ROLE OF MAGNETIC MICROSTRUCTURE FOR FERROMAGNETIC RESONANCE WITH A CONDUCTING FERROMAGNETIC FILM

1. NATURE OF THE SKIN EFFECT FOR FERROMAGNETIC RESONANCE WITH MAGNETIC STRUCTURE NEGLECTED

Yu. V. Kornev, N. I. Yurasov, and G. P. Bykov

UDC 538.245

In this study of the properties of a film of ferromagnetic metal in an electromagnetic field, a joint solution is found for the Maxwell equations and an equation for the change in the density of the magnetic moment. This latter equation is written in Landau–Lifshitz form [1a] with the relaxation term proposed by Gilbert. The conduction current density is described by the usual local relation with a homogeneous and isotropic static conductivity σ_0:

$$j = \sigma_0 E,$$

where **j** is the conduction current density; and **E** is the electric field.

The Landau–Lifshitz equation is

$$\frac{dM_s}{dt} = -\gamma [M_s,\ H + H_\alpha + H_{an}] + \frac{\alpha_0}{M_s}\left[M_s,\ \frac{dM_s}{dt}\right], \tag{1}$$

where $\mathbf{M_S}$ is the density of the saturation magnetic moment at the given temperature; $H_\alpha = \lambda_W M_S + \alpha\nabla^2 M_S$ is the exchange magnetic field for a cubic lattice; λ_W is the constant of the Weiss field; α_0 is the dimensionless damping parameter $\alpha_0 \ll 1$; α is the exchange constant from the Landau–Lifshitz equation [1a]; H_{an} is the anisotropy field, which can be expressed in terms of the corresponding anisotropy and anisotropy–dispersion tensors $\hat{\beta}$ and $\hat{\beta}_u$ by

$$H_{an} = \hat{\beta} M_s + \hat{\beta}_u \frac{\partial M_s}{\partial x_u}, \tag{2}$$

and γ is the magnetomechanical ratio. The first term in the expression for the exchange field has the form given above if the strong inequality $\omega\tau_0 \ll 1$ holds, where ω is the angular frequency of the exciting electromagnetic field; and τ_0 is the relaxation time for the magnetic moment density. In the opposite case of $\omega\tau_0 \gg 1$, the term $\lambda_W M_S$ becomes $\lambda_W M_{S0}$, where M_{S0} is the static magnetic moment density.

If the magnetization inhomogeneities, expressed by the differential terms of fields H_α and H_{an}, are distributed continuously, and if the magnet is bounded by planar surfaces, as in, e.g., the case of our plane-parallel film, the magnetic moment density can be written in a Fourier series. Assuming the vector $\mathbf{M_S}$ to be a function of the coordinates and time, we write

$$M_s = M_{s0} + \sum_\kappa M_\kappa e^{-i(\omega t - \kappa r)},$$

where **r** is the radius vector of a given point in the material; $\boldsymbol{\kappa}$ is the complex wave vector, and the summation is carried out over all types of oscillations. The magnetic field **H** can be written in a similar manner.

N. É. Bauman Moscow Higher Educational School. Translated from Izvestiya Vysshikh Uchebnykh Zavedenii, Fizika, No. 9, pp. 7-12, September, 1970. Original article submitted July 15, 1969.

Let us consider the Maxwell differential equations. The conduction current density for a metal in the frequency range $\omega < 10^{12}$ sec^{-1} is much higher than the displacement current density, so by neglecting the latter and combining the two Maxwell equations, we find a relation between M_K and H_K:

$$4\pi \boldsymbol{M}_\kappa = (-1)\left(1 + \frac{ic^2\kappa^2}{4\pi\sigma_0\omega}\right)\boldsymbol{H}_\kappa + \frac{ic^2\,\boldsymbol{\kappa}\,(\boldsymbol{\kappa}\boldsymbol{H}_\kappa)}{4\pi\sigma_0\omega}. \tag{3}$$

To obtain the dispersion equation $f(\kappa, \omega) = 0$, we must specify further the structure of field $H(r, t)$. In the dipole approximation the condition for ferromagnetic-resonance absorption is, according to [2]

$$\boldsymbol{H}_0[\boldsymbol{H}(\boldsymbol{r}, t) - \boldsymbol{H}_0] = 0,$$

where H_0 is the static magnetizing field; the expression in brackets is the sum over all types of oscillations:

$$[\boldsymbol{H}(\boldsymbol{r}, t) - \boldsymbol{H}_0] = \sum_{\kappa} \boldsymbol{H}_\kappa e^{-i(\omega t - \kappa r)}.$$

Starting from these considerations, we choose the field structure in the film:

$$\boldsymbol{H}\{(\boldsymbol{H}_\kappa)_x,\ (\boldsymbol{H}_\kappa)_y,\ (\boldsymbol{H}_0)_z\},$$

where (x, y, z) are a Cartesian coordinate system.

We first consider the case in which the field vector is parallel to the planes bounding the film of thickness d and in which the following conditions hold:

$$H_0 \sim M_{s0};\quad \boldsymbol{\kappa}\boldsymbol{H}_0 = 0;\quad [\boldsymbol{E}, \boldsymbol{H}_0] = 0;\quad \boldsymbol{H}_0 \parallel \boldsymbol{M}_{s0}.$$

The equality $[E, H_0] = 0$ eliminates the ordinary Hall effect. As in [3a, 4a], we are neglecting effects associated with the effect of the internal magnetic field on the orbital motion of conduction electrons; in other words, we are assuming the condition

$$\omega_c\tau \leqslant 0.5,$$

where ω_c is the gyrofrequency; and τ is the principal relaxation time of the conduction electrons.

The specified vector field M_{S0} and H_0 implies uniaxial anisotropy. Here we have neglected the second term in Eq. (2) and used the standard relation for the uniaxial-anisotropy field:

$$\boldsymbol{H}_{\text{an}} = \beta\,(\boldsymbol{M}_s\boldsymbol{n})\,\boldsymbol{n},$$

where β is the anisotropy constant; and n is a unit vector along the easy axis. We now assume

$$\boldsymbol{M}_{s0} \parallel \boldsymbol{n};\quad \nabla^2\boldsymbol{M}_{s0} = 0.$$

These conditions are widely used [3b, 4n, 5, 6]; their limitations will be discussed in Part II of this study. Substituting Eq. (3) into Eq. (1), and linearizing the latter, we find the dispersion equation

$$f(\kappa, \omega) = 0;$$

$$K^6 - GK^4 + DK^2 + F = 0,$$

where

$$K = -i\kappa\delta_0\varepsilon,\quad \delta_0 = (c^2/4\pi\sigma_0\omega)^{1/2},\quad \varepsilon^2 = \alpha/4\pi\delta_0^2,$$

$$G = 1 + 2(\omega_{H_0}/\omega_M) + 2i[\varepsilon^2 + \alpha_0(\omega/\omega_M)], \tag{4}$$

$$D = (\omega_{H_0}/\omega_M) + (\omega_{H_0}/\omega_M)^2 - (\omega/\omega_M)^2 - \alpha_0(\omega/\omega_M) - 4\alpha_0\varepsilon^2(\omega/\omega_M) + i\{[1 + 2(\omega_{H_0}/\omega_M)](\omega/\omega_M)\alpha_0 + 4\varepsilon^2[1 + (\omega_{H_0}/\omega_M)]\},$$

$$F = 2\varepsilon^2[2[1 + (\omega_{H_0}/\omega_M)](\omega/\omega_M)\alpha_0 + i[(\omega/\omega_M)^2 - (1 + (\omega_{H_0}/\omega_M)^2 + (\omega/\omega_M)^2\alpha_0^2]\},$$

$$\mathscr{H}_0 = H_0 + \beta M_{s0},\quad \omega_{H_0} = \gamma\mathscr{H}_0,\quad \omega_M = \gamma 4\pi M_{s0}.$$

We now replace the complex wave number κ by the complex quantity K:

$$\kappa_p = iK_p,\quad p = 1, 2, 3, 4, 5, 6. \tag{5}$$

The wavelength λ in the material is related to K by

$$\lambda = 2\pi/|\operatorname{Im} K|,$$

where Im K is the imaginary part of the complex quantity K. It is sufficient to use only the roots with $\operatorname{Re} K_p > 0$ for the analysis, since Eq. (4) is bicubic. Numerical values of the roots of Eq. (4) were calculated on

a Minsk-22 computer for iron and cobalt; the initial data were

$$\left.\begin{array}{l} \omega = 2\pi \cdot 10^{10}\ \text{sec}^{-1}; \quad \omega_M(\text{Fe}) = 3.78 \cdot 10^{11}\ \text{sec}^{-1}, \\ \qquad\qquad\qquad\quad \omega_M(\text{Co}) = 3.15 \cdot 10^{11}\ \text{sec}^{-1}; \\ \mathscr{H}_0 = 0, 100, \ldots 2000\ \text{Oe}; \quad \gamma = 1.761 \cdot 10^{7}\ \text{sec}^{-1} \cdot \text{Oe}^{-1}; \\ \qquad\qquad\qquad\quad \sigma_0(\text{Fe}) = 9.0 \cdot 10^{16}\ \text{sec}^{-1} \\ \qquad\qquad\qquad\quad \sigma_0(\text{Co}) = 1.39 \cdot 10^{17}\ \text{sec}^{-1} \end{array}\right\},$$

from [7a].

To analyze the effects of the physical constants and parameters on the coefficients in dispersion equation (4), we use the following inequalities and approximations:

$$(\omega/\omega_M) < 0.2; \quad \alpha_0 \sim 10^{-2}, \ 0 < \alpha_0 < 0.1; \quad \delta_{0(\text{Fe, Co})} \approx 10^{-4}\ \text{cm};$$
$$0 \leqslant (\omega_{H_0}/\omega_M) \leqslant 0.11; \quad \alpha \sim (10^{-12} - 10^{-11})\ \text{cm}^2; \ \varepsilon^2 < 10^{-3},$$

from which it can be shown that the effect of the dimensionless magnetic-damping parameter α_0 on the roots of Eq. (4) is slight, so we can set $\alpha_0 = 0$. Using a computer, we were able to study the solutions of Eq. (4) and establish the following behavior: 1) the basic contribution to the numerical value of roots K_p (p = 1, 2, 3) comes from the modulus of coefficient G; 2) coefficient D affects the root only one-fourth as much; and 3) the effect of coefficient F on the root is one-third that of G.

In calculating the effective static field $\mathscr{H}_0 = H_0 + \beta M_{S0}$, we used the series of values 0, 100, 200, . . . , 2000 Oe, which span the range of static magnetic fields used to observe ferromagnetic resonance in iron and cobalt films in microwave fields. The results of this calculation for cobalt are shown as graphical dependences of $|\text{Re}\, K_p|$ and $|\text{Im}\, K_p|$ on the effective magnetic field in Figs. 1 and 2, respectively. The relative error in the calculation of K_p (p = 1, 2, 3) is no worse than 1%. The dependence of K_p on the effective field $\mathscr{H}_0$ at fields higher than shown in the figures is difficult to find because of the limitations of the Minsk-22 computer. However, by analyzing the dependence of K_p on the coefficients in Eq. (4), we can show that $\text{Re}\, K_1$ has a modulus much greater than those of the other K_p (p = 2, 3) and increases slowly up to about $5 \cdot 10^3$ Oe. Between this field and $\mathscr{H}_0 = 1.5 \cdot 10^3$ Oe, this dependence is nonlinear. The quantity $|\text{Re}\, K_1|$ decreases slightly, reaching a minimum; then it increases to its former value.

From Eq. (4) we see that we have $|K_p| \sim \alpha^{-1/2}$, for p = 1, 2, 3, in a very good approximation. A numerical check shows that this behavior holds for α between 10^{-10} and 10^{-14} cm^2 and over the entire field range of practical interest. Carrying out this calculation on the basis of the data of [6b], we find $\alpha \sim 10^{-12}$ cm^2. For films a few hundred atomic layers thick the static magnetic moment density can be assumed equal to the value of the bulk material.

Since the α-values calculated from the experimental data show a scatter, and since it was assumed for the α-calculation that the static magnetization is uniform throughout the sample, we analyzed the theoretical evaluations of α in [1b, 8]: $\alpha \sim 10^{-12}$-10^{-11} cm^2. We believe these values to be more reliable. The results calculated for K_p (p = 1, 2, 3), $\text{Re}\, K_p > 0$, shown in Figs. 1 and 2, were obtained with $\alpha = 10^{-11}$ cm^2 – the upper limit on α.

We turn now to an analysis of these figures. Let us evaluate the possible anomalous behavior of the skin effect. If the real part, the imaginary part, or both parts of K_p are very large, it may turn out that the conditions for the normal skin effect will not be satisfied for this type of oscillation. The limits for this effect are given by

$$l\,|Re\,K_p| \ll 1, \tag{6}$$
$$l\,|\text{Im}\,K_p| \ll 2\pi, \tag{7}$$

where l is the mean free path of a conduction electron. A numerical evaluation of l taking into account only the phonon scattering mechanism near the Debye temperature yields

$$l = E_0 a/\pi N \kappa_B T,$$

where E_0 is the Young's modulus; a is the lattice constant; N is the number of atoms per unit volume; κ_B is the Boltzmann constant; T is the absolute temperature. Results calculated from this equation for polycrystalline ferromagnets (iron, cobalt, and nickel) at 293°K are shown in Table 1.

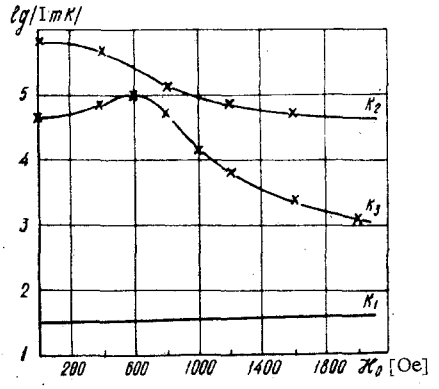

Fig. 1

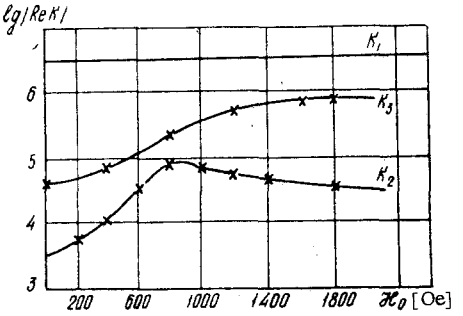

Fig. 2

TABLE 1. Mean Free Path of Conduction Electrons in Certain Transition Metals

Metal	l	$(ReK)_c$	$(ImK)_c$
Fe	$5{,}6\cdot10^{-6}$ cm	$1{,}8\cdot10^{5}$ cm^{-1}	$1{,}1\cdot10^{6}$ cm^{-1}
Co	$6{,}5\cdot10^{-6}$ cm	$1{,}5\cdot10^{5}$ cm^{-1}	$0{,}97\cdot10^{5}$ cm^{-1}
Ni	$5{,}9\cdot10^{-6}$ cm	$1{,}7\cdot10^{5}$ cm^{-1}	$1{,}1\cdot10^{6}$ cm^{-1}

The data used to calculate l were taken from [7b]. The last two columns in this table show the critical real and imaginary parts of K_p (p = 1, 2, 3), i.e., the values corresponding to equalities in (6) and (7). When these inequalities become equalities, the electric field changes by a factor of e over an electron mean free path, according to (6), while according to (7) the wavelength $2\pi / |\operatorname{Im} K_p|$ is equal to one electron mean free path. In both these cases the field is very nonuniform over an electron mean free path, so the local expression $\mathbf{j} = \sigma_0 \mathbf{E}$ which we used for the conduction current density in the calculations becomes invalid.

The actual mean free path of electrons in ferromagnets is slightly lower than shown in Table 1, because we have neglected the magnetic scattering mechanism. Since the range studied for iron and cobalt (293°K) lies far from the Curie point, however, the magnetic-scattering mechanism cannot reduce the value of l by more than 0.2 l. Since we have $|K_p| \sim \alpha^{-1/2}$ and $\alpha_{calc} = \alpha_{max}$ the calculated values of K_p are minimum values.

Taking this circumstance into account, and comparing the results in Table 1 with the curves for K (p = 1, 2, 3) in Figs. 1 and 2, we conclude that the skin effect behaves in an anomalous manner for the first mode (K_1), and there is evidence of an anomalous skin effect for the second (K_2) and third (K_3) modes in the field range of practical interest. However, in a large number of experiments, e.g., those reported in [9] and in the literature cited there, no anomalies were observed in the behavior of the skin effect over a broad range of microwave frequencies, 9-38 GHz, at room temperature, 293°K. The temperature dependence found there for the line width of ferromagnetic-resonance absorption was explained satisfactorily on the basis of the theory of the ordinary skin effect [4c]. We therefore turned to a possible effect of the microstructure of a ferromagnetic film, neglected above, on the nature of the skin effect; analysis of this factor will be reported in the second part of this study.

The authors thank S. A. Temirbulatov for assistance in this study.

LITERATURE CITED

1. L. D. Landau, Collected Works [in Russian], Vol. 1, Nauka, Moscow (137); (1969), p. 132.
2. S. V. Vonsovskii (editor), in: Ferromagnetic Resonance [in Russian], Fizmatgiz, Moscow (1961), p. 14.
3. W. S. Ament and G. T. Rado, Phys. Rev., 97, No. 6, 1559, 1560 (1955).
4. L. L. Hirst and R. E. Prange, Phys. Rev., 139, No. 3A, 859, 893, 894 (1965).
5. M. I. Kaganov and Yui-Lu, Izv. Akad. Nauk SSSR, Ser. Fiz., 25, No. 11, 1376 (1961).
6. R. F. Soohoo, Magnetic Thin Films, Harper and Row, New York (1965).
7. Handbook on the Physicochemical Properties of the Elements [in Russian], Kiev (1965), pp. 262; 11, 98, 112, 329.
8. A. I. Akhiezer, V. G. Bar'yakhtar, and S. V. Peletminskii, Spin Waves [in Russian], Nauka (1967), p. 25.
9. S. M. Bhagat, J. R. Anderson, and Ning Wu, Phys. Rev., 155, No. 2, 510-513 (1967).