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The influence of travel time variability on the value of time

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Abstract. Current benefits from travel time savings have only been related to the benefits from reducing mean travel time. Some previous attempts of including variability in the generalised cost function have mainly assumed commuters with fixed arrival time. This paper presents a comprehensive framework for valuing travel time variability that allows for any journey purpose and arrival time constraint. The proposed model is based on the expected utility approach and the mean-standard deviation approach. Stated Preference methods are considered the best technique for providing the data for calibrating the models. The values of time derived from the models are highly influenced by the value of travel time variability and it strongly depends on the probability distribution function travellers are faced with.

Introduction

The valuation of travel time savings is a very important aspect of cost-benefit analysis. It is also an important practical issue in terms of the allocation of resources to the transport sector as well as within the sector. In railway and road investments a substantial part of the total measured benefits come in the form of time savings. In the road sector in Britain, for instance, some 80% of the measured benefits come from travel time savings (Department of Transport 1976). In urban transport studies, travel time savings are considered a major indicator of improvement to the existing system (Gunn, Mackie and Ortuzar 1980). Within this framework, to omit a value for time savings from a feasibility analysis would exclude one of the main benefits and distort investments (Howe 1976).

A large number of studies have investigated ways of inferring values of travel time, for instance, the studies reported by Bruzelius (1979) and MVA, ITS and TSU (1987).

A general criticism of benefits of travel time savings derived from current studies is that the benefits have been only related to reductions in the expected (mean) travel time and any benefits from reductions in travel time variability have been completely ignored. Note that many transport improvements reduce travel time (mean travel time) and increase the reliability of transport services. There are even cases where travel time variability may be reduced without a reduction in mean travel time (for instance, an increase in the frequency of public transport).

The absence of travel time variability amongst the benefits of travel time savings may be a serious weakness of current studies. The consequences of ignoring effects of unreliability (variability) may affect not only the estimates of the value of travel time but it also assumes that there is no benefits from reducing unreliability.

Travellers' decision-making

In the real world travel decisions are taken in risky conditions. Travellers do not know *a priori* if public transport is going to be reliable on a specific day or if there will be serious congestion.

In general, travellers do not know with certainty the consequences of their choices. The most they can do is to attach probabilities to an expected level of travel time. If travellers know the probability associated with an outcome that is called a *risky* situation. If they do not know the probability that is called an *uncertain* situation. This paper analyses the risky situations.

In the transport field, Knight (1974) suggests that the evaluation of highway and public transport investment projects might include an "unreliability factor" in prediction of diversion to rival modes as well as in the evaluation of consequent changes in user benefits.

The importance of risk should be present in any travel decision. Gutman (1979) and Menashe and Gutman (1986) emphasize the importance of uncertainty in modal split modelling. According to these authors if travellers are risk-averse their mode choice is led by a more "certain" transport mode.

The economic literature has examples of different approaches to analyse decisions under uncertainty. Roy (1952) identifies two major schools in this subject: one, led by Shackle's ideas (Shackle 1949, 1955) and another by Von Neuman and Morgenstern (1947) and Friedman and Savage (1948).

Shacke's ideas are part of the two-parametric criteria that takes undesirable dispersions into account through representing the distribution by one parameter measuring a *mean return* and another parameter measuring *risk* and then assuming a utility function over these parameters. Although part of that criteria, Shackle's ideas do not use statistical distribution parameters as risk and return measures. He prefers to subjectively assess index numbers of the distributions to be evaluated. Thus, there are two ways in which the decision maker's preferences affect the evaluation of a probability distribution. The first is the usual one via the shapes of indifferent curves in a mean-standard deviation diagram. The second is through the formulation of these index numbers.

The alternative school is an extension of the theory of choice under certainty to problems involving expectation, by the assumption that individuals maximise expected utility. This *Expected Utility approach* was originally proposed by Bernoulli (1738). After the axiomatic foundation by Von Neuman and Morgenstern (1947) it became the most popular approach to formulating a preference function for the evaluation of probability distributions.

Studies within the transport field have assumed transport users to be utility maximisers. However, these studies do not take into account that there are probabilities related to alternative outcomes. Current studies implicitly assume a riskless environment. On the other hand, previous studies which have analyzed the benefits of reducing travel time variability have mainly considered commuters with fixed arrival time. Relatively little research has been conducted on non-commuters and those travellers who do not face arrival time constraints, but who may value reductions in travel time variability.

According to a framework suggested by Black and Towris (1990), three main steps have been identified which must be followed in order to build up a broad analysis of travel decisions under uncertainty. Firstly, it is useful to examine the importance of transport system unreliability (or the effects of travel time variability for transport users). Secondly, it is important to assess current approaches to evaluate the importance of travel time variability. Finally, it is necessary to build up a general framework of travel choice under uncertainty which should be consistent with an acceptable theoretical basis such as the expected utility approach.

The main aim of this paper is to develop a general model for valuing travel time variability that applies for any traveller facing variable travel times. An analysis of previous studies in the area suggests that the theoretical framework of existing models for valuing travel time variability heavily relies on the behaviour of commuters with fixed arrival times. A comprehensive model for valuing travel time variability must be able to cope not only with commuters with fixed arrival time but also with travellers under a wider set of journey purposes and arrival time constraints.

This paper also analyses travellers response to travel time variability according to four different categories:

- Commuters with fixed arrival time
- Commuters with flexible arrival time
- Non-commuters with fixed arrival time
- Non-commuters with flexible arrival time

Existing models

The standard model

The most usual way of assessing the value of travel time is through a utility function defined as a function of time and cost. The utility is assumed to be linear and the models are usually based on Revealed Preference (RP) or Stated Preference (SP) data with the following functional form:

$$U = \alpha t^{\beta} + \phi cost \qquad (1)$$

Previous studies using this approach assumed $\beta = 1$ and did not test whether the data would support some other values of β.

Studies that rely on data gained from RP data assume $t = E(t)$. The process of asking travellers about their actual travel time may not reflect reality since the actual time does not necessarily mean the travellers' expected travel time.

In recent years, SP studies have become popular for valuing travel time. However, in current SP studies, travellers are asked to choose between alternative t rather than alternative $E(t)$, and this suggests the choices have been taken under certainty.

The development of approaches that take into account travel time variability, should include both the analysis of a proper theoretical basis and consideration of a proper SP design. The following sections describe existing methods for considering travel time variability, propose alternative methodologies and discuss alternative ways of presenting the problem of travel time variability.

The safety margin hypothesis

The safety margin hypothesis discussed by Garver (1968), Thomson (1969) and Knight (1974) assumes that, in order to avoid the possibility of arriving late at a destination, individuals allow extra time, the so called safety margin. Some studies (for instance, Pells 1987) have assumed that the safety margin is the proper measure of travel time variability because it is the way people react to uncertain travel times.

Pells (1987) defines the safety margin as the difference between the mean arrival time and the work (destination) start time with activity-specific time values. For example, time at home has a higher value than early time at work. If this was not the case, an individual would be indifferent as to how early he or she arrived at work.

Pells show that there are two opposite forces acting: the need to restrict the frequency of late arrivals to some acceptable level and the maximisation of the amount of time at home relative to that spent at work early. To meet the first objective the individual allocates a safety margin to slack time; to meet the second objective the individual must keep the slack time allocation to a minimum.

Two choice models have been estimated by Pells. The first model uses data from an SP experiment where individuals are asked to assume that the time taken for the journey to work is constant from day to day. In option 1 the individual is presented with a scenario in which his/her departure time is restricted such that he/she arrives at work 20 minutes early every day. A given level of trip cost is also part of option 1. Option 2 offers the individual a later departure time at various levels of cost. By choosing option 2 early time is substituted for time at home. This process is referred to as "slack time substitution".

The second model also uses data gained in an SP experiment where respondents choose between an option in which they arrive at work on time with certainty and one in which they have some risk of being late. The latter alternative is cheaper than the former.

The Early time mode is defined as

$$U = \alpha \, EARLY + \delta \, Cost \tag{2}$$

and the Late time mode is defined as

$$U = \alpha \, LATE + \delta \, Cost + \eta \, Freq \tag{3}$$

where *Freq* = Frequency, or the number of late arrivals in a month.

Pells found that the cost of travel time variability on journeys to work is identified with the need to allocate a safety margin of slack time to the journey. He also notes that reductions in travel time variability are followed by a reduction in the amount of slack time allocated to the journey.

Pells' model is mainly concerned with commuters with a fixed arrival time. However, the model does not allow for travellers who have no arrival time constraints. Otherwise, the assumption that travel time is constant such as presented in the SP experiments is quite unrealistic.

The process of travellers allowing a safety margin is a result of variable travel times and also a clear consequence of the assumption that the early time at work is valued higher than the time at home.

For Polak (1987), a safety margin is the difference between the planned travel time (the amount of time which travellers allow to the journey) and the expected travel time (which is a result of travellers' previous experiences). The basic idea is that travellers have, *a priori*, an idea about the travel time which has the highest probability of occurring (expected travel time). The planned travel time may be simply the expected travel time (in this case the safety margin is zero) or the planned travel time may be larger than the expected travel time (and in this case the safety margin is bigger than zero).

There is a slight difference between Pells and Polak's definition of the safety margin. Polak is talking about variations in travel time and Pells is talking about the effects of variations in travel time, or the allocation of a slack time in order to prevent the probability of being late. Polak's definition is a more comprehensive definition since the size of the safety margin has been associated with the spread of travel time.

The mean-standard deviation approach – Jackson and Jucker's model

Since its application to problems of portfolio selection by Markovitz (1952a and 1952b) and Tobin (1957/1958), the mean-standard deviation criteria (μ, σ) has become the most frequently used two-parametric approach. The basic idea is to define utility as a function of expected travel time (mean travel time) and variability in travel time (standard-deviation).

In transport,the two-parametric approach was introduced originally by Jackson and Jucker (1982). It assumes that every traveller has an *a priori* estimate of the mean and the variance of travel time related to a specific origin-destination pair. The objective of each traveller is then to

$$\textit{Minimize } U = \alpha\, E(t) + \tau\, V(t) + \delta\, C \qquad (4)$$

where τ is a parameter that measures the influence of variance in travel time; $E(t)$ is the expected travel time for each origin-destination pair; $V(t)$ is the variance of travel time and C is cost. Jackson and Jucker assume that parameter τ should be positive. However, in theoretical terms τ may also be negative since some individuals may like variability provided that it is a fair gamble. These individuals have been defined as risk-prone according to expected utility theory.

A very basic assumption underlying Jackson and Jucker's approach is that travellers are trading-off mean travel time and travel time variability (variance).

From expected utility approach to a mean-standard deviation approach

Expected utility theory explores the observation that different individuals have different attitudes to risk. Some individuals are *risk averse*, continually paying high premiums to insure themselves against any possibility of bad outcomes, even when bad outcomes may have little chance of occurring. Individuals are *risk prone* when, instead of paying, they receive a risk premium for accepting risk. *Risk neutral* individuals are indifferent to risk.

The expected utility approach is presented in terms of a lottery. Related to any lottery there are two expectations:

1. the expected value (expected travel time), and

$$E(t) = \Sigma_t \, p(t) \, t \tag{5}$$

2. the expected utility

$$E[U(t)] = \Sigma_t \, p(t) \, U(t) \tag{6}$$

Expected travel time is the average payoff in travel time terms that results from the lottery. If the traveller entered the lottery many times she or he would expect to have on average the expected travel time $E(t)$ for each play of the lottery.

Related to the expected utility of a lottery is its certainty equivalent, t_c, that is the level of travel time accepted by a decision-maker to enter the lottery. The utility of the certainty is equivalent is equal to the expected utility, or

$$U(t_c) = E[U(t)] \tag{7}$$

More details about the certainty equivalence may be found in French (1986).

By assuming that uncertainty in travel time may be compared to a lottery, travellers win when there is a travel time smaller than the expected travel time. The expected value of the lottery is the sum of the outcomes, each multiplied by its probability (p) of occurrence. Formally,

$$E(t) = pt_1 + (1 - p)t_2 \tag{8}$$

where t_1 and t_2 are different travel times.

- *Risk neutrality.* A traveller is said to be risk neutral if he equates the expected value of the lottery and the utility of the expected value, i.e., if

$$U[pt_1 + (1 - p)t_2] = pU(t_1) + (1 - p)U(t_2) \tag{9}$$

  This condition shows that the traveller is only interested in the expected value of time and s/he is totally obvious to risk. S/he is indifferent between any two lotteries. If we specify a general (dis)utility function of the following form

$$U = \alpha t^{\beta} \tag{10}$$

the value of β is 1 for risk-neutral individuals.

- *Risk aversion.* A traveller is said to be risk averse if the utility of the expected value of utility. Formally,

$$U[pt_1 + (1 - p)t_2] < pU(t_1) + (1 - p)U(t_2) \tag{11}$$

This implies that a traveller prefers a certain outcome to an uncertain one with the same expected value.

If the equation above is valid for all $0 < p < 1$ and all t_1 and t_2 within the domain of the utility function, then the function is strictly convex which means that $\beta > 1$.

It is possible to say that risk-averse individuals do not take part in an unfavourable or barely fair gamble. If penalties for delays are high, risk-averse individuals are willing to pay high premiums to insure themselves against any possibility of delay. Travellers allowing a safety margin to their journal time is an example of individuals acting as risk-averse.

- *Risk proneness.* A traveller is said to be risk-prone relating to an uncertain travel time if the utility of its expected value is bigger than the expected value of its utility. Formally,

$$U[pt_1 + (1 - p)t_2] > pU(t_1) + (1 - p)U(t_2) \tag{12}$$

This condition implies that a traveller prefers an uncertain outcome (provided a fair gamble) to a certain one with the same expected value. The utility function is strictly concave.

The quadratic model

The polynomial of second degree has been suggested by Polak (1987) who provides an alternative approach to the traveller's decisions under risky conditions. The study is a combination of the risk preference approach and the arrival time cost approach and the result is a model that defines the safety margin as a function of the travel time distribution, the costs of time spent at work (destination) early and late, and the degree of risk aversion.

Polak's study consider utility as a polynomial of second degree:

$$U = \alpha_1 t + \alpha_2 t^2 + \delta C \tag{13}$$

where t is travel time.

The model proposed by Polak has already being proposed in the economic literature (Richter 1959/1960; Borch, 1963; Tobin 1957/1958 and 1967; Roy

1952; Sinn 1983) According to Sinn (1983), this approach is a combination of the expected utility approach and the mean-standard deviation approach. The literature shows that both criteria coincide if the utility function is a polynomial of second degree. Richter (1959/1960) shows that applying the expectation operator to a polynomial of second degree gives:

$$E[U(t)] = \alpha_1 E(t) + \alpha_2 E(t^2) \tag{14}$$

Applying some properties of variance and including a cost variable we have the model

$$E[U(t)] = \alpha_1 E(t) + \alpha_2 E^2(t) + \alpha_2 \sigma^2(t) + \delta C \tag{15}$$

An important point about Polak's model is that, if it is assumed to be the correct model, the omission of the term $[E(t)]^2$ may bias the estimates of α_2, with implies the utility is not just written as a function of mean and standard deviation. If the utility expression is an n^{th} degree polynomial in travel time, moments as high as the n^{th} must be considered. With a quadratic utility function, for instance, the expected utility is a function of only the first two moments of travel time.

According to Richter (1959/1960), under expected utility maximization there are two ways of restricting the order of relevant moments (as travel time allocation criteria):

- by restrictions on the utility function, and
- by restrictions on the probability distribution.

By considering similar arguments of portfolio selection such as used by Tobin (1957/1958), the choice of travel time allocation of an expected-utility-minimizing traveller can be analyzed in terms of the two parameters, mean and variance of his/her subjective probability distributions of travel time from alternative travel time if one or both of the following assumptions is met:

i) the traveller's utility function is quadratic or
ii) s/he regard time (t) as normally distributed.

In the absence of condition (i), the second is required. That is, indeed, an important point since the quadratic utility function has some undesirable properties such as having a maximum or minimum. Otherwise, this property allows one to search for the best fit model (that may have any shape) rather than assuming an approximation such as is the case of the polynomial of second degree.

Polak (1987) also proposed an exponential utility function

$$U(t) = -e^{\alpha t}$$

that in some circumstances would be more appropriate to the travel context. More details about this model may be found in Polak (1987).

Senna's model

This section introduces an alternative approach for defining the utility function (Senna 1991, 1992, 1994). The approach is based on the relationship between the expected utility approach and the mean-standard deviation approach such as Polak's model, but it provides a less restrictive functional form.

The analysis starts by assuming the basic function defined by equation 1:

$$U = \alpha t^{\beta} + \delta C$$

which covers the possibility of individuals being risk-averse, risk-neutral or risk-prone.

This function is not the only one that conforms with the expected utility approach. A polynomial of degree "n", for instance, would also be considered, by because of some obvious correlation between t and t^n, equation 1 is the most appropriate. Applying the expectation operator to equation 1 and also some additional work presented in appendix 1, we have the basis of Senna's model:

$$E(U) = \alpha E(t^{\beta}) = \alpha\{[E(t^{\beta/2})]^2 + [\sigma(t^{\beta/2})]^2\} + \delta C \tag{16}$$

The estimated function it is not two separable terms involving mean and variance as seems to be typically estimated in practice by models following Jackson and Jucker's approach.

The empirical study

To illustrate how travel time variability can be evaluated as a determinant of travel choice, we have designed a stated preference experiment. In this section we briefly review previous attempts to measure time variability and then outline the experimental design. We follow this with an application.

How to present the problem of travel time variability to the respondents

Previous studies have suggested different ways of presenting the idea of travel time variability. From a modelling point-of-view, any measure of spread such as standard deviation, variance or coefficient of variation may provide the necessary inputs to run the alternative models. However, from a respondents' point-of-view the technical dimension must be translated into common use language.

Jackson and Jucker (1982) suggest the concept of *usual time* as a location measure for hypothetical time distribution. Usual time is defined as "the approximate time it takes most of the time". This concept, the mode of the distribution, is a more intuitive concept than mean travel time (Jackson and Jucker 1982). The measure of variability was associated with the concept of "possible delays" which means delays that might occur due to accidents, congestion, etc.

The respondents were told to assume that they could not switch route to avoid delays or predict when the delays would occur. The dimension of variability was then related to both magnitude and frequency. Frequencies such as once a week, once every two weeks and once a month were assumed to present the idea of travel time variability in an appropriate way. An example of the commuting alternatives is presented in Table 1.

Table 1. Pairwise comparisons presented by Jackson and Jucker (1982).

Card		Alternative 1	Alternative 2
1	Usual time:	30 minutes	20 minutes
	Possible delays:	none	5 minutes a week

The main advantage of Jackson and Jucker's experiment is that they found a way of presenting respondents with a very simple representation of travel time and travel time variability which may be easily expressed by formal statistical terms such as mean and variance.

Bates et al. (1987) and Johnston et al. (1989) propose an SP experiment based on three variables: departure time, (mean) journey duration and travel time variability (measured by the standard deviation of the distribution of journey duration, given a departure time). Although the original proposal was to base the analysis on the mean-standard deviation of journey time, the final design presented respondents with their "usual time" (interpreted as the mode of the distribution) and a "bad travel time" (interpreted as the 90% percentile, or happening once a fortnight on average). This approach is similar to that of Jackson and Jucker.

Benwell and Black (1984) make a distinction between reliability and punctuality for a sample of rail travellers. Reliability is defined as the amount of lateness plus the probability of its occurring. Punctuality is defined as the probability of arriving on time. 75% of the respondents said that an early arrival time would cause no inconvenience, implying no arrival time constraints.

Benwell and Black emphasizes the quality of the visual presentation of the questions. 384 respondents were offered a profile of the lateness associated with a representative 10 trains journeys. Prior to the SP questions the respondents were presented with three simple options. All the alternatives have the same mean (M) of 5 minutes lateness:

A	{0,0,5,6,8,7,6,4,5,9}	M = 5	$\sigma = 2.86$
B	{0,0,0,0,0,0,25,5,10,10}	M = 5	$\sigma = 7.75$
C	{0,0,0,0,0,0,0,0,20,30}	M = 5	$\sigma = 10.25$

The results indicate that 56% of the respondents choose option C, 38% choose option A and only 6% choose option B. The analysis suggests that respondents prefer a low probability of lateness (punctuality) and, given unpunctuality, the respondents prefer the alternative,with the smallest amount of lateness.

The choice of alternative C as the most preferable, in spite of its high variability suggests that travellers prefer more variability than less. This is new evidence that respondents have few penalties for delays. These results are compatible with the concept of risk-proneness and some of the results found in the current study.

Pells (1987), whose approach is based on the safety margin hypothesis, developed two SP experiment, one trading-off early arrival time and costs, and the other trading-off late arrival time and costs. An example of the early experiment is presented in Table 2.

Table 2. Pairwise comparisons presented by Pells (1987).

Card	Alternative 1	Alternative 2
1	20 minutes early every day Cost: 40 pence per day	15 minutes early every day Cost: 50 pence per day

The SP experiment

The SP experiment design identify individuals' valuation of expected (mean) travel time and travel time variability. Three levels of mean travel time and three levels of travel time variability have been considered.

The first step in the design is to specify M1, M2 and M3 as three levels of difference in expected travel time representing, respectively, low, medium and high levels of difference in mean travel time. Now, consider V1, V2, and V3 to be the three levels of difference in travel time variability representing, respectively, low, medium and high levels of difference in travel time variability. The cost variable is defined by boundary (see appendix 3) values related to expected travel time and travel time variability. It is important to present in all cards an alternative related to certainty (or regularity) which is represented by either (i) a standard deviation equal to zero or (ii) all journeys presenting the same travel time. This simple procedure contrasts the idea of regular journeys with variable travel times. Otherwise, it would be too difficult to have variability in both options A and B.

To guarantee independence between mean travel time and travel time variability (orthogonality) a main effects test with 9 combinations is considered (i.e., 3^2).

Boundary values may be divided into *fixed* and *variable*. If the level of variability remains unchanged, then the boundary value of time is said to be fixed and vice-versa. If the level of variability also changes across the option, then the boundary value is called a variable boundary value. Fixed boundary values are defined for each of the nine replications (six related to variability and 3 related to mean travel time). Three additional replications are included in order to check the design for flexible boundary values. The design is orthogonal (from replication 1 to 9) in differences on the attributes between alternatives.

Table 3 presents the design for the differences between the attributes. The boundary values which are defined by equations 18 and 19 are the basis for the definition of the differences in costs (DC's) presented in Table 3.

The actual levels of M and V depend on the respondent's actual journey, obtained prior to revising the SP experiment. The questionnaires were customized according to five different levels of actual travel time. The main objective of customizing the questionnaire is to reduce the difference between the hypothetical question that has been asked and the actual travel time

Table 3. The basic design of the main SP experiment.

Cards	Option A		Option B		Difference in costs
	Mean	Variability	Mean	Variability	
1	M1	V1	M1	V2	DC1
2	M1	V1	M2	V1	DC2
3	M1	V1	M3	V3	DC10
4	M2	V1	M1	V4	DC11
5	M2	V1	M2	V2	DC3
6	M2	V1	M3	V1	DC8
7	M3	V1	M1	V1	–DC5
8	M3	V1	M2	V2	DC12
9	M3	V1	M3	V4	DC7
10	M1	V1	M1	V3	DC4
11	M2	V1	M2	V3	DC6
12	M3	V1	M3	V4	DC9

experienced by the respondents. Table 4 presents the different ranges assumed for each level of travel time.

The different levels of variability were considered in terms of percentages related to the different levels of mean travel time assumed by M2 in Table 4. An example from Table 4 is range (1) that comprises a man travel time varying from 10 minutes to 20 minutes.

Table 4. Ranges of travel time.

	Range (minutes of travel time)				
	(1) 10–20	(2) 20–30	(3) 30–40	(4) 40–50	(5) 50–70
M1	10	20	30	40	50
M2	15	25	35	45	60
M3	20	30	40	50	70

The different levels of variability for each range of travel time is presented in Table 5. The actual levels of attributes in Tables 4 and 5 are substituted in Table 3 according to the different levels of actual travel time. The final values presented to the respondents are customized by journey purpose, arrival time constraint and actual travel time.

Table 5. The different levels of variability presented to the respondents.

Levels of variability (standard deviation)	Mean travel time (minute)				
	15	25	35	45	60
V1 (0%)	0	0	0	0	0
V2 (20%)	3	5	7	9	12
V3 (30%)	4.5	7.5	10.5	13.5	18
V4 (50%)	7.5	12.5	17.5	22.5	30

The boundary values related to (mean) travel time (BVOT) are the result of $V_A - V_B = 0$ and then the value of time is only a function of mean travel time and cost. Similarly, the Boundary values of variability (BVOV) are the result of $t_A - t_B = 0$, and then the value of variability is only a function of variability.

In Table 3 the differences in costs related to mean time are defined by (fixed) boundary values which are presented in Table 6.

Table 6. Boundary values of (mean) travel time.

Card		Boundary values BVOT Cr$/minute	Difference in costs BVOT * $(t_1 - t_2)$ (Cr$)
2	M1 ► M2	0.6	DC2
6	M1 ► M3	3.8	–DC5
7	M2 ► M3	1.5	DC8

In Table 6 BVOT is calculated by

$$\text{BVOT} = \frac{C_1 - C_2}{T_1 - T_2} \tag{17}$$

The fixed boundary values associated with travel time variability are presented in Table 7.

Table 7. Boundary values of travel time variability.

Card		Boundry values BVOT (Cr$/minute)	Difference in costs (Cr$/minute)	
			Low	High
1	V1 ► V2	0.31	DC1	–
5	V1 ► V2	1.26	–	DC3
10	V1 ► V3	0.67	DC4	–
11	V1 ► V3	2.90	–	DC6
9	V1 ► V4	1.10	DC7	–
12	V1 ► V4	4.27	–	DC9

In Table 7 BVOV is calculated by

$$\text{BVOV} = \frac{C_1 - C_2}{V_1 - V_2} \tag{18}$$

DC10, DC11 and DC12 are differences in costs defined by flexible (variable) boundary values which are presented in cards 3, 4 and 8 of the proposed design. Thus, let us assume BVOV = k * VOT, where k may be any value such as 0.25, 1, or 2. Thus, BVOV is

$$\text{BVOT} = \frac{C_1 - C_2 - \text{K} \times \text{BVOT}\,(V_1 - V_2)}{T_1 - T_2} \tag{19}$$

The experiment

The survey was administrated on a sample of commuters and non-commuters with fixed or flexible arrival times. Each respondent was presented with a scenario in which travel time shows great regularity, implying the idea of no variability in travel time and a high probability of arriving at the destination at the expected arrival time. Together with the cost of the daily trip this forms option 1. Option 2 offers different levels of mean travel time to option 1 and different levels of variability. Costs are also associated with each option. Travellers choose between the two options and select an outcome on a semantic scale. An example is shown in Table 8.

Table 8. The proposed cards – an example of the cards presented to the respondents.

Option	Journeys (minutes)					Mean	Cost*	Card 1	Choice
	1	2	3	4	5				
A	20	20	20	20	20	20	35.00	Definitely choose A Probably choose A Indifferent	
B	20	30	45	20	35	30	27.00	Probably choose B Definitely choose B	

The numbers 1, 2, 3, 4 and 5 relate to the days of the week if a respondent is a commuter, or the first, second, . . . , fifth, trip to a specific destination if the respondent is a non-commuter. By presenting travel time on a day-to-day basis the experiment provides suitable data to obtain estimates of β. The choices are made on a semantic scale which is transformed either into a probabilistic scale (Bates and Roberts 1983; Bates 1984) or simply taken to represent discrete choice.

Pilot surveys were undertaken in Leeds (UK) and Porto Alegre (Brazil) to evaluate the suitability of alternative design responses and ways of collecting the data. The pilot studies suggest that:

- respondents preferred pairwise comparisons to a full ranking of 10 alternatives and each option should contain no more than 5 travel times;
- interviews at the workplace were more efficient than household surveys; in a single place it would be possible to interview several respondents; and
- questions related to an individual's non-work journeys could be asked to people at their workplace.

In the main survey, 500 questionnaires were distributed among workers whose workplace was located near the Assis Brasil corridor, in Porto Alegre city, Brazil. This corridor carries approximately 100,000 passengers/hour in peak periods.

The number of questionnaires returned was 319, representing 0.3% of the total number of travellers at the peak period. 18 questionnaires were eliminated because they were incompletely answered or respondents did not accept the trade-offs. The remaining 301 questionnaires represent approximately 64% of the total questionnaires distributed.

The survey covered respondents with different kinds of arrival time constraints, different penalties for delays and different levels of income and education. Commuters were asked to answer the hypothetical questions of the SP experiments considering the constraints of their actual job. Some respondents were asked to answer the questionnaire considering any other destination which would not be work. Thus, they could choose journeys to the theatre, cinema, football, shopping, to visit relatives, etc.

The modelling analysis

Table 9 presents the main empirical findings.

Table 9. Senna's model.

Variable	Commuters		Non-commuters	
	Fixed arrival time $\beta = 0.5$ ($\beta/2 = 0.25$)	Flexible arrival time $\beta = 1.4$ ($\beta/2 = 0.7$)	Fixed arrival time $\beta = 1.4$ ($\beta/2 = 0.7$)	Flexible arrival time $\beta = 1.4$ ($\beta/2 = 0.7$)
$\{[E(t^{\beta/2})]^2 + [\sigma(t^{\beta/2})]^2\}$	−0.567566 (−5.69)	−0.033141 (−5.56)	−0.025098 (−5.93)	−0.020173 (−5.41)
Cost	−0.033972 (−9.23)	−0.082343 (−6.17)	−0.056816 (−6.52)	−0.048951 (−5.41)
Log-likelihood	−1090.50	−205.57	−313.64	−399.15
Rho-bar square %	5.90	14.77	11.79	8.44
N.O. Respondents	143	31	45	53
Observations	1672	348	513	629

For all segmentation the parameters are of the right sign and highly significant. Rho-bar squared is relatively low, particularly for commuters with fixed arrival time. Indeed, SP models typically present low Rho-bar squared (Wardman 1991). The value of β varies from $\beta = 0.5$ (for commuters with fixed arrival time) to $\beta = 1.4$ (for the other journey purposes and arrival time constraints). Commuters with fixed arrival time have $\beta = 0.5$ which implies risk-proneness. Note that commuters are frequently travelling in the same route and this situation provides them information about the distribution of travel time. On the other hand, approximately 70% of commuters with fixed arrival time have no penalties for delays and they are therefore more comfortable to behave as risk-prone.

Commuters with fixed arrival time and non-commuters with fixed and flexible arrival time are risk-averse ($\beta = 1.4$).

The value of time and the value of travel time variability from the models

The value of time is defined by the marginal cost of time:

$$\text{VOT} = \frac{\partial U/\partial t}{\partial U/\partial C} \tag{20}$$

For the general function $U = \alpha t^{\beta}$ the value of time is:

$$\text{VOT} = \frac{\partial U/\partial t}{\partial U/\partial C} = \frac{\alpha\beta t^{\beta-1}}{\phi} \tag{21}$$

The values of time and variability depend on the function of form of the different models considered. Appendix 2 presents the value of time and variability functions for the specific models. For all the models discussed in this study, the value of time is a *function* rather than a value and both the value of time and the value of variability functions depend on the distribution of travel time. The majority of models designed to obtain values of travel time savings assume a constant value of time savings. There are exceptions (e.g., Truong and Hensher, 1985). In the models discussed in this paper, the value of time depends on the probability distribution function of travel time.

Since both the value of time and the value of variability depend on the probability distribution function of travel time, it is essential to characterise some different distributions in order to get some values of time and variability. Four schemes are presented in this section. They will be considered for estimating the value of time and variability.

Let us assume two examples of projects in urban transport in which changes in mean travel time and in travel time variability are the main benefits of introducing such schemes. The first example would be a bus system currently operating in mixed traffic. The proposed project would be the introduction of a bus lane (which would be operated either by segregated carriageway or segregated by right of way). The current system is associated with a set of travel times which characterise a given distribution of travel time. The introduction of such schemes would change the current distribution from state A (see Table 10) to state B which characterise another distribution.

Table 10. Proposed schemes been analyzed (travel time in minutes).

Scheme	State A					$E(t)$	$\sigma(t)$	State B					$E(t)$	$\sigma(t)$
1	5	10	15	25	20	15	7.07	10	15	20	15	15	15	3.16
2	20	35	10	20	15	20	8.36	5	20	25	10	15	15	7.07
3	20	35	10	10	25	20	9.48	15	15	15	15	15	15	0.00
4	20	5	5	15	30	15	9.48	15	15	15	15	15	15	0.00

The second example would be the introduction of route guidance. The introduction of such a scheme would reduce the peak daily travel times for users of the system, especially those travellers who are not completely acquainted with the network.

The introduction of any of the above schemes would change the current distribution from one state (for instance, State A in Table 10) to another (state B). The introduction of these schemes could either change mean travel time variability, or both.

Schemes 1 and 4 present no change in mean travel time. However, in scheme 1 there is a small reduction in variability ($\sigma(t)$) but with variability remaining at lower levels. In scheme 2 there is a larger reduction in variability with the new state (B) showing no variability.

A practical way of calculating the value of time is to estimate the monetary equivalent (ME) of changing the expected utility from State A to State B:

$$\text{ME} = \frac{\Delta U}{\delta} \rightarrow \frac{E(U_A) - E(U_B)}{\delta} \qquad (22)$$

Table 11 presents the values of travel time variability derived from equation 22.

The values of travel time variability presented in Table 11 were estimated in two steps. First, variability was assumed to be equal to zero and then the value of the expected travel time (mean travel time) is estimated. Second, the value of variability is the difference between the value of time (total value of time) and the value of expected travel time (value of mean travel time). The negative sign of the value of time from Senna's model for schemes 1, 3 and 4, for commuters with fixed arrival time means that the scheme implies

Table 11. Values of travel time variability.

Senna's model	Commuters fixed arrival time	Commuters flexible arrival time	Non-commuters fixed arrival time	Non-commuters flexible arrival time
Scheme 1	–1.19*	2.08	2.29	2.13
Scheme 2	–0.46*	0.14	0.16	0.15
Scheme 3	1.39	3.68	4.04	3.77
Scheme 4	–2.38*	4.48	4.91	4.57

* In these cases reductions in variability makes the situation even worse (disutility is higher).

a "disbenefit" for those travellers. In other words, commuters with a fixed arrival time are willing to pay Cr$ 1.46 and Cr$ 2.38 for avoiding schemes 1 and 4. They reject the scheme being proposed.

The values of variability are systematically higher for commuters and non-commuters with flexible arrival time and for non-commutes with fixed arrival time. The only exception is for scheme 2 where non-commuters with flexible arrival time is the only case that the value of variability is greater than the value of variability for commuters with fixed arrival time.

Table 12 presents the values of travel time variability as a percentage of the value of time (total value of time). The value of variability as a percentage of the value of time shows some interesting features. The value of variability represents at least 100% of the value of time for schemes 1 and 4. The exceptions are commuters with fixed arrival time in Senna's model, but even so variability represents 81% and 76% of the value of time.

Table 12. Values of travel time variability as a percentage of the value of travel time for the schemes being considered (in %).

Senna's model	Commuters fixed arrival time	Commuters flexible arrival time	Non-commuters fixed arrival time	Non-commuters flexible arrival time
Scheme 1	81	155	155	155
Scheme 2	4	1	2	2
Scheme 3	17	32	33	33
Scheme 4	76	153	153	153

Conclusions and further research

The objective of this paper was to describe a method for valuing travel time variability. The method includes the development of a general framework which allow us to estimate the value of travel time variability, not only for commuters with fixed arrival times but also for non-commuters and any commuter with a flexible arrival time. Some schemes in transport may be reducing travel time variability without reducing mean travel time at significant levels, and current studies are ignoring such benefits.

There is growing evidence that travel decisions are taken under risk, or under circumstances where the decision-makers do not know with certainty the outcome of his/her decision. This is the reason why a more comprehensive framework for understanding travellers' decisions should be considered. This study provides a general framework to value travel time variability. The analysis, however, has raised some questions which deserve attention in future research.

Travel time variability has been presented as a set of travel times associated with daily trips to the same destination or sporadic trips to non-work destinations. Long term effects of variability have been omitted in this work and more research about alternative ways of introducing such effects are recommended.

The design of SP experiments in the context of non-linear utility functions is an appealing way of incorporating uncertainty. More research on other forms of uncertainty is necessary.

The definition of boundary values following non-linear functions request further investigation. Some initiatives in this direction have already been identified (for instance, Hensher and Truong 1985) but additional research is highly suggested.

In this study, journey time has been assumed to be the time between departure and arrival time. A consideration of journey time in a more disaggregate form such as including waiting time and in-vehicle time is desirable.

The absence of travel time variability may be perceived in most of the travel time choice models. Further research is required in order to better understand the effects of including travel time variability in the utility function for other travel choice models.

A tendency of incorporating variability in transport models is already observable in the literature. However, more emphatic attention must be given to this problem. Route choice and mode choice, for instance, seem to be obvious subjects for further investigation.

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Appendix 1

Applying the expectation operator to the basic function $U = \alpha t^{\beta}$ we have

$$E(U) = \alpha E(t^{\beta}) \tag{A1.1}$$

It is also possible to show that

$$E(t^{\beta}) = E(t^{\beta/2} * t^{\beta/2}) \tag{A1.2}$$

However, one of the properties of the expectation operator when X and Y are not independent is that

$$E(XY) = E(X) * E(Y) + Cov(X, Y) \tag{A1.3}$$

and $Cov(X, Y) = \Gamma_{x,y} * \sigma_x * \sigma_y$.

In the present case, $X = Y = t^{\beta/2}$, which implies $\Gamma_{x,y} = 1$.

Similarly, $\sigma_x * \sigma_y = [\sigma(X)]^2 = [\sigma(t^{\beta/2})]^2$

and $E(X) * E(Y) = [E(X)]^2 = [E(t^{\beta/2})]^2$.

Thus, by substituting these values in equation 1 we finally have:

$$E(U) = \alpha E(t^{\beta}) = \alpha\{[E(t^{\beta/2})]^2 + [\sigma(t^{\beta/2})]^2\} \tag{A1.4}$$

Appendix 2. The mean-variable approach.

Before presenting the value of time and the value of variability function it is important to present the basic expression that will be frequently used in the paper. The expression refers to the derivative of an expected operator which is defined by:

$$\frac{\partial[E(t^{\beta})]^{n}}{\partial t} = n[E(t^{\beta})]^{n} * \beta t^{\beta-1} \tag{A2.1}$$

This expression is also valid for Variance because variance is defined by the expected value of $[t - E(t)]$. For Jackson and Jucker's model the value of time function is defined by

$$\text{VOT} = \frac{\dfrac{\partial E(U)}{\partial t}}{\dfrac{\partial E(U)}{\partial C}} = \frac{\alpha[E(t^{\beta})] * \beta t^{\beta-1} + 2\tau[\sigma(t)]^{2}}{\delta} \tag{A2.2}$$

and the value of variability is defined by

$$\text{VOV} = \frac{\dfrac{\partial E(U)}{\partial[\sigma(t)]^{2}}}{\dfrac{\partial E(U)}{\partial C}} = \frac{\dfrac{\partial E(U)}{\partial t} * \dfrac{\partial t}{\partial[\sigma(t)]^{2}}}{\delta} \tag{A2.3}$$

Note that

$$\frac{\partial[\sigma(t)]^{2}}{\partial t} = 2\tau[\sigma(t)]^{2} \quad \therefore \quad \frac{\partial t}{\partial[\sigma(t)]^{2}} = \frac{1}{2\tau[\sigma(t)]^{2}} \tag{A2.4}$$

The value of variability function then becomes:

$$\text{VOV} = \frac{\dfrac{\alpha[E(t^{\beta}) * \beta t^{\beta-1}] + 2\tau[\sigma(t)]^{2}}{2\tau[\sigma(t)]^{2}}}{\delta} \tag{A2.5}$$

In the case of Jackson and Jucker's model $\beta = 1$. A very important aspect to be emphasized is that the value of time and the value of travel time variability depend on the distribution of travel time.

Appendix 3

This appendix is based on an example presented by Fowkes (1991). Let us assume that the utility function is described by the attributes time (t) and cost (c):

$$U = \alpha t + \Phi c \tag{A3.1}$$

If two alternative are compared ($i = 1$ and $i = 2$) we have

$$U_1 - U_2 = \alpha(t_1 - t_2) + \Phi(c_1 - c_2) \tag{A3.2}$$

At the point of indifference between the alternatives $U_1 = U_2$,

$$\frac{\Phi}{\alpha} = \frac{t_1 - t_2}{c_1 - c_2} \tag{A3.3}$$

where Φ/α is the boundary value of time expressed in terms of cost. If there are no random effects it may be said that a traveller whose value of t in terms of c is greater than Φ/α will prefer the alternative with the smallest t, and vice-versa. The main feature of using boundary values is to define a range of (boundary) values to allow for different values people have, and to get reliable estimates of these.

The case presented above is a simple case involving only two variables. In the present case three variables have been considered. Thus, the generalised costs can be presented as

$$\text{VOT}.T_1 + \text{VOV}.V_1 + c_1 = \text{VOT}.T_2 + \text{VOV}.V_2 + c_2 \tag{A3.4}$$

where VOT is the value of mean travel time and VOV is the value of travel time variability. By rearranging this equation the general equation for the boundary value of time becomes

$$\text{BVOT} = \frac{c_1 - c_2 - \text{VOV}(V_1 - V_2)}{t_1 - t_2} \tag{A3.5}$$

where BVOT is the boundary value of time. Similarly, the boundary value of variability (BVOV) is defined by

$$\text{BVOV} = \frac{c_1 - c_2 - \text{BVOV}(t_1 - t_2)}{V_1 - V_2} \tag{A3.6}$$

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