TOPOLOGICALLY MASSIVE GAUGE THEORIES IN SUPERSPACE

B. M. Zupnik and D. G. Pak UDC 530.145

This paper presents a study of the geometrical properties of superfield D = 3, N = 1, 2 Yang–Mills theories, and derives explicit expressions for the topological mass term. In the case of the D = 3, N = 1 theory, the subject is addressed in the language of differential forms, and a special mapping of the Chern–Simons class of differential forms into the corresponding class of integral forms is used to construct the topological mass term. A superfield description is given for the D = 3, N = 2 Yang–Mills theory that arises upon dimensional reduction of the corresponding four-dimensional theory with N = 1 supersymmetry. The expression for the topological mass term is derived in compact form in prepotential terms.

Topologically massive three-dimensional gauge theories have been examined in [1, 2]. The study of supersymmetric generalization of these theories is of interest because of their possible application in the theory of membranes and domain walls. Geometrical properties related to the existence of topological invariants have been investigated in [3] by methods of algebraic geometry for the case of N = 0 gauge theories. The direct transference of this treatment to the case of superfield theories is difficult. This is due in particular to the fact that differential forms on supermanifolds cannot be integrated, and the geometry of real superspaces is inadequate to the internal geometry of SYM^N_D supergauge theories, except in the cases D = 2, 3 and N = 1. Differential forms on superspaces have been examined in [4, 5]. Additional geometric objects – integral and pseudodifferential forms – arise in supermanifold theory [6-8]. To construct the topological mass term in SYM^1_3 we use a special mapping of the Chern–Simons class of differential forms into the corresponding class of integral forms. We also have derived a simple expression for the topological mass term in the SYM^2_3 theory.

The formation of a three-dimensional supersymmetric N = 1 Yang–Mills theory using differential forms on the superspace $Z^A = (x^a, \theta^\alpha)$ is analogous to the existing formulation of the corresponding D = 4, N = 1 theory [9]. In contrast to the four-dimensional case in the SYM^1_3 theory, the spinor connectedness $A_\alpha(Z)$ is the main independent superfield. Let us examine the forms of the connectedness and curvature (strength) with values in the Lie algebra of the gauge group

$$A = ie^a A_a(Z) + ie^\alpha A_\alpha(Z), \tag{1}$$

$$F \equiv \frac{1}{2} e^A e^B F_{BA} = dA + A^2, \tag{2}$$

where covariant differentials (the displacement 1-forms) are used to preserve explicit supersymmetry

$$e^a = dx^a + \frac{i}{2} d\theta^\mu \theta^\nu \gamma^a_{\mu\nu},$$
$$e^\alpha = d\theta^\alpha,$$
$$de^a = \frac{i}{2} e^\mu e^\nu \gamma^a_{\mu\nu}, \quad de^\alpha = 0. \tag{3}$$

The use of the agreement $F_{\alpha\beta} = 0$ makes it possible to express the vector connectedness A_a in terms of the spinor connectedness A_α

Tashkent State University. Translated from Izvestiya Vysshikh Uchebnykh Zavedenii, Fizika, No. 12, pp. 13-17, December, 1988. Original article submitted May 20, 1986.

 0038-5697/88/3112-0962$12.50

$$A_a = -\frac{i}{2}\gamma_a^{\alpha\beta}(D_\alpha A_\beta + D_\beta A_\alpha + i\{A_\alpha, A_\beta\}) \equiv -\frac{1}{2}\gamma_a^{\alpha\beta} A_{\alpha\beta}. \quad (4)$$

The only independent tensor superfield in the theory is the spinor superfield $W_\alpha(Z)$, which enters into the strength component $F_{\alpha,\beta\gamma}$. The action of the Yang–Mills theory has the following form:

$$S_0 = \frac{1}{g^2}\mathrm{Tr}\int d^3x d^2\theta\, W^\alpha W_\alpha. \quad (5)$$

Let us determine the 3-form ω_3^1, which corresponds to the superfield analog of the Chern–Simons class

$$\omega_3^1 = \mathrm{Tr}\left(AdA + \frac{2}{3}A^3\right). \quad (6)$$

It is not hard to verify that the group variation of ω_3^1 is the complete differential of the 2-form ω_2^2

$$\delta\omega_3^1 = -2d\,\mathrm{Tr}(F\Lambda) \equiv d\omega_2^2, \quad (7)$$

where $\Lambda = \Lambda^\kappa(Z)T^\kappa$ is a parameter of the gauge group.

We will use the differential form ω_3^1 as an auxiliary construct for building the topological mass term. We know that differential forms are not integrable on supermanifolds, and that integral forms can serve as the initial object for the construction of integral density [6-8]. We will examine a special formula for constructing integral forms by using differential forms in SYM_3^1, which may prove unsuitable for other superspaces. Let us introduce the covariant quantities $\bar{d}x_a$ and $\bar{e}_\alpha$, which form an algebra analogous to the algebra of the covariant derivatives ∂_a and D_α. We will define the covariant integral forms in N = 1, D = 3 superspace as elements of an algebra with anticommuting generatrices e^a and $\bar{e}_\alpha$

$$\Sigma = \rho(Z)\, e^a e^b e^c e_\alpha \bar{e}_\beta \varepsilon_{abc}\varepsilon^{\alpha\beta} + \Sigma_1 \equiv \Sigma_0 + \Sigma_1, \quad (8)$$

where Σ_1 is a polynomial in e^a, and $\bar{e}_\alpha$ is of lower degree than Σ_0. Construct (8) is manifestly invariant with respect to transformations of planar supersymmetry; however, it is not hard to generalize this construct to the case of curvilinear coordinates by using the supertriad $E_M^A(Z)$ and the corresponding forms

$$e^A = dZ^M E_M^A(Z),\ \bar{e}_A = E_A^M(Z)\,\bar{d}Z_M. \quad (9)$$

By definition, the integral of the integral form contains only the Σ_0-term

$$\int\Sigma :\stackrel{\mathrm{def}}{=} \int\Sigma_0 = \int d^5Z\rho(Z). \quad (10)$$

Let us consider the mapping of the class of differential forms ω_3^1 into the class of integral forms Σ_3^1 that is specified by the following substitution of basic elements:

$$e^a \to \tilde{e}^a = e^a,\ e^\alpha \to \tilde{e}^\alpha = \gamma_a^{\alpha\beta} e^a \bar{e}_\beta. \quad (11)$$

The operation of external differentiation

$$\tilde{d} = \tilde{e}^a\partial_a + \tilde{e}^\alpha D_\alpha,\ \tilde{d}^2 = 0 \quad (12)$$

is defined naturally in the algebra of integral forms, and the structure equation of flat superspace is preserved

$$\tilde{d}\,\tilde{e}^a = \frac{i}{2}\tilde{e}^\mu\tilde{e}^\nu\gamma_{\mu\nu}^a,\ \tilde{d}\,\tilde{e}^\alpha = 0. \quad (13)$$

The topological mass term S_m is the integral of the form $\Sigma_3^1 = \tilde{\omega}_3^1$

$$S_m = \frac{m}{g^2}\int\Sigma_3^1 = \frac{m}{g^2}\int\Sigma_{03}^1. \quad (14)$$

Performing substitution (11) of the basic elements in form ω_3^1 (6) and using relation (4), we will not find it hard to reduce the action S_m to the form

$$S_m = \frac{m}{g^2} \int d^5 Z \, \mathrm{Tr} \, (A^{\alpha\beta} A_{\alpha\beta} + i A^\alpha \partial_{\alpha\beta} A^\beta). \tag{15}$$

The superfield equations of the SYM_3^1 massive gauge theory with the total action $S = S_0 + S_m$ have the form

$$\nabla^\gamma \nabla_\alpha W_\gamma - 3m W_\alpha = 0, \tag{16}$$

where ∇_α is the covariant derivative.

The D = 3, N = 2 supersymmetric Yang–Mills theory (SYM_3^2) arises naturally upon the dimensional reduction of the corresponding four-dimensional N = 1 theory. The reduced D = 3, N = 2 superspace contains three space–time coordinates x^m and two complex-conjugate spinor coordinates θ_α, $\bar\theta_\alpha$, which transform via the real representation of the SL(2, R) group. The group gauge transformation of the scalar chiral superfields Φ and the prepotential V into SYM_3^2 have the form

$$\Phi' = e^{i\Lambda} \Phi, \; e^{V'} = e^{i\bar\Lambda} e^V e^{-i\Lambda}. \tag{17}$$

The spinor covariant derivatives of SYM_3^2 in the chiral representation have the form

$$\bar\nabla_\alpha = \bar D_\alpha, \; \nabla_\alpha = D_\alpha + i A_\alpha = e^{-V} D_\alpha e^V. \tag{18}$$

In SYM_3^2 the main tensor superfield in terms of which all strength components are expressed is the real scalar superfield W

$$\{\nabla_\alpha, \bar\nabla_\beta\} = -i\gamma^m_{\alpha\beta} \nabla_m + \varepsilon_{\alpha\beta} W, \tag{19}$$

$$W = \frac{i}{2} \bar\nabla_\alpha A^\alpha = \frac{i}{2} \nabla^\alpha \bar A_\alpha. \tag{20}$$

The covariant operation of involution is related to the conventional conjugation operation by the equation

$$\bar A \equiv e^{-V} A^+ e^V. \tag{21}$$

The strength of W satisfies the Bianchi identity

$$\nabla^\alpha \nabla_\alpha W \equiv 0. \tag{22}$$

It is not difficult to represent the action of SYM_3^2 in the form of the integral over the complete superspace

$$S = \frac{1}{g^2} \mathrm{Tr} \int d^3 x d^4 \theta W^2. \tag{23}$$

To construct the invariant topological mass term, we require that its group variation have the form

$$\delta_\Lambda S_m = \frac{m}{g^2} \int d^7 Z \, \mathrm{Tr} \, (i \bar\Lambda \overset{+}{W} - i \Lambda W), \tag{24}$$

where

$$\overset{+}{W} = e^V W e^{-V}. \tag{25}$$

Let us note that $\delta_\Lambda S_m$ vanishes by virtue of the Bianchi identity. Let us transform equation (24) to a form that contains the covariant gauge variation $\nu(\delta_\Lambda V)$:

$$\delta_\Lambda S_m = \frac{m}{g^2} \int d^7 Z \, \mathrm{Tr} \, (\nu (\delta_\Lambda V) W), \tag{26}$$

$$\nu (\delta_\Lambda V) \equiv e^{-V} \delta_\Lambda e^V = e^{-V} i \bar\Lambda e^V - i\Lambda. \tag{27}$$

Let us assume that in the general case as well the variation of the action S_m has an analogous form, with the substitution of an arbitrary variation δV for $\delta_\Lambda V$. The variation δS determines the covariant variational derivative $\Delta S_m / \Delta V$:

$$\delta S_m = \frac{m}{g^2} \int dZ \, \mathrm{Tr} \, [\nu (\delta V) W] \overset{\mathrm{def}}{=} \int dZ \, \mathrm{Tr} \left[\nu (\delta V) \frac{\Delta S_m}{\Delta V} \right], \tag{28}$$

$$\frac{\Delta S_m}{\Delta V} = \frac{m}{g^2} W. \tag{29}$$

Let us examine the functional $S_m(t)$, which is obtained from the expression for S_m by the replacement $V \to tV$, such that $S_m(0) = 0$, $S_m(1) = S_m$. Let us select the particular case of variation of the prepotential in the form

$$\delta_t V = dtV = \nu(\delta_t V). \tag{30}$$

If we take into account the last relation, Eq. (28) becomes an ordinary differential equation with respect to $S_m(t)$ with the corresponding boundary conditions

$$d_t S_m(t) = \frac{m}{g^2} dt \int dZ \, \mathrm{Tr}\,[VW(tV)]. \tag{31}$$

Integrating this equation, we obtain the final expression for the topological mass term in the D = 3, N = 2 Yang–Mills theory

$$S_m = \frac{m}{g^2} \int_0^1 dz \, \mathrm{Tr}\,[VW(tV)]. \tag{32}$$

The lower terms in the expansion of S_m over powers of V have the form

$$S_m = \frac{m}{2g^2} \int dZ \, \mathrm{Tr}\left(VD^\alpha \bar{D}_\alpha V - \frac{1}{3} VD_\alpha V\bar{D}^\alpha V + \frac{1}{3} D_\alpha VV\bar{D}^\alpha V + \ldots\right). \tag{33}$$

Superfield methods of investigating topologically massive gauge theories can be used to study three-dimensional supergravities in superspace. In particular, the topological mass term of N = 1, D = 3 supergravity has a form analogous to relation (14) if the connectedness form with values in the tangent SL(2, R) group is used.

LITERATURE CITED

1. S. Deser, R. Jackiw, and S. Templeton, Phys. Rev. Lett., 48, 975 (1982).
2. S. Deser, R. Jackiw, and S. Templeton, Ann. Phys., 140, No. 2, 372 (1982).
3. B. Zumino, Y.-S. Wu, and A. Zee, Nucl. Phys., B239, No. 3/4, 477 (1984).
4. V. P. Akulov, D. V. Volkov, and V. A. Soroka, Pis'ma Zh. Éksp. Teor. Fiz., 22, No. 7, 396 (1975).
5. B. M. Zupnik and D. A. Leites, in: Multiple Processes at High Energies [in Russian], Fan, Tashkent, 3-26 (1976).
6. F. A. Berezin, Yad. Fiz., 30, No. 4(10), 1168 (1979).
7. D. A. Leites, The Theory of Supermanifolds [in Russian], Petrozavodsk (1983).
8. Yu. I. Manin, Gauge Fields and Complex Geometry, Nauka, Moscow (1984).
9. J. Wess, Springer Lecture Notes in Physics, Vol. 77 (1978), p. 81.