fibre pull-out test is shown in Fig. 3. The debonding zone b is considered stress-free, at least compared to the bonded part of the interface. This means that the debonded zone can be characterized as an interfacial crack with a crack length b ($b < l$) and a total crack surface of $2(2\pi ab)$. The fibre is assumed to be circular with diameter $2a$ and length l and linearly elastic with Young's modulus E_f and Poisson's ratio ν_f. The matrix is also considered linearly elastic with the corresponding properties E_m and ν_m.

The maximum pull-out will be reached when the interfacial crack growth criterion (Equation 5) is satisfied. That is, for the pull-out problem, using b as a crack length, Equation 5 becomes

$$\frac{1}{2}\frac{\delta C}{\delta b}(P_{cr})^2 = 2\pi a 2\gamma_i \qquad (6)$$

where γ_i is the specific interfacial work of fracture.

To the authors' knowledge no exact solution is available for $C = C(b, \ldots)$. However, some useful information about such solutions can be obtained from the analysis of the perfectly bonded fibre case ($b = 0$) performed by Phan-Thien and co-workers [16–19]. They have developed approximate solutions using an asymptotic analysis under the assumptions

$$\frac{E_f}{E_m}\left(\frac{a}{l}\right)^2 \ln\left(\frac{2l}{a}\right) \gg 1 \quad \text{and} \quad l \gg a \qquad (7)$$

and

$$\frac{E_f}{E_m}\left(\frac{a}{l}\right)^2 \ln\left(\frac{2l}{a}\right) \ll 1 \quad \text{and} \quad l \gg a \qquad (8)$$

corresponding respectively to either very stiff (rigid) or elastic (long) fibres. For these two cases they determined the following relationship between P and the displacement u of the fibre at the free edge of the matrix (considered as an elastic half-space): for rigid fibres

$$u = \frac{1+\nu_m}{E_m}\ln\left(\frac{2l}{a}\right)\frac{1}{\pi l}P \qquad (9)$$

while for elastic fibres

$$u = \frac{1-\nu_m^2}{E_m}\frac{1}{\pi a}P \qquad (10)$$

In addition, Phan-Thien [16] has also determined the displacement in the fibre direction of a rigid fibre of length l embedded in an infinite matrix and loaded in the fibre direction. This solution reads

$$u = \frac{1+\nu_m}{E_m}\ln\left(\frac{l}{a}\right)\frac{1}{\pi l}P \qquad (11)$$

4.1. Elastic fibre case

If the fibre is elastic (Equation 8 satisfied) then Phan-Thien has shown that the compliance of the fibre–matrix system is independent of the embedded length (see Equation 10). This must also be the case when $b \neq 0$ as long as $(l - b)/a \gg 1$. If it is assumed that the interfacial crack has grown some distance large enough for the influence of the free edge of the matrix to be ignored, then the displacement u of the debonded fibre can be expressed as

$$u = G(E_m, \nu_m, a)P + \frac{b}{E_f \pi a^2}P \qquad (12)$$

where G is independent of b and l. Substituting the value of compliance obtained from Equation 12 in Equation 5 we can obtain the interfacial crack growth criteria as

$$\frac{1}{2}\frac{(P_{cr})^2}{E_f \pi a^2} = 2\pi a 2\gamma_i \qquad (13)$$

Note that P_{cr} is independent of b, which means that a long elastic fibre will eventually be pulled out with a constant load.

4.2. Rigid fibre case

If the fibre can be considered rigid (Equation 7 satisfied) and if it can be assumed that b/a is large enough and $(l - b)/a \gg 1$, then the compliance of the debonded fibre–matrix system can be expressed as

$$u = \frac{1+\nu_m}{E_m}\ln\left(\frac{l-b}{a}\right)\frac{1}{\pi(l-b)}P \qquad (14)$$

Note that Equation 11 for a rigid fibre embedded in an infinite matrix was used to derive Equation 14, since the fibre is assumed to be rigid and the free-edge effect is assumed negligible. It is now possible to derive the expression for the critical pull-out load as

$$\frac{1}{2}\frac{1+\nu_m}{\pi E_m}\left[\ln\left(\frac{l-b}{a}\right) - 1\right]\frac{1}{(l-b)^2}(P_{cr})^2 = 2\pi a 2\gamma_i \qquad (15)$$

Equation 15 predicts that the crack growth for the rigid fibre case is unstable, since P_{cr} is a decreasing function of b. In other words, using a stiffer fibre with respect to the matrix or a shorter embedment length, one approaches the rigid case and consequently an unstable debonding process.

Equations 13 and 15 offer a different approach to evaluate the standard pull-out test. Usually the strength of the interface is characterized by a criterion in terms of shear stress. Since the precise distribution of shear stress along the interface is difficult to determine, an average value is often determined from the assumed shear distribution and the maximum pull-out load. Use of Equations 13 and 15 enables the characterizing of bond strength in terms of specific work of fracture.

Note that the use of Equations 13 and 15 to interpret the pull-out test requires specimens with an initial interfacial crack of a well-defined length. This initial crack must be long enough to ensure that surface effects can be ignored.

5. Uniaxial tensile specimen

To predict the tensile strength of fibre-reinforced cement-based composites whose ultimate strength is dictated by the strength of the relatively weak fibre–matrix interface, the analyses described for the single-fibre pull-out case have been applied. The specimen considered here is shown in Fig. 4. It is assumed that (a) one macroscopic, transverse matrix crack is formed; (b) interfacial cracks have initiated; and (c) the fibres are continuous (that is, sufficiently long in the sense of Equations 7 and 8) and aligned in the direction of the applied tensile stress. First, the case of elastic fibres (Equation 8) is considered.

The length of the specimen is $2l$ and the length of the uncracked (bonded) region is $2(l - b)$. Let the cross-section of the specimen be A and the volume fraction of fibres V_f. The compliance of the specimen shown in Fig. 4 can be expressed as

$$C = 2\left[\frac{b}{V_f E_f A} + (l - b)\frac{1}{E_c A} + H\right] \qquad (16)$$

where the first term describes the compliance of the debonded zone and the second term describes the compliance of the bonded region, with E_c defined as the Young's modulus of the composite material; by Rule of Mixtures $E_c = E_f V_f + (1 - V_f)E_m$. The third term H describes the compliance of the zone where the interfacial cracks end, and for simplicity it is assumed to be independent of b. The total interfacial crack surface Ω can be written as

$$\Omega = 4bAV_f\frac{2}{a} \qquad (17)$$

Using once more the criterion proposed in Equation 5, the following relationship for the interfacial crack growth and the tensile stress (σ_{cr}) is obtained:

$$\frac{1}{2}2A\left(\frac{1}{V_f E_f} - \frac{1}{E_c}\right)\sigma_{cr}^2 = 4AV_f\frac{2}{a}\gamma_i$$

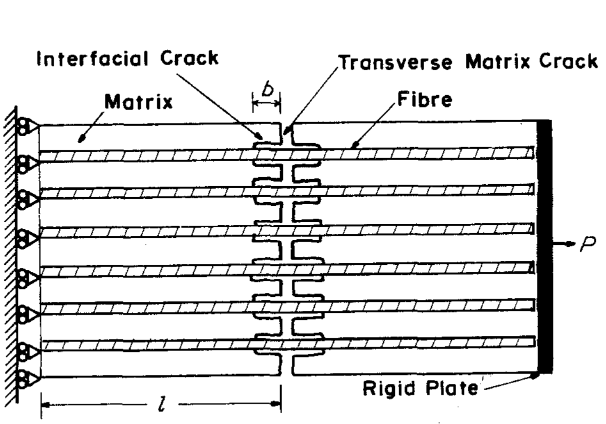

Figure 4 Idealized tensile specimen considered for the composite model.

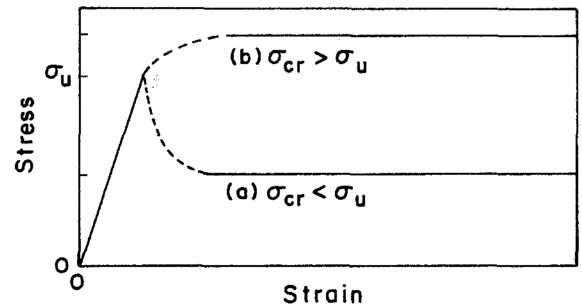

Figure 5 Possible model predicted solutions for composite tensile stress–strain behaviour.

or

$$\sigma_{cr} = 2V_f\left(\frac{E_c E_f}{E_c - V_f E_f}\,\frac{2\gamma_i}{a}\right)^{\frac{1}{2}} \qquad (18)$$

Note that if σ_{cr} is less than the tensile strength σ_u of the unreinforced matrix (which is approximately the same as the limit of proportionality for the commonly employed FRC composites), then the stress–strain curve labelled (a) in Fig. 5 is observed for the composites. On the other hand, if σ_{cr} calculated from Equation 18 is greater than σ_u then the stress–strain curve labelled (b) in Fig. 5 is predicted from the proposed analysis. Both of these types of curves have been reported for FRC composites (Fig. 1).

In the case of rigid fibres, the compliance of the specimen shown in Fig. 4 and the tensile stress at the critical growth of interfacial cracks can be written, respectively, as

$$C = 2\frac{1 + \nu_m}{\pi E_m}\ln\left(\frac{l - b}{a}\right)\frac{1}{(l - b)}\frac{\pi a^2}{V_f A} \qquad (19)$$

and

$$\sigma_{cr} = 2V_f\frac{l - b}{a}$$
$$\times\left\{\frac{E_m}{1 + \nu_m}\left[\ln\left(\frac{l - b}{a}\right) - 1\right]^{-1}\frac{2\gamma_i}{a}\right\}^{\frac{1}{2}} \qquad (20)$$

Comparing Equation 18 with Equation 20 it can be seen that for stiffer or shorter fibres the stiffness of the matrix and the length of the fibres become important in determining σ_{cr} and that the crack propagation tends to be unstable (maximum value of σ_{cr} obtained when $b = 0$).

6. Some comparisons with experimental results

To examine the applicability of the analysis proposed here, the prediction of the tensile strength of the composite was compared with the observed data for steel-fibre reinforced concrete.

The length and diameter of the most commonly used steel fibres are approximately 25 and 0.25 mm, respectively. Can these fibres be considered sufficiently long for the proposed analysis? The results of Sternberg and Muki [13–15] for pull-out of elastic fibres from elastic matrix with $E_f/E_m \leqslant 8$ show that when the ratio of embedment length to radius is about 20, then the fibre cannot "feel its own end". That is, making the fibre longer would make no difference in the compliance calculations. For the stiff fibre case much longer fibres are usually needed. However, Phan-Thien *et al.* [19] have shown that for $v_m \sim 0.1$, l/a values of about 50 are sufficient to ensure that Equation 7 is reasonably good. Thus, it can be concluded that one of the assumptions of the analysis (that fibres are long compared to their radius) is acceptable for steel fibres with an aspect ratio ($l/2a$) greater than about 25.

For steel fibres the ratio E_f/E_m is approximately 10. For this ratio and for $l/2a = 30$ to 100, the quantity $E_f/E_m(a/l)^2 \ln(2l/a)$ is less than 10^{-2}. Thus, steel fibres should be treated as elastic fibres in the sense of Equation 8.

Naaman and Shah [23] have conducted pull-out tests on straight and smooth steel fibres. They observed that the average peak pull-out loads (P_{cr}) for fibres with embedment length about 13 mm and diameters 0.4, 0.25 and 0.15 mm were 42, 26.5 and 6.20 N. For these values of P_{cr}, using Equation 13 the values of $\gamma_i = 13.3$, 21.7 and 5.5 J m^{-2} are obtained. Using these values of γ_i and for $V_f = 0.01$, values of $\sigma_{cr} = 3.51$, 5.67, 3.68 MPa are obtained by using Equation 18. These values of tensile strength for cement-based matrix reinforced with aligned fibres appear reasonable ([2, 3, 9] and Fig. 1). Also, note that the value of $\gamma_i = 6$ to 22 J m^{-2} for the steel fibre–matrix interface seems reasonable when compared to the values of 20 and 60 J m^{-2} observed for unreinforced matrix in Mode I crack propagation [5, 24].

It should be noted, however, than no initial crack was intentionally introduced in the tests reported [23], which means that the use of Equation 13 is not strictly correct. Since the critical pull-out load is larger, when b is not large enough to ensure that surface effects can be ignored, the values of γ_i found above may be too large. In the light of these considerations it is interesting to investigate the results reported by Burakiewicz [25], who made pull-out tests with straight and smooth steel fibres with embedment length 30 mm and diameter 0.38 mm and an initial crack of 5 mm. Burakiewicz found peak pull-out loads of about 30 N, which gives $\gamma_i = 8.31$ J m^{-2} when using Equation 13 in agreement with the considerations mentioned above.

7. Conclusions

An approximate but simple model is proposed to predict tensile strength of fibre-reinforced cement-based composites whose failure is governed by the strength of the fibre–matrix interface. The proposed analysis is based on the concept of fracture mechanics; it treats debonding as an interfacial crack and yields a closed-form equation to predict the ultimate strength of the FRC composite. The fibre–matrix interface strength is characterized in terms of specific fracture energy, and can be determined from the value of the maximum pull-out load obtained from single-fibre pull-out tests using the approach suggested here. There are several simplifying assumptions made which should be critically examined using a numerical analysis. These include (i) whether in a pull-out test, the interfacial crack is sufficiently long so that the free-edge effect is negligible; (ii) whether the cracked interface is stress-free; and (iii) whether the term H in Equation 16 is independent of b.

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References

1. A. N. GENT, G. S. FIELDING-RUSSELL, D. J. LIVINGSTON and D. W. NICHOLSON, *J. Mater. Sci.* **16** (1981) 949.
2. S. P. SHAH, P. STROEVEN, D. DALHUISEN and P. VAN STEKELENBURG, in Proceedings of RILEM Symposium on Testing and Test Methods of Fibre Cement Composites, Sheffield, September 1978 (Construction Press, 1978) p. 399.
3. S. P. SHAH and A. E. NAAMAN, *J. Amer. Concrete Inst.* **73** (Jan. 1976) 50.
4. R. BAGGOT, *Int. J. Cement Comp. Lightweight Concrete* **5** (1983) 105.
5. V. S. GOPALARATNAM and S. P. SHAH, *J. Amer. Concrete Inst.* **82** (May 1985) 310.
6. A. HILLERBORG, *Int. J. Cement Compos.* **2** (4) (1980) 177.
7. R. N. SWAMY, P. S. MANGAT, and C. V. S. K. RAO, in Special Publication SP–44 (American Concrete Institute, 1974) p. 1.
8. C. D. JOHNSTON and R. A. COLEMAN, in Special Publication (American Concrete Institute, Detroit, Michigan, 1974) p. 177.
9. S. P. SHAH and B. V. RANGAN, *J. Amer. Concrete Inst.* **68** (Feb. 1971) 126.
10. J. AVESTON, G. A. COOPER, and A. KELLY, in Proceedings of NPL Conference (IP Science and Technology Press, 1970) p. 63.
11. J. G. MORLEY and I. R. McCOLL, *J. Phys. D; Appl. Phys.* **8** (1975) 15.
12. D. J. HANANT, D. C. HUGHES and A. KELLY, *Phil. Trans. R. Soc.* **A310** (1983) 175.
13. E. STERNBERG and R. MUKI, *Int. J. Solids Struct.* **5** (1969) 587.
14. *Idem, ibid.* **6** (1970) 69.
15. *Idem, Z. Angw. Math. Phys.* **21** (1970) 552.
16. N. PHAN-THIEN, *Fiber Sci Technol.* **12** (1979) 235.
17. *Idem, ibid.* **13** (1980) 179.
18. N. PHAN-THIEN and C. J. GOH, *Z. Angw. Math. Mech.* **61** (1981) 89.
19. N. PHAN-THIEN, G. PANTELIS and M. B. BUSH, *Z. Angw. Math. Phys.* **33** (March 1982) 251.
20. P. LAWRENCE, *J. Mater. Sci.* **7** (1972) 1.
21. V. LAWS, *Composites* **13** (2) (1982) 145.
22. R. J. GRAY, *Int. J. Adhesion Adhesions*, **3** (1983) 197.
23. A. E. NAAMAN and S. P. SHAH, *J. Struct. Div. Amer. Soc. Civil Engrs* **102** (1976) 1532.
24. Y. S. JENQ and S. P. SHAH, *Eng. Fract. Mech.* **21** (1985) 1055.
25. A. BURAKIEWICZ, in Proceedings of RILEM Symposium on Testing and Test Methods of Fibre Cement Composites, Sheffield, September 1978 (Construction Press, 1978) p. 355.

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