COMPTON EFFECT IN AND POWER ABSORPTION BY AN ELECTRON GAS

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An analysis is made of Compton scattering by an electron gas. The power absorption and heating of an electron gas are calculated. The possible measurement of the intensity of intense coherent light beams is discussed.

1. Introduction

When a light wave passes through an electron gas it is scattered by electrons. In the optical range, this scattering occurs without any significant change in the frequency of the incident wave. Absorption of power from an optical-range light wave by an electron gas is therefore extremely slight and can be increased only by intensifying the incident wave.

There are two ways to calculate the power absorbed by an electron gas from a wave. The kinetic equation can be solved for the space charge in the field of an electromagnetic wave, as is the usual procedure in problems involving passage of radiation through a plasma. Alternatively, the Compton equation for the frequency of the scattered light could be used to calculate the power transferred from the wave to the gas.

Description of the motion of an electron gas in the field of an electromagnetic wave by means of the kinetic equation is a phenomenological procedure and yields no information about the mechanism for the interaction of an electron with the photons. A characteristic result found from the linearized kinetic equation [1, 3] is that power can be absorbed from the electromagnetic wave by an electron gas only through collisions of electrons with residual gas molecules present along with the electrons. Electron–electron collisions alone cannot lead to absorption, since these collisions are not accompanied by a change in the net momentum of the electron gas. In a two-photon process such as Compton scattering, on the other hand, there is always momentum transfer from photons to electrons. The frequency ω' of the scattered light is lower than that ω of the incident light, although it is true that (as can be seen in the familiar Compton equation) the frequency of the scattered light changes only when terms on the order of $\hbar\omega/mc^2$ are conserved; these terms are very small in the optical wavelength range.

A decrease in the frequency of the scattered light obviously implies that some of the energy of the light wave is absorbed by the electron gas, which becomes hotter as a result. The heating is observable only in intense incident light waves. In the last section below we will analyze the possibility of constructing a power meter for intense coherent radiation from an electron device operating by means of heating of the electron gas.

2. Power Absorption by an Electron Gas

Scattering of a light wave is generally different from Compton scattering by a free electron: in a real electron gas with a residual neutral gas background, the electron energy can change as a result of Coulomb interactions between electrons or as a result of electron collisions with neutral molecules. Let us evaluate the corresponding energy changes for the case of a free electron.

If there are no positive ions in the electron space charge, the Coulomb-interaction energy per particle can be found from [2]

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$$E_C = \sqrt{\pi} e^3 \sqrt{\frac{n}{\kappa T}}, \quad (1)$$

where n is the electron density in the electron gas; and T is the temperature of this gas. At $n \approx 10^9$ cm^{-3} and $T \sim 10^{3}$°K, corresponding to the conditions in ordinary electron devices, the Coulomb-interaction energy E_C of an electron is on the order of 10^{-5} eV, i.e., four orders of magnitude below the electron thermal energy at this temperature.

The energy acquired per unit time as a result of elastic collisions by an electron during the thermal motion of residual gas molecules (whose density is comparable to the electron density at achievable vacuum levels) is [1]

$$\Delta E_{el} = \frac{2m}{M} \left(\frac{mv^2}{2} \right) \nu, \quad (2)$$

where m and M are the masses of the electron and molecule; v is the electron velocity (in this case, the thermal velocity); and ν is the effective frequency at which electrons collide with gas molecules. According to the data in [1], this effective collision frequency ν_{eff} in air at a molecular density of 10^9 cm^{-3} (corresponding to a residual gas pressure of 10^{-7} torr) is 8.5 sec^{-1}. The energy acquired by an electron from residual gas molecules for the case of a thermal energy on the order of 0.1 eV ($\approx 10^{3}$°K) for the electrons in the electron gas is thus on the order of $E_{el} \approx 10^{-4}$ eV. Inelastic collisions are very improbable at thermal electron velocities. These estimates of E_C and E_{el} show that the energy of an electron in an electron gas differs very slightly from that of a free electron; the primary contribution to the energy is the rest energy, $mc^2 = 0.5 \cdot 10^6$ eV. Below we can therefore use the Compton equation for the frequency change of light scattered by a free electron:

$$\omega' = \frac{\omega}{1 + \frac{\hbar\omega}{mc^2}(1 - \cos\Theta)}, \quad (3)$$

where Θ is the angle through which the photon is scattered. Since we are not interested here in detecting photons scattered through specific angles, we use the average value $\overline{\cos\Theta} = 0$ and expand expression (3) in a series in terms of the small parameter $\hbar\omega/mc^2 \approx 4 \cdot 10^{-6}$ (small for the optical range), finding, within terms of a second order of smallness,

$$\omega' \approx \omega \left(1 - \frac{\hbar\omega}{mc^2} \right),$$

so the frequency decrease is

$$\Delta\omega = \omega - \omega' = \frac{\hbar\omega^2}{mc^2}. \quad (4)$$

The same result follows directly from the equation giving the change in the kinetic energy of an electron scattered through an angle φ:

$$\Delta E_{kin} = \hbar\Delta\omega = \frac{2mc^2 (\hbar\omega)^2 \cos^2\varphi}{(\hbar\omega + mc^2)^2 - (\hbar\omega)^2 \cos^2\varphi},$$

averaged over $\cos^2\varphi$.

Using (4) and knowing the cross section for scattering of the light by electrons, we can easily calculate the power absorbed by an electron from a wave during the scattering. The pertinent scattering cross section here is evidently $\sigma = 6.7 \cdot 10^{-25}$ cm^2 – the Thomson cross section, since Compton scattering of visible light does not involve a change in frequency. Accordingly, when a light wave of intensity J is incident on the electron gas the power absorbed from the wave per electron is

$$\Delta P = J\sigma \frac{\Delta\omega}{\omega} = J\sigma \frac{\hbar\omega}{mc^2}. \quad (5)$$

At high intensities, the Compton effect becomes nonlinear, and the scattering cross section and frequency change become dependent on the intensity. The frequency of the scattered light is given by the Compton equation except at very high intensities of the incident radiation [4]:

$$\omega' = \frac{\omega}{1 + \left(\frac{\hbar\omega}{mc^2} + \frac{1}{2} E\right)(1 - \cos\Theta)}, \tag{6}$$

where the parameter

$$E = \frac{e^2\rho}{m^2c^2\omega^2}$$

is approximately unity at a radiation density of $\rho = 10^8$ J/cm^3 at optical frequencies, corresponding to a radiation intensity of $J = 10^{19}$ W/cm^2. If such an intensity were achievable, the frequency change would be $\Delta\omega \simeq \omega/3$, according to (6); i.e., the possible power absorption would be five orders of magnitude higher than in the linear case.

3. Heating of the Electron Gas and Possible Measurement of the Power of Intense Coherent Radiation

The power absorbed by the electron gas heats this gas. Some of the energy absorbed from the wave is dissipated in collisions between electrons and residual particles. If the electron-gas temperature in the absence of the wave is T_0, and if this temperature is increased to T as a result of the interaction with the radiation, the energy transferred from electrons to neutral molecules per unit time is, according to (2),

$$\Delta E = \frac{3}{2} n\kappa (T - T_0) \frac{2m}{M} \nu. \tag{7}$$

The energy-balance equation per unit volume can thus be written as [5]

$$\frac{d}{dt}\left(\frac{3}{2} \kappa n T\right) = n\Delta P - \frac{3}{2} n\kappa (T - T_0) \frac{2m}{M} \nu. \tag{8}$$

From the preceding section we know that at a residual gas pressure on the order of 10^{-6}-10^{-8} torr the second term on the right side of (8) is negligible, and the heating of the electron gas over the irradiation time Δt is

$$\Delta T = T - T_0 = \frac{2}{3} \frac{\Delta P}{\kappa} \Delta t$$

or, when (5) is used,

$$\Delta T = \frac{2}{3} J\sigma \frac{\hbar\omega}{\kappa mc^2} \Delta t. \tag{9}$$

From this result we can determine the threshold intensity J_0 at which the electron gas warms by a certain amount. For the temperature to increase 300°K per 1 sec, the radiation intensity at $\lambda = 6934$ Å would have to be at least $J_0 = 1.6 \cdot 10^9$ W/cm^2, corresponding to an energy density of 0.05 J/cm^3.

Using Eq. (9) we can find the intensity J of the incident radiation from the measured heating of the electron gas.

We consider a planar triode with a grid voltage of $-U$. The current density j_0 which originally penetrates the grid is governed by the Boltzmann distribution of electrons emitted from the cathode at temperature T_0 with the grid at a cutoff voltage:

$$j_0 = j_{in} e^{-\frac{eU}{\kappa T_0}}, \tag{10}$$

where j_{in} is the current density near the cathode.

After the electron cloud between the grid and the cathode is irradiated, and after the electron temperature increases to T, the current density arriving at the anode becomes

$$j = j_{in} e^{-\frac{eU}{\kappa T}}. \tag{11}$$

From Eqs. (10) and (11) we find the increase in the electron temperature to be

$$\Delta T = \frac{eU}{\kappa}\left(\frac{1}{\ln\frac{j_0}{j_{in}}} - \frac{1}{\ln\frac{j}{j_{in}}}\right). \tag{12}$$

Comparing (9) and (12), we find (replacing the constant factors by their numerical equivalent) an expression for the intensity of the incident wave:

$$J\left(\frac{\mathrm{W}}{\mathrm{cm}^2}\right) = 0.29\cdot 10^{27}\,\frac{U}{\omega\Delta t}\left(\frac{1}{\ln\frac{j_0}{j_{in}}} - \frac{1}{\ln\frac{j}{j_{in}}}\right). \tag{13}$$

Here U is in volts.

By measuring the anode currents j_0 and j before and after irradiation of the space charge for a time Δt, we can thus determine the intensity J of incident radiation at a frequency ω.

We note in conclusion that during the irradiation of the tube by an intense light beam a photoelectric effect may occur at the envelope and at the electrodes. The resulting photocurrent may contribute to the increase in the anode current.

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