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An Option Pricing Approach to the Valuation of Real Estate Contaminated with Hazardous Materials

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Abstract

This paper uses option pricing to examine how the presence of hazardous materials affects real estate value. The property owner has two options. The first option is to remove the hazardous materials at the best time. The second option, embedded in the first one, is to redevelop the property at the best opportunity. The owner has three possible timing strategies with respect to the exercise of these two options: remove the hazardous materials first and retain the option to redevelop the property later, remove and redevelop at the same time, or do nothing. Conditions under which the presence of the hazardous materials may either expedite or postpone the decision to redevelop are also derived. If the regulatory environment does not allow the property owner to make optimal timing decisions with respect to the exercise of these options, then our results provide an indication of the cost of regulation as measured by the additional loss in property value.

Key Words: hazardous materials, option pricing, optimal timing

Properties containing asbestos and other types of hazardous materials[1] are a major environmental problem for the real estate industry in this era of environmental consciousness. The economic impact can be considerable in terms of reductions in rents and selling prices, increased vacancies, higher operating and transactions costs, and actual and potential lawsuits. The owner of contaminated property may also be held strictly liable for cleanup and related costs. Economic tradeoffs between the benefits gained from removing the hazardous materials and restoring the property to a normal state, on the one hand, and the often high costs of removal and disposal of these materials, on the other hand, render the choices available important factors affecting the value of the property.

The conventional approach to the valuation of income-producing properties containing hazardous materials is to use discounted cash flow (DCF) models. As done by Dewees (1986), the value of the property is first estimated as if clean. The discount from the "as if clean" value due to the presence of the hazardous materials is then estimated. Fisher, Lentz, and Tse (1992) developed a DCF model to show how the value of a commercial property containing asbestos varies as a function of the timing of the removal of the asbestos, and then to measure the additional loss in value resulting from removal at suboptimal times.

The major deficiencies of the DCF approach, as mentioned by Brennan and Schwartz (1985), are neglect of the stochastic nature of cash flows and of the value added by the capability of owners of real assets to respond to opportunities that may arise to improve cash flows and increase property values. The option pricing framework can explicitly incorporate into the estimate of property value the options available to a property owner to respond to stochastically varying cash flow expectations with actions that can increase the value of the property. Quigg (1993) found empirical evidence supporting the explanatory value of option pricing models in pricing land available for development.

1. Related Studies and Research Objectives

A number of recent studies have extended the financial option pricing model (OPM) to price real assets. Brennan and Schwartz (1985) demonstrated how the OPM can be employed to value natural resource projects and to derive optimal decisions with respect to the management of such projects. Paddock, Siegel, and Smith (1988) extended the OPM to value claims on an offshore petroleum lease, taking into account differences between financial and real assets. More recently, Williams (1991) utilized the option pricing framework to derive the optimal time and density of real estate development and also to estimate the value added by development. Capozza and Sick (1991a) employed the option pricing framework together with spatial models to price agricultural land which can be converted to urban uses through risky development. Capozza and Sick (1991b) also apply the option pricing framework to value leased properties, showing that the value of the option to upgrade or redevelop the property to a lessee is circumscribed since its claim to the property is only for a limited time. Quigg (1991) adopted the OPM framework to price land, where the value of the land includes an option to wait to develop the land.

This paper uses the option pricing framework to estimate the value of income-producing real estate containing hazardous materials, explicitly taking into account choices available to the property owner concerning how to improve the economic performance of the property. The owners of properties contaminated with hazardous materials have two options. The first is to remove the hazardous materials at the optimal time. The second is to redevelop the property to a higher and better use at the optimal time.[2] Following prevailing practice and generally in accordance with legal requirements, we assume that the second option cannot be exercised unless or until the first option has been exercised. Thus the option to remove the hazardous materials is a compound option in that removal of the materials releases an option to redevelop the property from its current use to a higher and better use. The analytical solutions obtained in this paper for the value of the property build on the option pricing results obtained for land development by Capozza and Sick (1991a), Quigg (1991), and Williams (1991), the exchange option approach of Fischer (1978) and Margrabe (1978), as well as standard results obtained from financial option pricing theory. The novel results in our paper pertain to the interrelations between the two options.

In a number of property contamination situations, the options available to the property owner regarding when to remove the hazardous materials from the property and how and when to redevelop the property are limited by regulatory authorities. This is especially likely if the contaminated properties fall under superfund, or superfund-type legislation, or if the hazardous materials pose a serious danger to building occupants.[3] In these cases, the type and timing of response actions may be mandated by public authorities. Yet, in other cases, such as cases in which buildings are burdened with asbestos-containing materials

(ACMs), the property owner generally retains a large amount of discretion.[4] Where the options available to the property owner to maximize the value of the property are restricted by law or regulation, the method of analysis employed in the paper can be used to assess the decrease in property value resulting from the loss of one or more of these options.

Section 2 relates the options that an owner has to deal with a contaminated property to the value of the property. Standard option pricing methodology is adapted to this real property situation to derive a general valuation model for the contaminated property. This option pricing model is then used to derive valuation models for alternative removal and redevelopment strategies. The model is also extended to develop optimal timing rules for the exercise of each option.

Section 3 examines the interactions between the two options. Conditions are derived under which the optimal time to remove the materials is hastened or postponed by the option to redevelop the property, and likewise the optimal time to redevelop is hastened or postponed by the need to first remove the materials. Section 4 describes the loss in property value due to the presence of the hazardous materials. Section 5 presents an illustrative example of the method of analysis developed in the earlier sections. Section 6 discusses the limitations of the model developed in the paper and offers suggestions for possible future extensions. Section 7 concludes the paper.

2. Property Valuation with the Option Pricing Model

Loosely following Capozza and Helsley's (1989) decomposition of the value of urban land, we decompose the value of a contaminated property into the following three additive components:[5] (1) the value based on the cash flows generated if the hazardous materials are retained in place, (2) the value of the option to remove the hazardous materials (the option to remove), and (3) the value of the option to redevelop the property to a higher and better use (the option to redevelop). The first component of property value represents the value of the property with the hazardous materials in the building, which would be depressed to the extent that the presence of the hazardous materials reduces the expected net cash flows of the property. The second component, the option to remove, provides the property owner with an opportunity to increase the value of the property by reducing, if not eliminating, the risks stemming from the hazardous materials. The third component, the option to redevelop, allows the property owner to increase the value of the property by renovating the property to upgrade the existing use (e.g., upgrading from an Class B to a Class A office building) or by converting the property to a higher and better use. Rather than explicitly model two redevelopment options (i.e., renovation and conversion), we assume one general redevelopment option.

2.1. Option Pricing Assumptions

Let the free and clear cash flow of a similar but clean property (i.e., a property free of any hazardous materials) be defined as x.[6] The cash flows of the contaminated property before and after removal are $\varphi_1 x$ and $\varphi_2 x$, respectively, where φ_1 and φ_2 and proportionality factors, $0 \leq \varphi_1 \leq \varphi_2 \leq 1$.[7] The factor φ_1 is expected to be less than φ_2 and greater than zero, and generally the factor φ_2 is expected to equal 1. If $\varphi_2 < 1$, the implication is that cleaning up the property is not expected to fully restore the cash flows of the property to normal (clean) levels, possibly due to lags in contract adjustments or to some form of lingering taint that remains with the property. Assume further that the cash flow of the property after redevelopment is some multiple of the cash flow of the property in its original (clean) use, ϕx, where $\phi \geq 1$.[8] The property is assumed to have an infinite economic life, and thus options on this property are perpetual.[9]

The exercise price of the option to remove is the total clean-up cost.[10] Let C be the total clean-up cost for the contaminated property. The total clean-up cost consists of the removal cost (the costs associated with removing and safely disposing the hazardous materials) and the restoration cost (the costs associated with restoring the property to a normal functional state).[11] The exercise price of the option to redevelop is the total redevelopment cost.[12] Let R be the total redevelopment cost. For purposes of simplification, we assume that the removal cost and the restoration costs are proportional to the redevelopment cost R; they are defined, respectively, as $\alpha_1 R$ and $\alpha_2 R$, where α_1 and α_2 are positive fractional proportions. Therefore, the total clean-up cost, C, is $\alpha_1 R + \alpha_2 R$.

The property owner would exercise the option to remove only if the expected increase in cash flows and reduction in economic risks after removal outweigh the expected clean-up costs. The option to redevelop would be exercised only if the increase in future cash flows expected from redeveloping the property outweigh the expected total redevelopment cost.

Following Williams (1991), the property cash flows, x, and the redevelopment cost, R, are assumed to follow a stochastic Wiener process, expressed as

$$dx/x = \mu_x dt + \sigma_x dz_x, \text{ and}$$

$$dR/R = \mu_R dt + \sigma_R dz_R,$$

where μ_x and μ_R are the instantaneous growth rates of x and R.

Our approach to the construction of a hedge position for the cash flows x is closely related to that of Margrabe (1978). We assume a portfolio of traded securities, P, exists that will allow us to form a hedge position for the cash flow, x, of the clean property. This traded portfolio is assumed to follow a stochastic process similar to x, as follows:

$$dP/P = \mu_P dt + \sigma_P dz_P,$$

where μ_P is the instantaneous return on the portfolio. Moreover, to allow for the possibility that traded securities offer only an imperfect hedge against the variability of cash flows of a real asset, as in Quigg (1991) and Shimko (1992), the stochastic element dz_P is assumed to be only partially correlated with dz_x in accordance with the following relation:

$$dz_x = \rho_{xp} dz_p + (1 - \rho_{xp}^2)^{1/2} dz_{pe},$$

where ρ_{xp} and ρ_{xe} are the correlation coefficients. The random variable dz_{pe}, normally distributed with mean zero and variance dt, is assumed to be unsystematic; therefore, it will not be priced.

Our approach to the construction of a hedge position for the redevelopment cost R follows Fischer (1978). For the redevelopment cost R, we assume a portfolio of traded securities, K, which is partially correlated with the redevelopment cost such that

$$dK/K = \mu_k dt + \sigma_k dz_k, \text{ and}$$

$$dz_R = \rho_{Rk} dz_k + (1 - \rho_{Rk}^2)^{1/2} dz_{ke},$$

where μ_k is the instantaneous growth rate of K, dz_k is N(0, dt), and ρ_{Rk} is the correlation coefficient. The random variable dz_{ke}, like dz_{pe}, is N(0, dt), unsystematic, and not priced.[13]

2.2. *Valuation of the Property as Clean*

Because we assume that the hazardous materials must be removed from the property before it can be redeveloped, the option to redevelop can be said to be embedded in the option to clean. Consequently, we must backwards solve for the value of the contaminated property. That is, we must first value the property after clean up has been completed, and then work backwards from the value after clean up to obtain the value of the contaminated property before removal.

The value of the clean property includes an option to redevelop. The value of this property is a function of the underlying cash flows (x) and the exercise price of the option to redevelop (R), V(x, R). Applying Ito's lemma to V(x, R), the value of the property evolves according to

$$dV = [xV_x\mu_x + RV_R\mu_R + 0.5(x^2V_{xx}\sigma_x^2 + R^2V_{RR}\sigma_R^2 + 2xRV_{xR}\sigma_{xR})]dt$$
$$+ xV_x\sigma_x dz_x + RV_R\sigma_R dz_R, \quad (1)$$

where σ_{xR} is the covariance between x and R and is given by $\rho_{xR}\sigma_x\sigma_R$.

Next we form a risk-free hedge position by holding one call (the property), *m* shares of the hedge portfolio P short, and *n* shares of K short. Gains (losses) to the long position come from the increase (decrease) in the call value and the increases (decreases) in the instantaneous cash flow from the property. However, increases (decreases) in the prices of the underlying hedge portfolios, P and K, produce losses (gains) in the short position. The hedge position can be represented by:

$$dH = dV - m\,dP - n\,dK + \varphi_2 x\,dt. \quad (2)$$

Applying standard option pricing arguments to equation 2 leads to the following differential equation describing the behavior of the property value per dollar of redevelopment cost:

$$0 = 0.5\sigma^2 Z^2 Q_{ZZ} + \delta Z Q_Z - \gamma Q + \varphi_2 Z, \quad (3)$$

where Q(Z) is the value of the call option per dollar of the redevelopment cost, Q(Z) = V(x, R)/R, and Z is the clean cash flow per dollar of the redevelopment cost, Z = x/R. The steps leading from equation 2 to equation 3 are detailed in subsection 1 of the Appendix.

The parameters σ^2, γ, and δ in equation 3 are defined respectively as follows:

$$\sigma^2 = \sigma_x^2 + \sigma_R^2 - 2\sigma_{xR},$$

$$\gamma = \omega_K - \mu_R, \text{ and}$$

$$\delta = g - (\mu_R - \mu_K + r),$$

where g is the risk-adjusted growth rate of the cash flow x, defined as $\mu_x - (\mu_P - r)\beta_x$ with β_x given by $\rho_{xP}\sigma_x/\sigma_P$. The term $(\mu_P - r)\beta_x$ is the risk premium of the risky cash flow stream, x. ω_K is the risk-free rate plus the risk premium for the uncertain redevelopment cost, that is, $\omega_K = r + (\mu_K - r)\beta_R$, where $\beta_R = \rho_{RK}\sigma_R/\sigma_K$. The boundary conditions for (3) are

(i) $Q(Z^*) = \phi Z^*/(r - g) - 1$,[14]

(ii) $Q(Z) \geq \text{Max}[0, \phi Z/(r - g) - 1]$ for $Z \leq Z^*$,

(iii) $Q_Z^* = \phi/(r - g)$, and

(iv) $Q(0) = 0$.

The initial condition assumes that the property owner must first determine the optimal ratio, Z^*, of the clean cash flow to the redevelopment cost. Then, when Z exceeds Z^*, the property owner will exercise the option to redevelop. We refer to Z^* as the optimal redevelopment point, the point at which it is optimal to exercise the redevelopment option. The economic payoff of the decision to redevelop at this point is the present value of the cash flow per dollar of redevelopment cost minus one dollar of redevelopment cost. The second condition states that it is not optimal to exercise the option before Z reaches Z^* because the property in its current clean condition is worth more than the payoff after redevelopment. The third condition gives the change in property value for every unit change in the ratio Z^*. The last condition states that the property is worth nothing if the cash flow to the property is zero.[15]

Assuming a value-maximizing property owner and using the four boundary conditions above to solve equation 3 for Q, the following solution is obtained for the value of the clean property with an option to redevelop:

$$V = \begin{cases} \dfrac{\varphi_2 x}{r - g} + \left(\dfrac{\phi - \varphi_2}{r - g}\right)^q \left(\dfrac{(q - 1)^{q-1}}{q^q}\right) \dfrac{x^q}{R^{q-1}} & Z \leq Z^* \quad (4.1) \\[2ex] \dfrac{\phi x}{r - g} - R & Z > Z^* \quad (4.2) \end{cases}$$

The steps detailing the derivation of equation 4 are in subsection 2 of the appendix.

The parameter q is restricted to be greater than 1. The value of q is given by

$$q = 0.5\left(\frac{\sigma^2 - 2\delta}{\sigma^2} + \sqrt{\frac{(2\delta - \sigma^2)^2 + 8\gamma\sigma^2}{\sigma^4}}\right). \tag{5}$$

The restriction $q > 1$ implies that the risk-adjusted growth rate g of the cash flow x must be less than the risk-free rate of return r, that is, $r > \mu_x - (\mu_P - r)\beta_x$.[16] This implies $r + (\mu_P - r)\beta_x > \mu_x$.[17]

Equation 4.1 is the value of cleaned property before redevelopment. The first term in equation 4.1 is the value of the clean property without the option to redevelop, while the second term is the value of the option to redevelop. Equation 4.2 is the payoff (the net present value) from redevelopment at Z^*. Following redevelopment, the option to redevelop is extinguished. A new option to redevelop will be created only if the highest and best use should change such that the cash flow stream would be elevated to a higher level by redevelopment.

The optimal point for the exercise of the redevelopment option is given by

$$Z^* = \frac{r - g}{\phi - \varphi_2}\frac{q}{q - 1}. \tag{6}$$

If Z^* is finite, it becomes optimal to exercise the option to redevelop and attain a higher level of cash flow ϕx when Z reaches Z^*. If at the time of valuation $Z < Z^*$, the expected waiting time, τ_z, for $Z > Z^*$, and hence for exercising the option to redevelop, is

$$\tau_z = \left(\frac{\ln(Z^*) - \ln(Z)}{m_x - m_R}\right) \quad \text{for } m_x > m_R, \tag{7}$$

where $m_x = \mu_x - 0.5\sigma_x^2$, $m_R = \mu_R - 0.5\sigma_R^2$, and $(m_x - m_R)$ is the instantaneous rate of change of the variable Z, with Z being conditional on the values of x and R at the time of valuation.

If Z^* is infinite, as is the case when $\phi = \varphi_2$, the probability that Z will ever reach Z^* is zero. Therefore, it is never optimal to exercise the option because there is no gain in cash flow by redeveloping the property. In this case, the value of the option to redevelop is zero.

2.3. Valuation of the Contaminated Property Before Removal

Now that the value of a property "as clean" has been determined, we turn our attention to the valuation of the contaminated property before removal. The owner of a contaminated property is assumed to be able to pursue at any point during the life of the property the following three general timing strategies with respect to the exercise of the options to remove and redevelop: 1) to exercise the option to remove but retain the option to redevelop at a later time (the sequential timing strategy); 2) to exercise both options simultaneously where removal becomes part of the redevelopment process (the simultaneous timing strategy); and 3) to do nothing[18] but retain the right to exercise both options at a later time, either sequentially or simultaneously. In the sequential timing strategy, the property owner removes the hazardous materials at time $\tau_1 \geq 0$ and then redevelops the property at time $\tau_2 > \tau_1$. In the simultaneous timing strategy, the property owner jointly removes the hazardous materials and redevelops the property at time τ_3, where $\tau_3 \geq 0$. The property owner is assumed to pursue the strategy that maximizes the expected present value of the property.

2.4. Property Valuation with a Decision to Remove and Redevelop Sequentially

To derive the value of a contaminated property when the property owner is assumed to make sequential decisions about when to remove and redevelop, we must first obtain the value of the property upon completion of removal. As described by equation 4.1, the value of the clean property is the present value of the new cash flow stream, $\varphi_2 x$, plus the value of the redevelopment option. This value obtained, the next step is to derive the value of the property as contaminated.

Let Y be the ratio of the clean cash flow, x, to the clean up cost, C, (i.e., x/C), and let Y^* be the value of that ratio at which it is optimal to exercise the option to remove. We refer to Y^* as the optimal point of removal. The value of the contaminated property when the option to remove and the option to redevelop are exercised sequentially is expressed respectively for the cases $Y \leq Y^*$ and $Y > Y^*$ by equations 8.1 and 8.2 below:

$$V_1 = \begin{cases} \dfrac{\varphi_1 x}{r-g} + \left(\dfrac{(q-1)^{q-1}}{q^q}\right)\left(\left(\dfrac{\varphi_2-\varphi_1}{r-g}\right)^q \left(\dfrac{x}{C}\right)^{q-1} + \left(\dfrac{\phi-\varphi_2}{r-g}\right)^q \left(\dfrac{x}{R}\right)^{q-1}\right) x & Y \leq Y^* \quad (8.1) \\[2ex] \dfrac{\varphi_2 x}{r-g} + \left(\dfrac{(q-1)^{q-1}}{q^q}\right)\left(\dfrac{\phi-\varphi_2}{r-g}\right)^q \left(\dfrac{x}{R}\right)^{q-1} x - C & Y > Y^* \quad (8.2) \end{cases}$$

where C, the total clean-up cost, is given by $(\alpha_1 + \alpha_2)R$. The derivation of equation 8 is presented in subsection 3 of the appendix.

At any time before $Y > Y^*$, the value of the contaminated property, as given by equation 8.1, consists of three components: (1) the present value of the cash flow stream of the property as contaminated, $\varphi_1 x$, (2) the value of the option to remove, and (3) the value of the embedded option to redevelop after removal of the hazardous materials. After Y^* is attained and the property is cleaned up, the value of the property (equation 8.2) consists of the present value of the new cash flow stream, $\varphi_2 x$, and the value of the option to redevelop. The exercise price of the option to remove under the sequential timing strategy is the total clean-up cost. Under this strategy, the option to redevelop will be exercised after removal when $Z > Z^*$. From equation 4.2, the payoff from redevelopment is the present value of the cash flow stream made possible by redevelopment, ϕx, net the cost of redevelopment R.

The optimal point of removal, Y^*, is given by

$$Y^* = \frac{r-g}{\varphi_2 - \varphi_1}\frac{q}{q-1}. \tag{9}$$

If at the time of valuation $Y < Y^*$, then the expected waiting time, τ_y, for $Y > Y^*$, and hence for exercising the option to remove, is

$$\tau_y = \left(\frac{\ln(Y^*) - \ln(Y)}{m_x - m_R} \right) \quad \text{for } m_x > m_R. \tag{10}$$

2.5. Property Valuation with Cleanup and Redevelopment as Simultaneous Decisions

By choosing to remove and redevelop simultaneously, the property owner can lower the total cost of removal. With redevelopment at the time of removal, the property owner saves the costs of restoration, $\alpha_2 R$, since these costs are assumed to be included as part of the costs of redevelopment (as renovation costs or as the cost of constructing new improvements). Thus, the combined cost of removal and redevelopment as a joint action is $(1 + \alpha_1)R$.[19]

Let W be the ratio of the clean cash flow, x, to the combined removal and redevelopment cost, $(1 + \alpha_1)R$, and let W^* be the value of that ratio at which it is optimal to exercise the joint option to remove and redevelop. The value of the contaminated property for the simultaneous exercise of the two options for the cases $W \leq W^*$ and $W > W^*$ is expressed respectively by equations 11.1 and 11.2 below:

$$V_2 = \begin{cases} \dfrac{\varphi_1 x}{r - g} + \left(\dfrac{\phi - \varphi_1}{r - g} \right)^q \left(\dfrac{(q-1)^{q-1}}{q^q} \right) \dfrac{x^q}{(\alpha_1 R + R)^{q-1}} & W \leq W^* \quad (11.1) \\[2ex] \dfrac{\phi x}{r - g} - (\alpha_1 R + R) & W > W^* \quad (11.2) \end{cases}$$

The derivation of equation 11 is explained in subsection 4 of the appendix.

Equation 11.1 represents the value of the contaminated property if the property owner considers removal and redevelopment as a joint decision but it is not yet optimal to exercise the joint option. At any time before $W > W^*$, the value of the contaminated property consists of the value of the cash flows before removal and redevelopment plus the value of the joint option. The payoff from the joint exercise (equation 11.2) is the present value of the cash flows after redevelopment net of the total redevelopment cost.

The value of W^* is as follows:

$$W^* = \frac{r - g}{\phi - \varphi_1} \frac{q}{q - 1}. \tag{12}$$

If at the time of valuation $W < W^*$, then the expected waiting time, τ_w, for $W > W^*$, and hence for exercising the option to remove and redevelop simultaneously, is

$$\tau_w = \left(\frac{\ln(W^*) - \ln(W)}{m_x - m_R} \right) \quad \text{for } m_x > m_R. \tag{13}$$

2.6. Optimal Actions

The question addressed in this subsection is, when is each of the alternative strategies identified in the previous subsections for improving the value of contaminated property optimal? Table 1 gives the valuation formulas relevant for determining which of the alternative actions maximizes the expected present value of the property under different states of the world.

The decision whether to remove and redevelop sequentially or simultaneously depends on which action will produce the higher property value. For example, if $Y > Y^*$ and $W > W^*$, then two courses of action are possible. The property owner can make either a sequential decision to remove first and wait to redevelop at a later time, or a decision to remove and redevelop simultaneously. The question of which alternative to undertake is resolved by comparing the property values given by equations 8.2 and 11.2 for the property in question.

From valuation formulas (8) and (11), we can see that the usual comparative statics for the value of an option apply: (1) $\partial V/\partial R < 0$, (2) $\partial V/\partial \alpha_1 < 0$, (3) $\partial V/\partial \alpha_2 < 0$, (4) $\partial V/\partial x > 0$, and (5) $\partial V/\partial \sigma^2 > 0$. The last result is true provided the risk-adjusted growth rate g of the cash flow x is kept constant. Briefly stated, assuming all else constant, the lower the redevelopment cost, R, the lower the removal cost, $\alpha_1 R$, the lower restoration cost, $\alpha_2 R$, the higher the cash flow, x, and the higher the uncertainty, σ^2 (as long as g does not change), the higher the value of the contaminated property.

The effect of the uncertainty parameter, σ^2, requires some clarification. Unlike undeveloped land, an income-producing property is not generally analogous to a pure option. The economic return of an income-producing property consists of two components, a cash flow component and an option component. The effect of variance on the value of the option component is unambiguously positive while the effect on the cash flow component is unam-

Table 1. Decision rules.

States of the World at Time of Valuation		Optimal Action at Time of Valuation	Formula for Deriving Property Value
$Y \le Y^*$	$W \le W^*$	If (11.1) > (8.1), wait until $W > W^*$ to remove and redevelop simultaneously; otherwise wait until $Y > Y^*$ to remove and redevelop sequentially	MAX[(8.1), (11.1)]
	$W > W^*$	Remove and redevelop now if (11.2) > (8.1); otherwise wait until $Y > Y^*$ to remove and redevelop sequentially	MAX[(8.1), (11.2)]
$Y > Y^*$	$W \le W^*$	If (11.1) > (8.2), wait until $W > W^*$ to remove and redevelop simultaneously; otherwise remove now and wait until $Z > Z^*$ to redevelop	MAX[(8.2), (11.1)]
	$W > W^*$	Remove and redevelop now if (11.2) > (8.2); otherwise remove now and wait until $Z > Z^*$ to redevelop	MAX[(8.2), (11.2)]

biguously negative. Therefore, the effects of the variance of the redevelopment cost and of the cash flows on the value of the property as a whole are indeterminate. Proof of this result is in subsection 5 of the appendix.

3. Accelerate or Postpone Redevelopment

As shown in the preceding section, the owner of a clean property exercises the option to redevelop when $Z > Z^*$. At the time of valuation, the redevelopment of a clean property may be optimal because the anticipated increase in cash flows is expected to outweigh the cost to redevelop, R. For a contaminated property, however, the optimal conditions for redevelopment depend not only on the additional cash flows expected from redevelopment and the redevelopment cost, but also on the removal and restoration costs. With a contaminated property the cost to clean up may be so high as to make redevelopment infeasible even though $Z > Z^*$ for the clean property. On the other hand, however, the decision to clean up may make a joint decision to remove and redevelop optimal, even though redevelopment of a clean property is not yet optimal, i.e., $Z \leq Z^*$. This situation may occur if the restoration cost component of the total clean-up costs are high. Thus, the two options are interrelated in that the decision to exercise the option to remove may be influenced by the option to redevelop, and the decision to exercise the option to redevelop may be influenced by the need to remove the hazardous materials before redevelopment.

To illustrate how the presence of the hazardous materials may accelerate the decision to redevelop, consider two properties that are similar except for the presence of hazardous materials. The property owner of a clean property will delay the decision to redevelop as long as Z, the ratio of the current level of the cash flow stream, φx, to the redevelopment cost, R, is less than Z^*. Alternatively, consider an otherwise similar property whose market value is severely depressed by the presence of hazardous materials, and suppose the opportunity to improve the cash flows, and hence the property value, by removal is substantial. The owner of the contaminated property may decide to remove and redevelop simultaneously while the owner of a clean property will decide to wait. By comparing the expected waiting time of action, τ_z and τ_w, we obtain the following results:

Proposition. If at the time of valuation $Z < Z^*$ and $W < W^*$, then the owner of the contaminated property expects to accelerate the redevelopment of the property relative to a comparable clean property (i.e., $\tau_w < \tau_z$) whenever the cost of removal satisfies:

$$\frac{C_1 + R}{R} < \frac{\phi - \varphi_1}{\phi - \varphi_2},$$

where $C_1 = \alpha_1 R$. On the other hand, the owner expects to postpone redevelopment (i.e., $\tau_w > \tau^2$) whenever the cost of removal satisfies:

$$\frac{C_1 + R}{R} > \frac{\phi - \varphi_1}{\phi - \varphi_2}.$$

The proof is presented in subsection 6 in the appendix.

From the above results, the economic factors that determine whether the redevelopment of the contaminated property is to be accelerated or postponed relative to the redevelopment of a clean property are as follows: (1) the magnitude of the removal cost relative to the magnitude of the redevelopment cost, and (2) the gain in cash flows of the contaminated property by removal relative to the gain in cash flows of a clean property by redevelopment. Basically the owner of a contaminated property expects to accelerate the redevelopment of the property if the ratio of the gain in cash flows from cleaning up the contaminated property to the gain from redeveloping a clean property exceeds the increase in total redevelopment costs contributed by the clean-up costs.

4. Loss in Property Value

The loss in property value because of the presence of the hazardous materials can be divided into two levels. The first level is the loss in value assuming the property owner makes optimal decisions. This is the minimum possible loss in value. It can be measured by comparing the "as clean" value from equation 4, the benchmark, with the value of the contaminated property from equations (8, 11). Table 2 summarizes the loss in property value under different states of the world where $Z \leq Z^*$ for the clean property. A similar table can be constructed for the case of $Z > Z^*$.

The second level is the *additional* loss in value resulting from suboptimal behavior. This level exists if a property owner removes the materials prematurely without adequately considering the costs and benefits involved, or if a property owner, because of legislation or regulation, is not permitted to make optimal decisions with respect to the contaminated property. In the latter case, this additional loss in value results from a reduction or elimination of the value of decision. Indeed, the value of the options foregone can be used as a measure of the cost of regulation. If the benefit to the public (the increase in social welfare) is not increased by at least the amount of the loss in value, then the excess of the cost of regulation over the benefit from the regulation represents a deadweight loss.

To illustrate, suppose at the time of valuation, the contaminated property satisfies $Y < Y^*$ and $W < W^*$. The property value based on optimal actions is max(8.1, 11.1). If the property owner is required to clean up immediately, the property value becomes max(8.2, 11.2). In this case, the property value can be negative if the clean-up cost exceeds the value of the income after cleanup. The loss due to regulation is then, max(8.2, 11.2) − max(8.1, 11.1).

5. An Illustrative Example

5.1. Numerical Example

This section presents a short example using hypothetical values to illustrate the application of the valuation methods developed above. Let P be the hedge portfolio for the cash flow, x, of an "as clean property" and K be the hedge portfolio for the redevelopment cost, R.

Table 2. Loss in property value.

States of the World	Derivation	Loss in Property Value
$Z \le Z^*$, $Y \le Y^*$, $W \le W^*$	(4.1)–(8.1)	The minimum of $\frac{(\varphi_2-\varphi_1)x}{r-g} - \left(\frac{1}{\alpha_1+\alpha_2}\right)^{q-1}\left(\frac{\varphi_2-\varphi_1}{r-g}\right)^q\left(\frac{(q-1)^{q-1}}{q^q}\right)\frac{x^q}{R^{q-1}}$
		or
	(4.1)–(11.1)	$\frac{(\varphi_2-\varphi_1)x}{r-g} + \left(\left(\frac{\phi-\varphi_2}{r-g}\right)^q - \left(\frac{1}{1+\alpha_1}\right)^{q-1}\left(\frac{\phi-\varphi_1}{r-g}\right)^q\right)\left(\frac{(q-1)^{q-1}}{q^q}\right)\frac{x^q}{R^{q-1}}$
$Z \le Z^*$, $Y \le Y^*$, $W > W^*$	(4.1)–(8.1)	The minimum of $\frac{(\varphi_2-\varphi_1)x}{r-g} - \left(\frac{1}{\alpha_1+\alpha_2}\right)^{q-1}\left(\frac{\varphi_2-\varphi_1}{r-g}\right)^q\left(\frac{(q-1)^{q-1}}{q^q}\right)\frac{x^q}{R^{q-1}}$
		or
	(4.1)–(11.2)	$\frac{\varphi_2 x}{r-g} + \left(\frac{\phi-\varphi_2}{r-g}\right)^q\left(\frac{(q-1)^{q-1}}{q^q}\right)\frac{x^q}{R^{q-1}} - \frac{\phi x}{r-g} + (1+\alpha_1)R$
$Z \le Z^*$, $Y > Y^*$, $W \le W^*$	(4.1)–(8.2)	The minimum of $(\alpha_1+\alpha_2)R$
		or
	(4.1)–(11.1)	$\frac{(\varphi_2-\varphi_1)x}{r-g} + \left(\left(\frac{\phi-\varphi_2}{r-g}\right)^q - \left(\frac{1}{1+\alpha_1}\right)^{q-1}\left(\frac{\phi-\varphi_1}{r-g}\right)^q\right)\left(\frac{(q-1)^{q-1}}{q^q}\right)\frac{x^q}{R^{q-1}}$
$Z \le Z^*$, $Y > Y^*$, $W > W^*$	(4.1)–(8.2)	The minimum of $\frac{(\varphi_2-\varphi_1)x}{r-g} - \left(\frac{1}{\alpha_1+\alpha_2}\right)^{q-1}\left(\frac{\varphi_2-\varphi_1}{r-g}\right)^q\left(\frac{(q-1)^{q-1}}{q^q}\right)\frac{x^q}{R^{q-1}}$
		or
	(4.1)–(11.2)	$\frac{\varphi_2 x}{r-g} + \left(\frac{\phi-\varphi_2}{r-g}\right)^q\left(\frac{(q-1)^{q-1}}{q^q}\right)\frac{x^q}{R^{q-1}} - \frac{\phi x}{r-g} + (1+\alpha_1)R$

Consider the following market information: $\mu_P = 15.00\%$, $\sigma_P = 20\%$, and $r = 5\%$. Also assume that the current net cash flow of an "as clean" property is \$300,000, with $\mu_x = 10\%$, $\sigma_x = 20\%$, and $\rho_{xP} = 0.75$. Further assume that the current net cash flow for the contaminated property is only 40% of that of the "as clean" property ($\varphi_1 = 0.4$), and that cleanup will fully restore the cash flow level to that of the "as clean" property ($\varphi_2 = 1$). The property, if redeveloped, will produce a cash flow level twice as large as the "as clean" cash flow ($\phi = 2$). The current redevelopment cost R is \$5,000,000, with $\mu_R = 7.00\%$, $\sigma_R = 20\%$, $\mu_K = 15\%$, $\sigma_K = 4\%$, $\rho_{xR} = 0.6$, and $\rho_{RK} = 0.8$. Hence, ω_K, which is equal to $r + (\mu_K - r)\beta_R$ where $\beta_R = \rho_{RK}\,\sigma_R/\sigma_K$, is 12.5%. Suppose the clean-up and restoration costs are respectively 30% and 20% of the redevelopment cost R, i.e., $\alpha_1 = 0.3$ and $\alpha_2 = 0.2$.

The questions to be addressed with this example are 1) What is the value of the contaminated property? 2) What is the optimal strategy regarding cleanup and redevelopment? 3) What is the impact of the presence of hazardous materials on the value of the property if the regulatory environment does not allow the property owner to choose the optimal timing for cleanup?

From the above assumptions, the risk-adjusted growth rate, g, = 2.5%. The value for q as determined from equation 5 is 1.3279. Z, defined as x/R, is 0.06; and Z^* (from equation 6) is 0.1012. For an "as if clean" property, the value as determined from equation 4.1 is \$19,612,305. The expected waiting time for redevelopment as determined from equation 7 is 17.4 years.

The value of the contaminated property depends on the optimal course of action. First, consider sequential actions. We have $Y = x/(\alpha_1 R + \alpha_2 R) = 0.12$, and Y^* (from equation 9) = 0.1687. Since Y is less than Y^*, equation 8.1 is used to calculate the value of the property, which is found to be \$17,261,054. From equation 10, the expected time of cleanup is 11.3 years. Next consider simultaneous actions. Now we have $W = x/(R + \alpha_1 R) = 0.0462$, and, from equation 12, $W^* = 0.0633$. Since W is less than W^*, equation 11.1 is applied to finding the value of the property, which is \$17,837,783. From equation 13, the expected waiting time for simultaneous cleanup and redevelopment is 10.5 years.

Given the above results, it is obvious that the property owner should adopt the simultaneous strategy because it yields a higher value for the property. Also, the property owner will redevelop the contaminated property sooner than the "as clean" property. The loss in property value due to the presence of hazardous materials under this optimal strategy is \$1,774,522 (\$19,612,305 − \$17,837,783), or approximately 9% of the "as if clean" value; and the expected waiting time for redevelopment is decreased approximately 41% from 17.4 to 10.5 years.

5.2. *The Effect of Regulatory Constraint*

Next suppose a prospective property owner will be allowed no discretionary choice by regulation regarding when to clean up the contaminated property, and will instead be compelled to remove the materials immediately following purchase. Then the property value assuming immediate cleanup under sequential strategy is \$17,112,304 (equation 8.2). If the property owner chooses to cleanup and redevelop simultaneously, the value to the prospective owner is \$17,500,000 (equation 11.2). In this case, the property owner will adopt the simultaneous strategy. The loss in property value is now \$2,112,305 (\$19,612,305 − \$17,500,000), which is \$337,783, or 19.04%, greater than the loss when the property owner is permitted to make the optimal decision. This additional loss of value, approximately 1.72% of the "as if clean" value, is a measure of the extra cost imposed on the property owner by regulation. Aggregated across the economy, the total regulatory cost may be quite large. Supplied with estimates of such regulatory costs, policymakers can then use their informed judgment to determine if the benefits to social welfare provided by the regulation outweigh the costs.

6. Limitations and Extensions

As is generally true of real option pricing models that attempt closed-form solutions to valuation and decision problems involving complex economic phenomena, the model development in this paper is based on assumptions that simplify the economic reality being represented. The valuation of the contaminated property under the variety of conditions examined is accomplished with only two state variables, the free and clear cash flows of a similar but clean (i.e., uncontaminated) property and the redevelopment costs. Proportionality assumptions are employed to represent all other cash flows and costs, which implies that the risks of the cash flows and costs before and after cleanup and/or redevelopment remain the same.

A question that can be examined by further research is, to what extent do these proportionality assumptions limit the "real world" applicability of the model developed in this paper? One approach to examining this question would be to reduce the reliance on proportionality by adding at least two more state variables to the option pricing models. In this more complete model, the cash flows associated with redevelopment as conversion to a different use and redevelopment as renovation of the existing use can be modeled as separate stochastic processes. Although it is plausible to assume that the cash flows of a property redeveloped by renovation are perfectly correlated with the cash flows of a property not redeveloped, it is probably not plausible to extend that assumption to properties redeveloped by conversion to a different use. Realism also suggests that the costs associated with cleaning up a contaminated property and the costs associated with redevelopment of the property should be modeled as different stochastic processes.

Valuation results with this more complex model can be compared to those obtained with the simpler model employed in this paper in order to examine the extent and the direction of divergence in the results. This approach would allow the researcher to measure how much accuracy is potentially lost by resorting to the proportionality assumptions in lieu of enhancing the model's ability to more realistically reflect the "real world" by adding more state variables. Even adding just two additional state variables, however, would make the model exceedingly complex, solvable only by numerical analysis. As implied by Friedman (1953), considerable simplification of reality is generally needed to develop models that provide tractable economic explanations and clear, insightful economic intuition.

The paper assumes that the property has no debt financing. The exact legal standing of lenders with respect to properties with hazardous materials under CERCLA is complicated, and a discussion of the risks and the recourse available to lenders under the environmental laws and regulations is beyond the scope of this paper. If the property were financed with debt and the problems posed by the hazardous materials became more costly than the owner was willing to bear, the owner may be able to default on the loan and transfer the risks to the lender. This default option could affect the value of the equity position in the property, in a manner analogous to the effect of the option to abandon. The value of the option to abandon, however, may be offset by the additional loan costs charged by lenders as the price for granting property owners this option. An extension of this paper would be to include debt financing so that these problems could be explicitly examined.

Another extension would be to conduct empirical tests of the theoretical results obtained in this paper. One set of tests could attempt to determine if the value of the contaminated property reflected the existence of clean-up and redevelopment options, and then to determine the value placed on these options by investors in the real estate market. The data set for such tests would include the selling prices of contaminated and of otherwise comparable clean properties, the cash flows of comparable contaminated and clean properties, property cash flows before and after redevelopment, and the clean-up and redevelopment costs. If the cash flows are not observable, the cash flows can be backed out from estimates of property prices using capitalization rates. An example of how this set of tests can be performed is contained in Quigg (1993).

Optimal timing hypotheses derived from Section 3 could possibly form the bases of a second group of tests. One set of hypotheses could test how values are affected by selecting (and failing to select) the optimal timing strategy. A second set of possible hypotheses may be derived from the proposition in Section 3 regarding the optimal time to redevelop a contaminated property relative to a comparable "clean" property. However, the necessary data set for such tests may be more difficult to obtain than for the first set of tests mentioned. Not only would the types of data mentioned for the first set of tests be needed, but for contaminated properties, data would be required about how long from the time of purchase it took the property owners to clean up or redevelop. For otherwise comparable but clean properties, information about the length of time from purchase to redevelopment would be needed.

7. Conclusion

The preceding analysis assumes that the property owner has two options for improving the value of the property. The first option is to remove the hazardous materials (i.e., to clean up the property) and the second option is to redevelop the property to a higher and better use. The second option is embedded in the first option in the sense that it cannot be exercised unless the first one is exercised. The owner is assumed to have three possible timing strategies with respect to the exercise of these options at any time during the life of the property. Criteria were developed for determining the value-maximizing strategy. From this analysis it can easily be shown that the size of the loss in property value due to the presence of the hazardous materials depends on the states of the world determining optimality conditions for the exercise of the removal and redevelopment options. Finally, the loss in property value is shown to increase if property owners are not permitted (because of law or regulation) to make optimal decisions with respect to contaminated properties. The value of the options foregone can be used as a measure of the cost of regulation.

The analysis in this paper explicitly assumed an income-producing property. However, the model can be easily applied to nonincome-producing properties such as vacant land by setting the cash flow component of the model to zero, or even by allowing for negative values to reflect cash outlays for property taxes, insurance, and other possible property expenses in the absence of offsetting cash inflows. Thus the model we developed is general because it allows for income effects.

Our specification of the problem can be generalized to other situations in which properties are encumbered by a condition that makes the property owner subject to laws or regulations that necessitate some future costly action with respect to the property. One example is properties that are in violation of current building codes but whose violations have been grandfathered. Like properties burdened with hazardous materials, the value of such properties may be adversely affected by the costly actions required to remove or otherwise cure the encumbrance and by unfavorable market acceptance due to the presence of the encumbrance. The analysis in this paper can be used to determine when the property owner should take costly actions to remove those encumbrances.

Appendix

1. Derivation of Equation 3

By substituting dV in equation 1 into equation 2, we obtain the following change in hedge position

$$\begin{aligned} dH = {} & [xV_x\mu_x + RV_R\mu_R + \varphi_2 x + 0.5(x^2V_{xx}\sigma_x^2 + R^2V_{RR}\sigma_R^2 + 2xRV_{xR}\sigma_{xR})]dt \\ & + xV_x\sigma_x(\rho_{xp}dz_P + (1 - \rho_{xp}^2)^{1/2}dz_{pe}) + RV_R\sigma_R(\rho_{RK}dz_k + (1 - \rho_{RK}^2)^{1/2}dz_{ke}) \\ & - mP(\mu_P dt + \sigma_P dz_P) - n(\mu_K K dt + \sigma_K K dz_K). \end{aligned} \quad \text{(A1)}$$

To establish a risk-free hedge position, the short positions *m* and *n* must repsectively satisfy:

$$xV_x\sigma_x\rho_{xp}dz_P = mP\sigma_P dz_P$$

and

$$RV_R\sigma_R\rho_{RK}dz_K = n\sigma_K K dz_K,$$

which in turn imply that

$$m = (xV_x/P)(\rho_{xp}\sigma_x/\sigma_P) = (xV_x/P)\beta_x,$$

where

$$\beta_x = \rho_{xp}\sigma_x/\sigma_P,$$

and

$$n = (RV_R/K)(\rho_{RK}\sigma_R/\sigma_K) = (RV_R/K)\beta_R,$$

where

$$\beta_R = \rho_{RK}\sigma_R/\sigma_K.$$

Substituting m and n and taking the expected value of dH produce the expected change in value of the hedge position as follows:

$$E(dH) = [xV_x\mu_x + RV_R\mu_R + \varphi_2 x + 0.5(x^2V_{xx}\sigma_x^2 + R^2V_{RR}\sigma_R^2 + 2xRV_{xR}\sigma_{xR})]dt - xV_x\mu_P\beta_x dt - RV_R\mu_K\beta_R dt. \tag{A2}$$

The expected return on the riskless hedge position is the risk-free rate of return, r. Therefore the expected change in the hedge position is

$$E(dH) = r[V - mP - nK]dt = r[V - xV_x\beta_x - RV_R\beta_R]dt. \tag{A3}$$

Setting (A2) and (A3) equal to each other yields

$$0 = xV_x[\mu_x - (\mu_P - r)\beta_x] + RV_R(\mu_R - \omega_K + r) + \varphi_2 x - rV + 0.5(x^2V_{xx}\sigma_x^2 + R^2V_{RR}\sigma_R^2 + 2xRV_{xR}\sigma_{xR}), \tag{A4}$$

where $\omega_K = r + (\mu_K - r)\beta_R$. To solve for the property value, V, requires some simplification of (A4). Let Q(Z) be V(x, R)/r where Z = x/R. Then (A4) can be rewritten as

$$0 = Q_Z Z[\mu_x - (\mu_P - r)\beta_x] + (Q - ZQ_Z)(\mu_R - \omega_K + r) + \varphi_2 Z - rQ + 1/2(Q_{ZZ}Z^2\sigma_x^2 + Q_{ZZ}Z^2\sigma_R^2 - 2Q_{ZZ}Z^2\sigma_{xR}) \tag{A5}$$

To simplify further, we let $g = \mu_x - (\mu_P - r)\beta_x$, $\sigma^2 = \sigma_x^2 + \sigma_R^2 - 2\sigma_{xR}$, $\delta = g - (\mu_R - \omega_K + r)$, and $\gamma = \omega_K - \mu_R$. Substituting these four parameters into (A5) yields equation (3).

2. Derivation of Equation 4

The solution to equation 3 is the sum of the general solution and the particular solution. Rewrite equation 3 as

$$0 = Q_{ZZ} + (2\delta/\sigma^2)(1/Z)Q_Z - (2\gamma/\sigma^2)(1/Z^2)Q + (2\phi_2/\sigma^2)(1/Z). \tag{A6}$$

The standard complete solution Q from ordinary differential equation is given by:

$$Q(Z) = \frac{\varphi_2 Z}{r - g} + A_1 Z^p + A_2 Z^q, \tag{A7}$$

where

$$p, q = 0.5\left(\frac{\sigma^2 - 2\delta}{\sigma^2} \pm \sqrt{\frac{(2\delta - \sigma^2)^2 + 8\gamma\sigma^2}{\sigma^4}}\right).$$

The next step is to solve for the unknown A_1 and A_2 by using the boundary conditions. The boundary condition (i) implies that

$$Q(Z^*) = \frac{\varphi_2 Z^*}{r - g} + A_1 Z^{*p} + A_2 Z^{*q} = \frac{\phi Z^*}{r - g} - 1, \tag{A8}$$

and boundary condition (ii) produces

$$\frac{\varphi_2}{r - g} + pA_1 Z^{*p-1} + qA_2 Z^{*q-1} = \frac{\phi}{r - g}. \tag{A9}$$

Now set $A_1 = 0$. Solving equations A8 and A9 yields

$$Z^* = \frac{r - g}{\phi - \varphi_2} \frac{q}{q - 1}$$

and,

$$A_2 = \left(\frac{\phi - \varphi_2}{r - g}\right)^q \left(\frac{(q - 1)^{q-1}}{q^q}\right).$$

Substituting $A_1 = 0$ and A_2 into equation (A7) produces the valuation formula (4).

3. Derivation of Equation 14

Following the derivation for equation 3 presented in section 1 of this appendix, the differential equation describing the value of the contaminated property is,

$$0 = 0.5\sigma^2 Y^2 Q_{YY} + \delta Y Q_Y - \gamma Q + \varphi_1 Y, \tag{A10}$$

where Y is now defined as the ratio of the cash flow x to the total clean-up cost C, and Q is the ratio of the value of the contaminated property to the clean-up cost, i.e., $V_1(x, C)/(C)$. To simplify our notation, we let

$$\Phi = \left(\frac{\phi - \varphi_2}{r - g}\right)^q \left(\frac{(q - 1)^{q-1}}{q^q}\right) \left(\frac{C}{R}\right)^{q-1}, \tag{A11}$$

so that the value of the property after removal can be expressed from equation 8.1 as $\varphi_2 x/(r - g) + \Phi Y^{q-1}x$. The boundary conditions for the contaminated property become:

(i) $Q(Y^*) = \varphi_2 Y^*/(r - g) + \Phi Y^{*q} - 1$

(ii) $Q(Y) \geq \varphi_2 Y/(r - g) + \Phi Y^q - 1$, for $Y \leq Y^*$,

(iii) $Q_Y = \varphi_2/(r - g) + q\Phi Y^{q-1}$, and

(iv) $Q(0) = 0$.

The first condition implies that at the optimal point for the removal of the hazardous materials, the value of the property after cleanup is the present value of the net cash flow, $\varphi_2 x$, plus the value of the option to redevelop, minus the clean-up cost C. This condition results directly from equation 4. The second condition states that it is not optimal to exercise the option to remove whenever $Y \leq Y^*$. The third and fourth conditions are parallel to the boundary conditions for equation 4.

To obtain the value of the contaminated property, solve for Q with these four boundary conditions. Conditions (i) and (ii) above imply respectively that

$$Q(Y^*) = \frac{\varphi_1 Y^*}{r - g} + B_1 Y^{*p} + B_2 Y^{*q} = \frac{\varphi_2 Y^*}{r - g} + \Phi Y^{*q} - 1, \text{ and}$$

$$Q_{Y*} = \frac{\varphi_1}{r - g} + qB_2 Y^{*q-1} = \frac{\varphi_2}{r - g} + q\Phi Y^{*q-1}.$$

Setting B_1 to zero and solving these two equations for Y^* and B_2 produces

$$Y^* = \frac{r - g}{\varphi_2 - \varphi_1} \frac{q}{q - 1}, \quad \text{and} \quad B_2 = \left(\frac{\varphi_2 - \varphi_1}{r - g}\right)^q \left(\frac{(q - 1)^{q-1}}{q^q}\right) + \Phi.$$

Substituting $B_1 = 0$ and B_2 into the differential equation for the value of the contaminated property

$$Q(Y) = \frac{\varphi_2 Y}{r - g} + B_1 Y^p + B_2 Y^q,$$

yields equation 14.

4. Derivation of Equation 11

Following the same procedures leading to equation 4, the differential equation describing the value of the contaminated property is given by

$$0 = 0.5\sigma^2 W^2 Q_{ww} + \delta W Q_w - \gamma Q + \varphi_1 W, \tag{A12}$$

where Q is now defined as the ratio of the value of the contaminated property to the total redevelopment cost $(\alpha_1 R + r)$, and W is defined as $x/(\alpha_1 R + R)$. The boundary conditions are similar to those for equation 3:

(i) $Q(W^*) = \phi W^*/(r - g) - 1$,

(ii) $Q(W) \geq \text{Max}[0, \phi W/(r - g) - 1]$ for $W \leq W^*$,

(iii) $Q^*_W = \phi/(r - g)$, and

(iv) $Q(0) = 0$.

With these boundary conditions, the derivation of equation (11) is exactly the same as that of equation 4.

5. *The Effects of σ_R^2 and σ_x^2 on the Value of the Property*

To show that the effects of σ_R^2 and σ_x^2 on the value of the property are indeterminate, we use equation 8.1 to determine the signs of the partial derivatives $\partial V/\partial\sigma_R^2$ and $\partial V/\partial\sigma_x^2$. The results obtained apply to valuation formula 14 as well. From (8.1), the partial derivative $\partial V/\partial\sigma_R^2$ with sign indicated below each component is given by

$$\left(\frac{-\varphi_2 x}{(r-g)^2}\right)\underbrace{\left(\frac{\partial(r-g)}{\partial\sigma_R^2}\right)}_{(=0)} + \left(\frac{\phi-\varphi_2}{r-g}\right)^q\left(\frac{(q-1)^{q-1}}{q^q}\right)\left(\frac{x}{R}\right)^{q-1} x \underbrace{\left(\ln\frac{Z}{Z^*}\right)}_{(-)}\underbrace{\left(\frac{\partial q}{\partial\sigma_R^2}\right)}_{(\pm)}$$

Note that the redevelopment cost uncertainty has no effect on the value of the cash flow x as $\partial(\gamma-\delta)/\partial\sigma_R^2$ is zero. The sign of the second term represents the effect of σ_R^2 on the option value. It is clear that the effect of the cost uncertainty depends on the behavior of q with respect to σ_R^2. From equation 6, one can show that $\partial q/\partial\sigma_R^2$ is positive when σ_R^2 is small and becomes negative as σ_R^2 increases. The value of q asymptotically approaches one as σ_R^2 tends to infinity. As a result, the net impact of σ_R^2 on the property value is that the property value at first declines and then increases as σ_R^2 increases.

The effect of σ_x^2 on the property value is similar, and $\partial V/\partial\sigma_x^2$ is given by

$$\underbrace{\left(\frac{-\varphi_2 x}{(r-g)^2}\right)}_{(-)}\underbrace{\left(\frac{\partial(r-g)}{\partial\sigma_x^2}\right)}_{(+)} + \left(\frac{\phi-\varphi_2}{r-g}\right)^q\left(\frac{(q-1)^{q-1}}{q^q}\right)\left(\frac{x}{R}\right)^{q-1} x \left(\underbrace{\frac{-q\partial(r-g)}{\partial\sigma_x^2}}_{(-)} + \underbrace{\left(\ln\frac{Z}{Z^*}\right)}_{(-)}\underbrace{\left(\frac{\partial q}{\partial\sigma_x^2}\right)}_{(\pm)}\right)$$

The behavior of $\partial q/\partial\sigma_x^2$ is similar to that of $\partial q/\partial\sigma_R^2$. The first product term shows the negative effect of σ_x^2 on the value of the cash flow stream x. The variance of the cash flow x tends to have more negative effect on the property value because the partial $\partial q/\partial\sigma_x^2$ tends to zero as σ_x^2 tends to infinity.

6. *Proof of Proposition*

Suppose the total removal and redevelopment cost, $(C_1 + R)$, for the contaminated property satisfies the condition as stated in the proposition; that is, $(C_1 + R)/R < (\phi - \varphi_1)/(\phi - \varphi_2)$. Since C_1 is equal to $\alpha_1 R$, this inequality implies that $(1 + \alpha_1)/(\phi - \varphi_1) < 1/(\phi - \varphi_2)$. Multiplying both sides by $(r - g)[q/(q - 1)]$ yields

$$(1 + \alpha_1)[(r - g)/(\phi - \varphi_1)q/(q - 1)] < [(r - g)/(\phi - \varphi_2)q/(q - 1)].$$

Recognizing $W^* = [(r - g)/(\phi - \varphi_1)][q/(q - 1)]$ and $Z^* = [(r - g)/(\phi - \varphi_2)][q/(q - 1)]$, we can rewrite the inequality as

$$(1 + \alpha_1)W^* < Z^*.$$

Dividing both sides by Z and recognizing $W = Z/(1 + \alpha_1)$ produces

$$W^*/W < Z^*/Z.$$

Taking natural log and dividing by $(m_x - m_R)$ on both sides, we have

$$\tau_W = \left(\frac{\ln(W^*) - \ln(W)}{m_x - m_R}\right) < \left(\frac{\ln(Z^*) - \ln(Z)}{m_x - m_R}\right) = \tau_Z.$$

Q.E.D.

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Notes

1. We use the term *hazardous materials* generally to also encompass toxic substances.
2. For tractability, we assume that the owner of the contaminated property does not have a third option to abandon the property. This simplification can be justified by assuming that the authorities will not allow the owner to abandon a contaminated property. Given the legal rules of strict liability and joint and several liability applicable to environmental cases, plus public policy, it can be argued that any deep-pocket investor will not be able to escape legal liability by abandonment. Thus, we assume that the property owner must either assure that the hazardous materials are safely contained or pay to have the materials removed.
3. The Comprehensive Environmental, Response, Compensation and Liability Act, (42 U.S.C. Section 9601 *et seq.*), also known as CERCLA and "superfund," severely limits the ability of properties owners to decide what to do with the hazardous materials on the property. CERCLA (or superfund) authorizes the EPA to order the cleanup of waste sites, active facilities, or container vessels where there is a release or the threat of a hazardous/toxic substance that presents an imminent and substantial danger to public health or the environment. Both before and after removal of the hazardous materials, decisions with respect to the operation and development of the property are subject to a maze of regulations. The majority of states have enacted similar statutes (referred to as mini-superfund statutes), many of which, most notably the New Jersey Environmental Cleanup Responsibility Act ("ECRA"), are stricter than federal law.
4. Asbestos in commercial buildings, though regulated by a web of overlapping federal and state statutes and regulations, is nonetheless regulated under laws less severe than superfund or superfund-type legislation. Generally, the removal of asbestos and other indoor potentially hazardous substances such as radon is not required by law. One exception is that when a building with ACMs is going to be demolished or substantially renovated and the amount of asbestos exceeds a specified threshold amount, then EPA regulations promulgated under the National Emission Standards for Hazardous Air Pollutants (NESHAPS) require removal prior to demolition or renovation (40 CFR 61 Part M). Federal, state, or local regulation is, however, much more demanding with respect to asbestos in schools than it is with asbestos in commercial buildings.

5. Capozza and Helsley (1989) analyzed the price of urban land by decomposing the value of the land into four additive components, namely, the value of land rent, the cost of conversion, the value of accessibility, and the value of expected future rent increases.
6. This similar but clean property serves as a proxy for what the cash flow of the contaminated property would be if it were clean.
7. For a previous example of the use of the proportionality assumption for modeling cash flows see Williams (1991). Williams assumes the cash flows of the undeveloped property are a fixed proportion of the cash flows of the developed property.
8. Possible implications of the proportionality assumption used to define the cash flows of the property in different states of the property are discussed in Section 6 of the paper (Limitations and Extensions).
9. Samuelson (1965) and McKean (1965) pioneered the development of analytical solutions applicable to perpetual options with early exercise.
10. We are referring to the option to remove taken by itself, that is, considered separately from the option to redevelop. As we will see, when the two options are exercised jointly, part of the total redevelopment cost described below (the restoration cost) can be avoided or charged to the redevelopment activity.
11. The latter includes the costs associated with replacing or repairing important building components that were removed or damaged during removal of the materials as well as replacing the removed materials with functional substitutes in cases where these materials performed important functions, as in the case of building with ACMs, PCBs, and other such hazardous but functionally important materials. It also includes the cost of restoring land to some sort of acceptable condition in the case of landfills or dump or spill sites.
12. The total redevelopment cost is assumed to include, where appropriate, the cost of gutting or demolishing an existing structure as well as the cost of constructing new improvements.
13. By assuming dz_{ke}, is unsystematic and will not be priced, this setup is the same as assuming a hedge security M, as in Fischer (1978), which is perfectly correlated with R with the returns of M following $dM/M = \mu_M dt + \sigma_R dz_R$ where $\mu_M = r + \rho_{Rk}\sigma_R/\sigma_k\ (\mu_k - r)$.
14. The value of a redeveloped property F is a function of the cash flow stream x only. The differential equation describing the value of the property is $0 = 0.5\sigma^2 x^2 F_{xx} + \delta x F_x - \gamma F + \phi x$ subject to the condition that $F(0) = 0$. A unique solution for F is $\phi x/(\gamma - \delta)$. This result is needed for the boundary conditions of equation (3).
15. In actual situations involving hazardous materials that legal regime of strict and joint and several liability for damages caused by the "responsible party" (a term of legal art) prevails, and the extent of liability often exceeds that of the value of the affected property.
16. This can easily be shown by setting $q > 1$ from equation 5. Rearranging and cancelling terms lead to $r > g$.
17. $g < r$ implies $\mu_x < r + \beta(\mu_P - r)$, the usual assumption for cash flow valuation models. In a stochastic context, this implies that the *trend* of the cash flow growth rate has to be less than the CAPM discount rate, although the periodic deviation from the trend allows for the periodic growth rate to exceed the CAPM discount at any particular point in time.
18. By do nothing, we mean to continue to use or to hold the property in its contaminated state (i.e., with the hazardous material present). We do not mean to imply that nothing at all will be done with the hazardous materials. We assume that the property onwer will take all actions with respect to safely maintaining and operating the property with the materials present that are required by law and regulation. The cost of these actions are assumed to be charged against the operating cash flows.
19. This assumption is most appropriate for buildings containing asbestos and other kinds of hazardous building materials. In the latter case, the costs associated with restoration will be included as part of the renovation, and thus included in redevelopment costs. In the case of raw land where EPA regulations require that contaminated land be restored to specified conditions, land restoration/reclamation costs may be reduced or avoided if the property is developed.

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