AVERAGE ENERGY OF ELECTRONS EJECTED FROM ATOMS DURING ELECTRON-COLLISION IONIZATION

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Plasma formation under various conditions is usually due primarily to the ionization of atoms in molecules from their ground states by electron collision. It is important to know the average energy of the ejected electrons as well as the number of ionization events. The number of such events is usually characterized by the total ionization cross section, while the electron energy distribution is characterized by the differential cross section. Several cross sections can be determined experimentally, so it is possible to check the theory in this case; on the other hand, it is difficult to determine differential cross sections from experiment, so various methods are used in which a theoretical description is found for the differential cross section for electron-collision ionization, the cross sections are compared, and then they are used to calculate the average energies of the ejected electrons.

The energy acquired by an atomic electron during an ionization event may have any value between the ionization energy v_0 and the energy v of the incident electron. If the atomic electron acquires an energy between x and x + dx, it is convenient to write the cross section to this reaction as

$$dq(x) = af(x)\,dx, \tag{1}$$

where a is a proportionality coefficient which is independent of the electron energy.

The total ionization cross section is

$$q = a\int_{v_0}^{v} f(x)\,dx. \tag{2}$$

The probability that an atomic electron will acquire an energy between x and x + dx during an ionization event is

$$dw(x) = \frac{f(x)\,dx}{\int_{v_0}^{v} f(x)\,dx} = g(x)\,dx, \tag{3}$$

where g(x) is the normalized distribution of ejected atomic electrons with respect to the energy transferred to them during the ionization. The average kinetic energy of the ejected electrons is

$$\bar{v} = \frac{\int_{v_0}^{v} (x - v_0) f(x)\,dx}{\int_{v_0}^{v} f(x)\,dx}. \tag{4}$$

The problem of finding this average energy reduces to one of finding the function f(x), but neither classical nor quantum mechanics gives a satisfactory description of the ionization process. The agreement between the theoretical results and experimental data does not appear very convincing. Several attempts have been made to construct semiempirical equations, of which the most successful is apparently the Drawin equation [1]. This equation gives the total ionization cross section in best agreement with experiment for a wide range of v [2]. The total ionization cross sections are given by expressions like [3]

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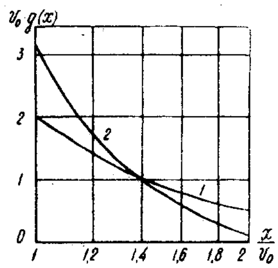

Fig. 1

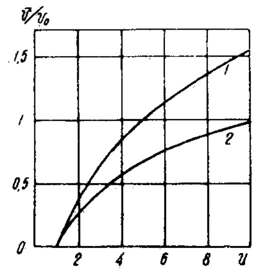

Fig. 2

Fig. 1. Distribution of ejected electrons for $v = 2v_0$. 1) Function (8); 2) function (14).

Fig. 2. Average kinetic energy of ejected electrons. 1) Function (9); 2) function (15).

$$q = \frac{a}{v_0^2} F(u), \tag{5}$$

where $u = v/v_0$, and the form of F(u) depends on the method used to calculate the cross section.

Below we compare the calculated distribution function g(x) and the average energy $\bar{v}$ found on the basis of the F(u) functions obtained by Thomson and Drawin, since both these functions are assumed to hold over a wide range of v.

In a classical analysis, Thomson found F(u) to be

$$F_1(u) = \frac{4(u-1)}{u^2}. \tag{6}$$

The total Thomson ionization cross section is found by substituting the known function f(x) [4] into Eq. (2):

$$f_1(x) = \frac{4}{vx^2}. \tag{7}$$

The distribution g(x) is

$$g_1(x) = \frac{vv_0}{(v - v_0)x^2}. \tag{8}$$

The average kinetic energy $\bar{v}$ of the ejected electrons is

$$\bar{v}_1 = v_0\left(\frac{u \ln u}{u-1} - 1\right). \tag{9}$$

As $u \to 1$, i.e., as $v \to v_0$, we have $\bar{v}_1 \to 0$; if, on the other hand, we have $u \gg 1$, i.e., $v \gg v_0$, we have $\bar{v}_1 \approx v_0(\ln u - 1)$.

Drawin wrote the function F(u) as

$$F_2(u) = 2.66\, b\, \frac{(u-1)}{u^2} \ln(1.25\, cu), \tag{10}$$

where we have b = 0.7-1.3 and c = 0.8-3 for various atoms. However, the ionization is described quite well if we assume b = c = 1 for the various atoms.

Since Eq. (10) is empirical, we must find the function f(x); for this purpose we use Eq. (2), substituting Eq. (5) into its left side, using Eq. (10):

$$\frac{2.66\, b}{v}\left(\frac{1}{v_0} - \frac{1}{v}\right) \ln\left(1.25\, c\, \frac{v}{v_0}\right) = \int_{v_0}^{v} f_2(x)\, dx. \tag{11}$$

Since ionization energy v_0 must appear only as an integration limit in Eq. (2) for the total ionization cross section, we must have

$$\frac{d}{dv_0} \int_{v_0}^{v} f_2(x)\, dx = -f_2(v_0). \tag{12}$$

Differentiating the left side of Eq. (11) with respect to v_0, and replacing v_0 by x in the function $f_2(v_0)$, we find

$$f_2(x) = 2.66b\left(\frac{\ln(3.4\,c\,v/x)}{vx^2} - \frac{1}{v^2x}\right). \tag{13}$$

Distribution function g(x) is

$$g_2(x) = \frac{vv_0}{v - v_0}\left(\frac{\ln(3.4\,c\,v/x)}{x^2} - \frac{1}{vx}\right)\frac{1}{\ln(1.25\,c\,v/v_0)}. \tag{14}$$

Figure 1 illustrates the difference between functions $g_1(x)$ and $g_2(x)$; here we have assumed c = 1.

Let us use the function $f_2(x)$ to determine $\bar{v}$:

$$\bar{v}_2 = v_0\left(\frac{u\ln u\ln(3.4c\sqrt{u})}{(u-1)\ln(1.25cu)} - \frac{1}{\ln(1.25cu)} - 1\right).$$

As $u \to 1$, i.e., as $v \to v_0$, we have $\bar{v}_2 \to 0$; if, on the other hand, we have $u \gg 1$, i.e., $v \gg v_0$, and $c \approx 1$, we have

$$\bar{v}_2 = v_0\left(\frac{1}{2}\ln u - \frac{1}{\ln u}\right).$$

Figure 2 shows the functions $\bar{v}_1(u)$ and $\bar{v}_2(u)$ in relative energy units; here we have assumed c = 1.

The average energy is an increasing function of u, and this is true of both the $\bar{v}_1(u)$ and $\bar{v}_2(u)$ dependences. Calculations carried out on the basis (15) for all values of u yield average energies lower than those found through the use of Eq. (9). The discrepancy increases with increasing u, and at $u \gg 1$ the energy $\bar{v}_1$ has become roughly twice as large as $\bar{v}_2$.

By using function $f_2(x)$ and the proper value of the coefficient c, we may be able to find a highly accurate average kinetic energy for the electrons ejected from atoms in electron-collision ionization, both for various atoms and for a wide range of u.

LITERATURE CITED

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