PHASE OF THE CHARGE IN ELECTRODYNAMICS

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The equations of a free electromagnetic field are covariant with respect to field-intensity transformations [1-3]:

$$E \to E\cos\theta - H\sin\theta;\quad H \to E\sin\theta + H\cos\theta. \tag{1}$$

For a volume containing charge, the Maxwell equations are covariant with respect to transformations (1) only if we consider, along with electric charge e, magnetic charge e_+ and if we introduce the transformation law

$$e \to e\cos\theta - e_+\sin\theta;\quad e_+ \to e\sin\theta + e_+\cos\theta. \tag{2}$$

Since e and e_+ transform according to (2), the question of the possible existence of magnetic monopoles must be reformulated; strictly speaking, the problem is to determine whether charges having phases different from the phases of ordinary charges exist [4]. Below we derive a conservation law corresponding to transformations (2), and we analyze a relation between the phases of the charges of interacting particles which follows from this law.

For this case it is useful to write the Maxwell equations in spinor form [5-7]. These equations can be obtained from the Lagrangian

$$\Lambda = -F^{\gamma}{}_{\dot\beta}\bar{F}^{\beta}{}_{\gamma} - J_{\dot\alpha\beta}A^{\beta\dot\alpha} - J^{\beta\dot\alpha}A_{\dot\alpha\beta}, \tag{3}$$

where

$$J_{\dot\alpha\beta} = \sigma_{\dot\alpha\beta n}(J^n + iJ^n_+);\quad A_{\dot\alpha\beta} = \sigma_{\dot\alpha\beta n}(A^n + iA^n_+);$$

J^n and A^n are the ordinary 4-current and 4-potential, and J^n_+ and A^n_+ are the magnetic potentials (the Latin indices are tensor indices, in contrast with the spinor Greek indices). For the electromagnetic field tensor we have the corresponding spinors

$$F^{\gamma}{}_{\beta} = \partial^{\gamma\dot\alpha}A_{\dot\alpha\beta};\quad \bar{F}^{\beta}{}_{\gamma} = \partial_{\gamma\dot\alpha}A^{\beta\dot\alpha}, \tag{4}$$

where

$$\partial_{\gamma\dot\alpha} = \sigma_{\gamma\dot\alpha n}\partial^n.$$

The Lorentz conditions for the electric and magnetic potentials,

$$\partial_n A^n = 0;\quad \partial_n A^n_+ = 0,$$

are equivalent to the conditions

$$F^{\beta}{}_{\beta} = 0;\quad \bar{F}^{\beta}{}_{\beta} = 0.$$

The nonvanishing elements of $\sigma_{\dot\alpha\beta n}$ and of the metric spinor $\gamma_{\beta\gamma}$ are

$$\sigma_{\dot 000} = 1;\quad \sigma_{\dot 123} = \sigma_{\dot 231} = \sigma_{\dot 312} = -\sigma_{\dot 321} = -\sigma_{\dot 132} = -\sigma_{\dot 213} = i;$$
$$\sigma_{\dot 011} = \sigma_{\dot 101} = \sigma_{\dot 110} = \sigma_{\dot 022} = \sigma_{\dot 202} = \sigma_{\dot 220} = \sigma_{\dot 033} = \sigma_{\dot 303} = \sigma_{\dot 330} = 1;$$
$$\gamma_{00} = -\gamma_{11} = -\gamma_{22} = -\gamma_{33} = 1.$$

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Varying $A^{\beta\dot{\alpha}}$ and $A_{\dot{\alpha}\beta}$ we find the field equations:

$$\partial_{\gamma\dot{\alpha}} F^{\gamma}{}_{\beta} = J_{\dot{\alpha}\beta}; \quad \partial^{\gamma\dot{\alpha}} \bar{F}^{\beta}{}_{\gamma} = J^{\beta\dot{\alpha}}. \tag{5}$$

We introduce the complex charges

$$q = e + ie_{+}; \quad q^{*} = e - ie_{+}.$$

According to (2), we have

$$q \to qe^{i\Theta}; \quad q^{*} \to q^{*}e^{-i\Theta}. \tag{6}$$

Transformations (6) correspond to the current transformations

$$J_{\dot{\alpha}\beta} \to J_{\dot{\alpha}\beta}e^{i\Theta}; \quad J_{\beta\dot{\alpha}} \to J_{\beta\dot{\alpha}}e^{-i\Theta}.$$

The transformation law for the potentials must be the same:

$$A_{\dot{\alpha}\beta} \to A_{\dot{\alpha}\beta}e^{i\Theta}; \quad A_{\beta\dot{\alpha}} \to A_{\beta\dot{\alpha}}e^{-i\Theta}. \tag{7}$$

It follows from (4) and (7) that

$$F^{\gamma}{}_{\beta} \to F^{\gamma}{}_{\beta}e^{i\Theta}; \quad \bar{F}^{\beta}{}_{\gamma} \to \bar{F}^{\beta}{}_{\gamma}e^{-i\Theta}.$$

The charge phase, which changes under transformations (6), must generally vary. Using (3), we find, by varying this phase,

$$J_{\dot{\alpha}\beta}A^{\beta\dot{\alpha}} - J^{\beta\dot{\alpha}}A_{\dot{\alpha}\beta} = 0. \tag{8}$$

Applying the Noether theorem to transformations (6), we find

$$\partial_{\gamma\dot{\alpha}} (F^{\gamma}{}_{\beta}\delta A^{\beta\dot{\alpha}} + \bar{F}_{\beta}{}^{\gamma}\,\delta A^{\dot{\alpha}\beta}) = 0.$$

Since we have

$$\delta A^{\beta\dot{\alpha}} = - iA^{\beta\dot{\alpha}}\delta\Theta; \quad \delta A^{\dot{\alpha}\beta} = iA^{\dot{\alpha}\beta}\delta\Theta,$$

we have the conservation law

$$\partial_{\gamma\dot{\alpha}} (F^{\gamma}{}_{\beta}A^{\beta\dot{\alpha}} - \bar{F}_{\beta}{}^{\gamma}A^{\dot{\alpha}\beta}) = 0,$$

from which, taking field equations (5) into account, we can derive Eq. (8).

Let us consider the physical meaning of Eq. (8) for the simplest case – that of a constant electromagnetic field. We assume that particles having charges

$$q_1 = |q_1|\,e^{i\alpha_1}; \quad q_2 = |q_2|\,e^{i\alpha_2}$$

are interacting. The complex scalar potential φ at the position of the first particle is proportional to the charge of the second. From Eq. (8), which takes the form

$$\overset{*}{q}_1\varphi_1 - \overset{*}{\varphi}_1 q_1 = 0$$

in this case, we find

$$\sin(\alpha_1 - \alpha_2) = 0. \tag{9}$$

According to Eq. (9), the charge phases must either be equal or differ by π; in particular, if one of the interacting charges is real, the second must also be real (either positive or negative).

LITERATURE CITED

1. C. Y. Rainish, Trans. Amer. Math. Soc., 27, 106 (1925).
2. D. R. Brill and J. A. Wheeler, in: Current Problems in Gravitation [Russian translation], D. Ivanenko (editor), IL (1961).
3. C. Misner and J. A. Wheeler, "Appendix 1," in: J. A. Wheeler, Gravitation, Neutrinos, and the Universe [Russian translation], IL (1962).
4. H. Harrison et al., Amer. J. Phys., 31, 249 (1963).
5. O. Laporte and G. E. Uhlenbeck, Phys. Rev., 37, 1380 (1931).
6. J. R. Oppenheimer, Phys. Rev., 38, 725 (1931).
7. W. L. Bade and H. Jehle, Rev. Mod. Phys., 25, 714 (1953).