NONISOTHERMAL DISPLACEMENT OF OIL BY A SOLUTION OF AN ACTIVE ADDITIVE

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In earlier work [1, 2] mathematical models have been constructed for processes of displacement of oil from a porous medium by a solution of an active additive, i.e., an additive capable of changing the hydrodynamic characteristics of the fluid and the medium. An additive of this kind that was considered was a polymer that in the dissolved state influences the properties of the displacing fluid and in the adsorbed state the permeability of the porous medium. Self-similar solutions were obtained corresponding to the problem of frontal displacement from a homogeneous porous medium, and a number of numerical calculations were made. It is natural to generalize this treatment by introducing into the problem a second active factor, which is here taken to be the temperature of the injected fluid. The analysis of the nonisothermal displacement of oil by a solution of an active additive can be transferred without significant modifications to the general problem of displacement of oil by a solution carrying two active agents. The names "additive" and "temperature" are retained here only for convenience of exposition.

1. Basic Equations

The system of equations of motion consists of the equations of two-phase flow in the porous medium, the balance of the active additive in the dissolved state, the kinetics of the sorption process, and the thermal balance:

$$m\frac{\partial s_i}{\partial t}+\operatorname{div}\mathbf{u}_i=0,\quad i=1,2 \tag{1.1}$$

$$m\frac{\partial}{\partial t}(cs_1)+\operatorname{div}(c\mathbf{u}_1)+q=0 \tag{1.2}$$

$$\mathbf{u}_i=-k\mu_i^{-1}f_i\operatorname{grad}p,\quad \frac{\partial a}{\partial t}=q \tag{1.3}$$

$$m\frac{\partial}{\partial t}(s_1C_1T+s_2C_2T)+\frac{\partial}{\partial t}(C_3T)+\operatorname{div}(C_1\mathbf{u}_1T+C_2\mathbf{u}_2T)+R=0 \tag{1.4}$$

Here, t is the time, s_i is the saturation of the pore space by the phase i, $\mathbf{u}_i$ is the flow velocity of phase i, m is the porosity of the medium, k is the permeability of the medium, f_i are the relative phase permeabilities, μ_i are the viscosities of the phases, p is the pressure, c is the concentration of the active additive in the solution, a is the amount of adsorbed additive, q is the intensity of the sorption process, T is the temperature, C_i are the specific heats of the water, oil, and rock, respectively, and R is the intensity of heat transfer per unit volume of the stratum. In what follows, the subscripts 1 and 2 are appended to the symbols which represent the displacing and displaced fluids, respectively.

Considering large-scale motions, we ignore the capillary discontinuity of the pressure, the diffusion transport of the additive, and heat conduction. We also ignore the heat losses to the rocks surrounding the stratum and assume that the specific heats are constants. We do not take into account the nonlinearity of the flow for the polymer solution. We restrict ourselves to the one-dimensional problem of frontal displacement, assuming that the motion takes place along the x axis, and that the total flow rate $U = u_1 + u_2$ does not depend on the time.

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Introducing in the usual manner the Buckley—Leverett function F, which is equal to the fraction of the displacing fluid in the total flow, we obtain the canonical system of equations of one-dimensional displacement:

$$u_1=FU, \quad u_2=(1-F)U; \quad F=\mu_2 f_1(\mu_2 f_1+\mu_1 f_2)^{-1}=F(s,\ c,\ T) \tag{1.5}$$

$$m\frac{\partial s}{\partial t}+U\frac{\partial F}{\partial x}=0 \tag{1.6}$$

$$\frac{\partial}{\partial t}\left(cs+\frac{a}{m}\right)+\frac{U}{m}\frac{\partial}{\partial x}(cF)=0, \quad a=a(c) \tag{1.7}$$

$$\frac{\partial}{\partial t}(sT+bT)+\frac{U}{m}\frac{\partial}{\partial x}(FT+hT)=0, \qquad b=(mC_2+C_3)[m(C_1-C_2)]^{-1};\ h=C_2/(C_1-C_2) \tag{1.8}$$

The system (1.6)-(1.8) admits discontinuous solutions, and on the discontinuities the following obvious conditions must be satisfied:

$$mV[s]=U[F(s,\ c,\ T)] \tag{1.9}$$

$$mV[sc+a/m]=U[cF(s,\ c,\ T)] \tag{1.10}$$

$$mV[T(s+b)]=U[T(F+h)] \tag{1.11}$$

where V is the velocity of the discontinuity; the square brackets denote the discontinuities of the corresponding quantities.

The main applied and theoretical significance is associated with the problem of decay of a discontinuity with initial and boundary conditions of the form

$$s=s_0, \quad c=c_0=0, \quad T=T_0=0 \quad (t=0), \qquad s=s^\circ, \quad c=c^\circ=1, \quad T=T^\circ=1 \quad (x=0) \tag{1.12}$$

Such a choice of the initial and boundary conditions corresponds to an admissible normalization of the temperature and the concentration in Eqs. (1.7) and (1.8). This problem admits a self-similar solution $s = s(\xi)$, $c = c(\xi)$, $T = T(\xi)$, $\xi = mx/Ut$, the solution to the boundary-value problem

$$\xi\frac{ds}{d\xi}=\frac{dF}{d\xi}, \quad \xi\frac{d}{d\xi}\left(cs+\frac{a}{m}\right)=\frac{d(cF)}{d\xi}, \quad \xi\frac{d}{d\xi}(T(s+b))=\frac{d}{d\xi}(T(F+h)), \quad s(\infty)=s_0$$
$$c(\infty)=c_0=0, \quad T(\infty)=T_0=0, \quad s(0)=s^\circ, \quad c(0)=c^\circ=1, \quad T(0)=T^\circ=1 \tag{1.13}$$

2. Analysis of the Construction of the Self-Similar Solution

The self-similar solution to the problem (1.13) is "fitted together" out of sections of continuous variation of the unknown functions, which are joined by discontinuities. In the case of a convex sorption isotherm ($a''(c) < 0$), the solution can consist of the following elements: sections of continuous variation of s with $s'(\xi) \neq 0$ for constant c and T and $\xi = F_{,s}(s,\ c,\ T)$; sections of constancy of s, c, and T; discontinuities of s with constant c and T; connected discontinuities of c and s or T and s. A decisive circumstance is that the solution in the case of convex sorption isotherm does not contain sections of continuous variation of the concentration and the temperature.

Even the simplest examples show that the solution is not constructed uniquely from these elements. A physically meaningful solution is identified by the additional requirement that the discontinuities be stable.

It follows from the above that it is sensible to seek the distributions $c(\xi)$ and $T(\xi)$ in the form of "steps":

$$c(\xi)=1, \quad \xi<\xi_c, \quad c(\xi)=0, \quad \xi>\xi_c, \qquad T(\xi)=1, \quad \xi<\xi_T, \quad T(\xi)=0, \quad \xi>\xi_T \tag{2.1}$$

The form of a solution with such concentration and temperature distributions is completely determined by the behavior of the function F(s, c, T) as a function of s for c = 0 and 1 and T = 0 and 1. Thus, instead of a function of three variables it is sufficient to consider a family of four functions of one variable s:

$$F_{ij}(s)\equiv F(s,\ i,\ j), \quad i,j=0,1 \tag{2.2}$$

For what follows, it is convenient to transform the jump conditions (1.9)-(1.11), reducing them to the form

$$\xi=\frac{F^+-F^-}{s^+-s^-},\quad F^\pm=F(s^\pm,c^\pm,T^\pm) \tag{2.3}$$

$$\xi=F^\pm\left(s^\pm+\frac{a^+-a^-}{(c^+-c^-)m}\right)^{-1},\quad [c]\neq 0 \tag{2.4}$$

$$\xi=\frac{F^\pm+h}{s^\pm+b},\quad [T]\neq 0 \tag{2.5}$$

The relations (2.4) and (2.5) differing in the choice of the signs are consequences of each other and the relation (2.3). In particular, at a "complete" discontinuity of the concentration ($c^- = 1$, $c^+ = 0$)

$$\xi=F^\pm(s^\pm+a(1)/m)^{-1} \tag{2.6}$$

It is easy to give a graphical interpretation of the jump conditions. On the s, F plane we plot the curves $F^\pm(s, c^\pm, T^\pm)$. If there is a discontinuity of the concentration, then the conditions (2.3) and (2.4) are satisfied simultaneously, which corresponds to transition of the representative point on the s, F plane from the curve F^- to the curve F^+ along a straight line passing through the point $0_c = (-s_a, 0)$, where $s_a = (a^+ - a^-)/(c^+ - c^-)m$.

Similarly, if there is a discontinuity of the temperature, the representative point goes over from the curve F^- to the curve F^+ along the straight line passing through the point $0_T = (-b, -h)$.

Finally, at a discontinuity of the saturation, which takes place without a change in the concentration and the temperature, only the condition (2.3) is satisfied. The curves F^- and F^+ merge, and the discontinuity corresponds to transition from one point of this curve to another.

We take the stability condition for the discontinuities in the form in which it is usually stated in the theory of shock waves [3, 4]: *the number of characteristics leaving a discontinuity line must be one less than the number of relations satisfied at the discontinuity* (i.e., in the given case equal to two). A feature of the considered problem is that the function F(s) has sections of convexity ($F_{,ss} < 0$). Therefore, retaining the previous formulation of the stability conditions, we shall also regard the characteristics whose velocity is equal to the velocity of the discontinuity as incoming characteristics.

In the cases for which there exists a rigorous theory (Eq. (1.6) with c = const, T = const), such a formulation is identical to the well-known condition for stability of a generalized solution [5]. Applied to the present problem, this condition can be regarded as heuristic.

The characteristics of the system (1.7)-(1.9) are given by the relations

$$\frac{dx}{dt}=\nu_1=\frac{\partial F}{\partial s},\quad \frac{dx}{dt}=\nu_2=F\left(s+\frac{a_{,c}}{m}\right)^{-1},\quad \frac{dc}{dt}=0,\quad \frac{dx}{dt}=\nu_3=\frac{F+h}{s+b},\quad \frac{dT}{dt}=0 \tag{2.7}$$

We shall call the characteristics of the first, second, and third families the s, c, and T characteristics, respectively, and discontinuities at which two characteristics of the corresponding family arrive s, c, and T discontinuities, respectively.

Suppose the instantaneous state is characterized by the triplet of values (s°, c°, T°). On the plane (s, F), we plot the curve $F(s, c^\circ, T^\circ)$. Then the three characteristic velocities ν_i are given, respectively, by the angular coefficients of the tangent to the curve F(s) at the point $s = s^\circ$ and the rays described to this point from the points ($-a_{,s}$/m, 0) and (−b, −h). Thus, the characteristic velocities ν_i also admit a perspicuous graphical interpretation on the s, F plane.

The above makes it natural to attempt a graphical construction of the self-similar solution. We construct the curves $F_{ij}(s) = F(s, i, j)$ for i, j = 0, 1. To construct the solution, it is necessary to find a continuous path from the point (s°, $F_{11}(s_0)$) to the point (s_0, $F_{00}(s_0)$) consisting of pieces of sections of the curves F_{ij} and segments of straight lines corresponding to the discontinuities. At the same time, the discontinuities must satisfy the stability condition formulated above. Motion along the F_{ij} curves corresponds to sections in which the saturation changes. On them, the derivative

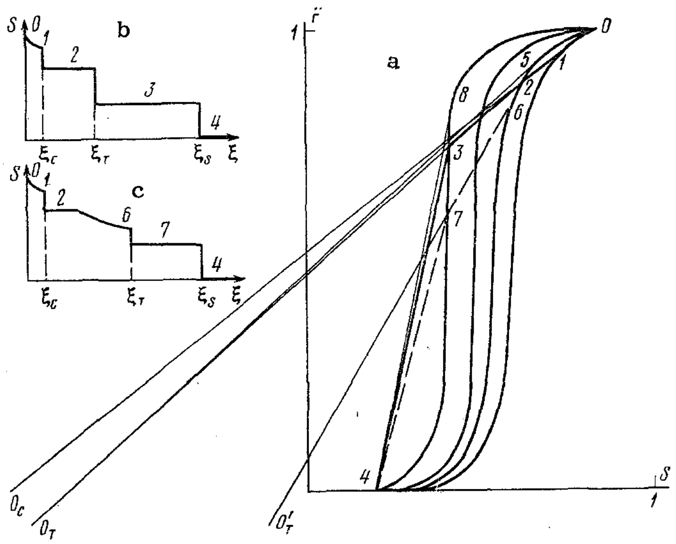

Fig. 1

dF/ds is equal to the self-similar variable ξ and, therefore, must increase along the path. The condition that ξ increases together with the stability condition for the discontinuities gives the necessary condition for joining a discontinuity to a continuous part of the solution.

3. Construction of Self-Similar Solution

The structure of the self-similar solution is determined by the mutual disposition of the F_{ij} curves on the plane (s, F). We shall assume that the derivatives $F_{,c}$ and $F_{,T}$ do not change sign. The pieces out of which the solution is "constructed" are determined by the signs of these derivatives. The simplest cases are those when these derivatives have the same signs.

Suppose first $F_{,c} \leqslant 0$, $F_{,T} \leqslant 0$. Then the solution can be constructed by direct generalization of the graphical procedure used in [1] (the problem of one "thickening" additive). The disposition of the F_{ij} curves on the s, F plane is shown in Fig. 1. To the curve F_{11} we draw the tangents from the poles 0_c and 0_T, corresponding to possible c and T discontinuities. With a view to the subsequent continuation of the solution, we actually make a transition to the tangent with the least slope (in this case, the c tangent, and the transition occurs to point 2 on curve F_{01}). The further development of the solution is governed by whether or not it is possible to have a discontinuity (a T discontinuity) directly from point 2 to curve F_{00}. If point 2 lies below point 5, at which the T ray touches the curve F_{01}, then the T discontinuity is from point 2 to point 3 on curve F_{00} along the straight line 0_T-2 (shown in Fig. 1 by the continuous curve). If point 2 lies above point 6, then the T discontinuity occurs along the straight line 6-7 (the broken line), and it is preceded (corresponding to smaller ξ) by the continuous section 2-6 of motion along the curve F_{01}. The s discontinuity is constructed similarly (the s ray is drawn from point 4 with coordinates $(s_0, F_{00}(s_0))$.

The path 0 1 2 3 4 of the representative point in Fig. 1 corresponds to the dependence $s(\xi)$ shown in Fig. 1b; in Fig. 1c, we have plotted the dependence $s(\xi)$ for the path 0 1 2 6 7 4.

To list all possible types of structure of the solution, we introduce the following notation. Sections of the type of simple waves on which the values of all the three variables remain constant (s = const, c = const, T = const) will be denoted by the letter P; sections of continuous variation of s with motion of the representative point along the curve F_{ij} will be called S waves and denoted by the letter S; the discontinuities will be denoted as follows: an s discontinuity by J, a c discontinuity by JC, and a T discontinuity by JT. Then 0 1 2 3 4 corresponds to the formula

$$(1,1)-S-JC\to(0,1)-P-JT\to(0,0)-P-J\to(0,0) \tag{3.1}$$

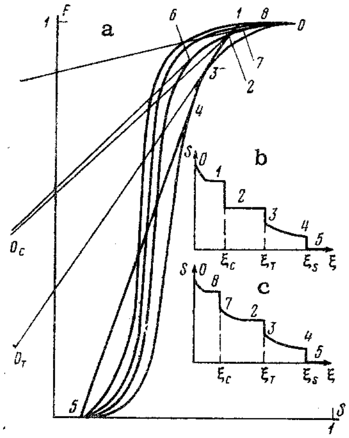

Fig. 2

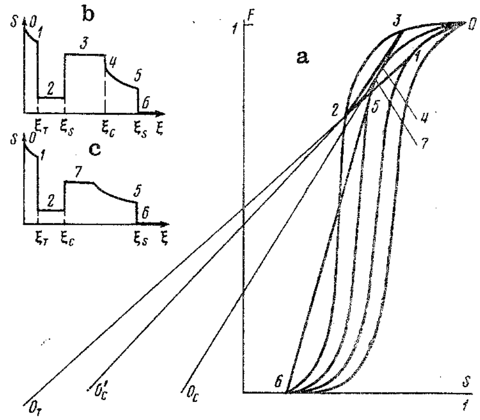

Fig. 3

In the brackets, we specify here the pair of (c, T) values that determines the curve F_{ij} on which the representative point is situated after the corresponding discontinuity.

The complete set of possible variants of the solution under the condition $F_{,c} \leqslant 0$, $F_{,T} \leqslant 0$ can be represented by the branching formula

$$(1,1)-S-\left\{\begin{matrix} JC\to(0,1)-P-(S)-JT \\ JT\to(1,0)-P-(S)\to JC \end{matrix}\right\}\to(0,0)-P-(S)-J\to(0,0) \tag{3.2}$$

Here, the S waves shown in the brackets may be absent. In this case, there are altogether eight possible types of solution. Note that the mutual disposition of the curves F_{10} and F_{01} does not influence the structure of the solution.

We now consider the case $F_{,c} \geqslant 0$, $F_{,T} \geqslant 0$ (Fig. 2). In this case, the construction begins at the "end" — at the point $(s_0, F_{00}(s_0))$ — and is a generalization of the procedure described in [2] for one additive that increases the mobility of the displacing fluid. The solution ends with an S discontinuity (4-5), which is preceded by an S wave (3-4), and it by a T discontinuity (2-3), as in Fig. 2a, if the T tangent to the curve F_{00} has a greater angular coefficient than the c tangent, and a c discontinuity otherwise. The further development of the solution is determined by the position of point 6, at which F_{01} touches the c ray, relative to point 2. If point 6 is below point 2, then the c discontinuity is along the straight line 1-2; if point 7 lies above point 2, then the c discontinuity is along the tangent 8-7 to the curve F_{01} and it is preceded by the continuous section 7-2 of motion along F_{01}. The corresponding dependences $s(\xi)$ are shown in Figs. 2b and 2c.

The complete set of possible solution types corresponds to

$$(1,1)-S-P-\left\{\begin{matrix} JC\to(0,1)-(S)-P-JT \\ JT\to(1,0)-(S)-P-JC \end{matrix}\right\}\to(0,0)-S-J\to(0,0) \tag{3.3}$$

There are altogether four possible types. The mutual disposition of the curves F_{10} and F_{01} does not influence the structure of the solution.

We now investigate the more complicated situation when the derivatives $F_{,c}$ and $F_{,T}$ have opposite signs. We consider the case $F_{,c} \geqslant 0$, $F_{,T} \geqslant 0$. At the same time

$$F(s,1,0)\geqslant F(s,1,1)\geqslant F(s,0,1);\quad F(s,1,0)\geqslant F(s,0,0)\geqslant F(s,0,1) \tag{3.4}$$

We draw the c and T tangents to F_{11} and assume that the angular coefficient of the T tangent is smaller. Then the initial section of the solution is the s wave 0-1 and the T discontinuity 1-2 to point 2, which is the lower intersection of the T ray 0_T-1 with the curve F_{10}. We now draw the c tangent to the curve F_{00}. The form of the solution is determined by the possibility of joining the T and c discontinuities corresponding to transition along these tangents. Suppose the upper point of intersection 3 of the c tangent to F_{00} with F_{10} lies below point 2. Then the solution can be readily

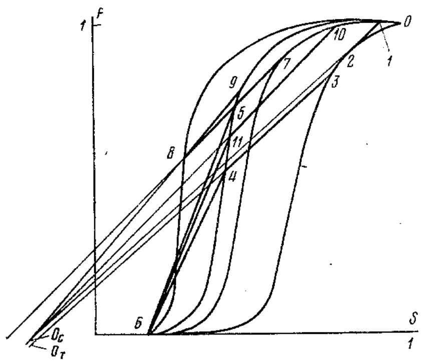

Fig. 4

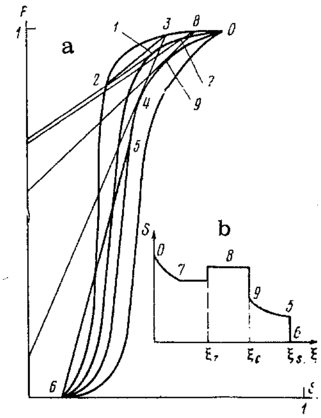

Fig. 5

constructed to the end and has the simple form

$$(1,1)-S-JT\rightarrow(1,0)-P-(S)-P-JC\rightarrow(0,0)-S-J\rightarrow(0,0) \qquad (3.5)$$

We begin to shift the point 0_c to the left, thereby decreasing the slope of the c tangent (in what follows, it is assumed that $F(s,\ 0,\ 0) \geqslant F(s,\ 1,\ 1)$). At the same time, point 3 will approach point 2, and when they merge the S wave indicated by the brackets in (3.5) will disappear. With further displacement of point 3 to the right, the "leading" and "trailing" parts of the solution no longer match one another and there is an additional S discontinuity 2-3 joining them, this being accompanied by an increase in the saturation with an increase in the self-similar variable (path 0 1 2 3 4 5 6 (Fig. 3)). The formula for the solution is

$$(1,1)-S-JT\rightarrow(1,0)-P-J\rightarrow(1,0)-P-JC\rightarrow(0,0)-S-J\rightarrow(0,0) \qquad (3.6)$$

Naturally, for the existence of a solution of this type it is necessary that the velocity of the S discontinuity 2-3 be less than the velocity of the c discontinuity 3-4.

We now shift the point 0_c to the left. When the lower point of intersection of the c tangent to F_{00} with F_{10} rises above point 2, a solution of the form (3.6) becomes impossible. Then the S-discontinuity—simple wave—c-discontinuity structure (2-3-4) is replaced by just a c discontinuity, which is no longer determined by the c tangent but by the c ray 0_c-2-7. The corresponding solution has the structure

$$(1,1)-S-JT\rightarrow(1,0)-P-JC\rightarrow(0,0)-P-(S)-J\rightarrow(0,0) \qquad (3.7)$$

and corresponds to the path 0 1 2 7 (5) 6 (Fig. 3).

Finally, shifting the point 0_c further to the left, we arrive at a situation in which the T tangent to F_{11} is steeper than the c tangent. In this case the T tangent for the curve F_{01} is also steeper than the c tangent and there exists a self-similar solution corresponsing to the path 0 1 2 3 4 (5)-6 (Fig. 4) with the structure

$$(1,1)-S-P-JC\rightarrow(0,1)-S-JT\rightarrow(0,0)-P-(S)-J\rightarrow(0,0) \qquad (3.8)$$

A singular case in these constructions arises when the straight line passing through the points 0_c and 0_T lies above the curve F_{01} but intersects the curve F_{11}. In this case, solutions of two types can be constructed: with first discontinuity of the temperature (the path 0 1 2 3 4 6 (Fig. 4)) and the structure (3.8) and with first discontinuity of the concentration (path 0 7 8 9 5 6) and structure (3.7). One can also have a disposition of the points 0_c and 0_T for which a solution with the structure (3.6) is obtained. It is readily verified that both solutions are admissible with respect to all the criteria listed above. Here, one can also have a construction containing a double (c, T) discontinuity from the upper point of intersection 10 of 0_c, 0_T with the curve F_{11} to its last point of intersection 11 with F_{00}. Depending on the mutual disposition of the points 11 and 5 on curve F_{00}, the further continuation of the solution contains P, a simple wave, or a combination of it with an S wave, ending with an S discontinuity. The structure of the solution

$$(1,1)-S-P-JTC\rightarrow(0,0)-P-(S)-J\rightarrow(0,0) \qquad (3.9)$$

Figure 4 shows the path 0 10 11 6, which does not contain an S wave. In this case, the disposition of the s characteristics has an unusual form. In the neighborhood of the double (c, T) discontinuity, both s characteristics leave the discontinuity. The stability of the discontinuity is ensured by the circumstance that both the T and c characteristics arrive on it, i.e., altogether four characteristics, as is required by the stability condition. Thus, in the given case the self-similar solution is not unique.

If the curve F_{00} lies below the curve F_{11}, there are some particular features of the construction associated with the circumstance that now the c and T tangents drawn to F_{00} lie to the right of and lower than the corresponding tangents to F_{11}.

Suppose the position of the point 0_T is fixed, and, as before, we move the point 0_c to the left. First, we obtain a solution with the structure (3.5), and then (3.6) (path 0 1 2 3 4 5 6 (Fig. 5)). In this case, the lower point of intersection of the c tangent to F_{00} with F_{10} always lies below the lower point of intersection of the T tangent to F_{11} with F_{10}, though a solution with the structure (3.6) is only possible when the angular coefficient of the T tangent to F_{11} is less than the angular coefficient of the straight line 2-3 (Fig. 5). The construction becomes impossible when the c tangent to F_{00} intersects F_{10} further to the right than the T tangent to F_{11}. A new structure corresponding to the path 0 7 8 9 5 6 (Fig. 5) arises. The point 8 is the upper intersection of the c tangent to F_{00} with F_{10}, and the transition 7-8 occurs along the straight line 0_T8. The formula for the solution is

$$(1,1)-S-P-JT\rightarrow(1,0)-P-JC\rightarrow(0,0)-S-J\rightarrow(0,0) \tag{3.10}$$

At the T discontinuity, there is an increase in the saturation.

When the angular coefficient of the T tangent to F_{00} becomes greater than that of the c tangent to F_{00}, we obtain a solution with the structure (3.9) in which a discontinuity of the temperature comes first.

There again exists a range of parameters of the problem for which all three constructions leading to three different self-similar solutions are possible: with discontinuity of the concentration first, with discontinuity of the temperature first, and with double (c, T) discontinuity. This happens if the straight line 0_c0_T intersects the curve F_{00} but not F_{01}.

If, preserving the mutual disposition of the curves F_{11} and F_{00}, we reverse the signs of the derivatives $F_{,c}$ and $F_{,T}$, then for the cases for which the solution above was constructed uniquely the new structure is obtained by replacing c by T and T by c. The incomplete symmetry of the variables c and T is due to the circumstance that the point 0_c always lies above the point 0_T, and the incomplete symmetry is manifested when the solution is not unique. The unique solution containing double (c, T) discontinuity with the structure (3.9) corresponds to the three-valued case.

4. Numerical Analysis of the Solution

The calculations were based on corresponding generalizations of the difference scheme used earlier in [2] to calculate isothermal displacement of oil by a solution of an active additive, which, in its turn, was based on experience in solving the Buckley—Leverett problem [6, 7]. To eliminate "spreading" of the contact discontinuity of the temperature, an artificial nonlinearity was introduced in the specific heat of the rock. The results of the calculations agree with the above qualitative analysis of the structure of a self-similar solution. In the cases of nonuniqueness of the analytic solution the numerically realized regime corresponded to greater velocities of the discontinuities (i.e., with discontinuity of the concentration first).*

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CHARGING OF DISPERSE PARTICLES IN TWO-PHASE MEDIA WITH UNIPOLAR CHARGE

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Charging of disperse particles with good conduction in two-phase media with unipolar charge is considered in the case when the volume concentration of the particles is low. For this, in the framework of electrohydrodynamics [1, 2], a study is made of the charge of one perfectly conducting liquid particle in a gas (or liquid) with unipolar charge in a fairly strong electric field. The influence of the inertial and electric forces on the motion of the gas is ignored, and the velocities are found by solving the Hadamard–Rybczynski problem. We consider the axisymmetric case when the gas velocity and electric field intensity far from the particle are parallel to a straight line. The analogous problem for a solid spherical particle was solved in [3-6] (in [3], the relative motion of the gas was ignored, while in [4-6] Stokes flow around the particle was considered). The two-dimensional problem of the charge of a solid circular, perfectly conducting cylinder in an irrotational flow of gas with unipolar charge was studied in [7].

1. In two-phase media consisting of disperse particles and a gas (or liquid) with unipolar charge, the particles may be charged, collecting ions from the surrounding gas (or liquid). To study this phenomenon in the case of a low volume concentration of the disperse particles, we consider the charging of a single spherical liquid (or solid) particle in a quasihomogeneous (over distances of the order of the particle radius a) incompressible viscous electrohydrodynamic flow. We shall assume that the particle is perfectly conducting and that all ions which reach its surface remain on it. In considering the electrohydrodynamic flow perturbed by the particle, we ignore in the Navier–Stokes equation the inertial terms and the electric Coulomb forces compared with the viscous term, the perturbation of the electric field due to the change in the charge density in the neighborhood of the particle compared with the unperturbed electric field, the diffusion of the ions over distances of order a compared with their collective transport and migration under the influence of the electric field, and also the departure of the particle's shape from spherical due to the influence of the inertial, Archimedean, and electric forces.

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