INVESTIGATION OF REGIONS OF UNBOUNDED GROWTH OF THE PARTICLE CONCENTRATION IN DISPERSE FLOWS

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Some examples of the motion of a disperse mixture in which regions of unbounded growth of the particle concentration arise are considered. It is shown that integrable and nonintegrable singularities of the concentration can exist. The distribution function for the distances between the particles found by Chernyshenko [9] is used to determine the conditions for the absence of interaction of the particles. It is shown that in the case of integrable singularities of the concentration the model of noninteracting particles is valid in a wide range of the determining parameters, since, despite the infinite value of the concentration, the distance between the particles remains much greater than the particle diameter.

To simulate the motion of disperse mixtures of the gas—particle type, wide use is made of the approximation of interpenetrating continua [1]. In many applications, the low value of the volume concentration of the particles makes it possible to simulate the particle medium by a continuum devoid of self-stresses, and expressions valid for a single particle in an unbounded fluid are used to determine the interphase force and energy interaction [2]. In the solution of a number of problems of flow of a disperse mixture past bodies [3-5] and the investigation of swirling flows of dusty gas [6] in the framework of the model of [2] it was found that there develop in the flow lines or surfaces on the approach to which the number concentration of the particles increases unboundedly. In a number of cases (as noted earlier in [7-8]) the presence of singularities of the concentration is due to the intersection of particle trajectories.

In view of the growth of the particle concentration near the singularities, it is necessary to consider the limits of applicability of the model of noninteracting particles and its possible modification by the introduction of "sheet" type discontinuities [8].

1. Motions with Integrable Singularities of the Particle Concentration

We adopt the usual assumptions of the model of a dusty gas [2], namely, the medium of the particles consists of identical spheres of radius σ, the volume concentration of the particles is negligibly small, the drag of a test particle satisfies Stokes's law, and Brownian motion and interaction of the particles are absent. Allowance for the influence of the particles on the motion of the carrier phase is not a fundamental complication for the exposition that follows, and therefore for simplicity we shall assume that the velocity field of the carrier phase is given.

The equations of steady motion of the particle medium in dimensionless form are [4]

$$(\mathbf{V}_s\nabla)\mathbf{V}_s=\beta(\mathbf{V}-\mathbf{V}_s),\ \operatorname{div} n_s\mathbf{V}_s=0 \qquad (1.1)$$

Here, $\mathbf{V}$ is the velocity of the carrier phase, and the parameters of the particles are identified by the subscript s. As the scales for making the various quantities dimensionless we have taken the characteristic linear dimension L of the problem, the characteristic velocity v_0' of the carrier phase, the characteristic particle concentration n_{s0}' (here and in what follows, the prime identifies the dimensional scales of the

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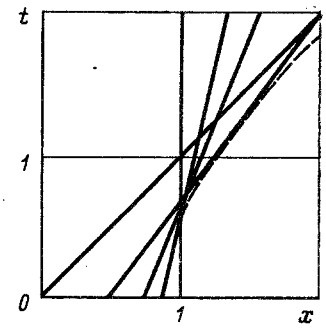

Fig. 1

quantities), $\beta = 6\pi\sigma\mu L/mv_0'$, m is the mass of a particle, and μ is the viscosity of the carrier phase.

Suppose the solution of (1.1) for $n_s(X)$ has a singularity at the point x_0 ($n_s \to \infty$ as $x \to x_0$). We denote by $S(x_0, r)$ the sphere of radius r with center at x_0. Suppose that in the limit $r \to 0$ the relation

$$N(\mathbf{x}_0, r) = \int_S n_s \, d\mathbf{x} = kr^\gamma + o(r^\gamma) \tag{1.2}$$

holds.

In (1.2), all the variables are dimensionless, as in (1.1); k and γ are positive constants.

We shall say that the exponent γ is the order of the singularity of the particle concentration. This definition is valid for integrable singularities of the particle concentration; for nonintegrable singularities, the integral in (1.2) does not exist. Below, we give three examples of motions that are typical elements of three-dimensional motions of dusty gas in which integrable singularities of the particle concentration arise.

A. We consider one-dimensional unsteady motion of a particle medium. For simplicity, we suppose that over the considered time scale the drag of the carrier phase can be ignored, i.e., the particles are characterized by a large inertia. Suppose that at t = 0 the particles occupy the region $0 \leqslant x_0 \leqslant 1$, have constant concentration, and the velocity distribution

$$u_s(x_0, 0) = 1 - x_0^2$$

All the variables are dimensionless. As scales we have chosen the length L of the region occupied by the particles at the initial time, the maximal velocity v_{s0}' of the particles at the initial time, the initial concentration n_{s0}', and the time scale L/v_{s0}'. In Lagrangian coordinates, the law of motion of the particle medium and the continuity equation are

$$x = x_0 + (1 - x_0^2)t, \quad n_s = 1/|1 - 2x_0 t| \tag{1.3}$$

A diagram of the motion of the particles in the x, t plane is shown in Fig. 1. At $t = \frac{1}{2}$, there is a "pile up" in the particle medium, and the velocity field is no longer well defined. (We note that solutions of such type were already considered in [7].) It follows from (1.3) that the concentration becomes infinite on the envelope of the particle trajectories, the equation of which is

$$x = 1/4t + t, \quad t \geqslant 1/2$$

In Eulerian coordinates in the region occupied by the particles we have for x < 1 and above the line t = x

$$n_{s1} = 1/(1 - 4xt + 4t^2)^{1/2} \tag{1.4}$$

In the remaining region occupied by the particles, $n_s = 2n_{s1}$. We assume that the particles moving along the direction to the envelope of the trajectories and from it belong to different noninteracting continua; this is true for only very rarefied systems. We determine the order of the singularity of the particle concentration on the envelope. From (1.4), we obtain

$$n_s \sim 1/t^{1/4} x_1^{3/4}, \quad x_1 = 1/4t + t - x$$

The function N(r) in (1.2) at the points of infinite concentration at small r takes the form

$$N(r) \sim 8\pi r^{5/2}/5t^{1/4}, \quad t \geqslant 1/2$$

We note that solutions for the particle concentration similar to those considered above can be realized when damped shock waves pass through a dusty gas. For example, Men'shov [10] has noted a tendency for "pile ups" to occur in a particle medium in the case of a strong explosion in a dusty gas.

A similar type of concentration singularity develops on the envelope of the trajectories of particles reflected from the surface of a blunt body in a dusty gas stream [5, 9].

B. We consider the problem of the motion of particles in the neighborhood of a stagnation point of an irrotational flow. Various studies have drawn attention to the existence of singularities of the particle concentration at the stagnation point of the flow of a gas suspension past a blunt body in the regime when there is no inertial deposition of particles. Unbounded growth of the particle concentration at the stagnation point was apparently established for the first time in [11]; a similar result was obtained in [3, 5]. Below, we determine the order of the singularity in the concentration of the solid phase in the neighborhood of a stagnation point. Suppose that in the region $y' \leqslant L$, $x' \leqslant L$ of the dimensional coordinates the velocity field of the carrier phase has the form $u' = \delta x'$, $v' = -2^j \delta y'$ (in the case of plane symmetry $j = 0$, for axial symmetry $j = 1$). At $y' = L$ we specify constant particle velocities $v_s'(x', L) = -v_0'$, $u_s'(x', L) = 0$ and constant concentration n_{s0}'. To simplify the calculations, we restrict ourselves to the case $v_0' = L\delta$. As scales for making the coordinates dimensionless we take L, for the velocity components v_0', and for the concentration n_{s0}'. The equations of motion (1.1) of the particles in the form of relations along the characteristics and the boundary conditions become in dimensionless form

$$\ddot{x} + \beta(\dot{x} - x) = 0, \ x(0) = x_0, \ \dot{x}(0) = 0, \quad \ddot{y} + \beta(\dot{y} + 2^j y) = 0, \ y(0) = 1, \ \dot{y}(0) = -1$$

Here, the coordinates x, y of the particles are functions of the dimensionless time of motion $t = t'v_0'/L$ of a particle along its trajectory. For $\beta > 2^j 4$, the particles do not reach the surface of the body in a finite time, and the solution has the form

$$x = x_0[b\exp(at) + a\exp(-bt)]/(a+b), \quad y = [(d-1)\exp(-ct) + (1-c)\exp(-dt)]/(d-c)$$
$$a = \beta(-1 + \sqrt{1 + 4/\beta})/2, \ b = \beta(1 + \sqrt{1 + 4/\beta})/2, \quad c = \beta(1 - \sqrt{1 - 2^j 4/\beta})/2, \ d = \beta(1 + \sqrt{1 - 2^j 4/\beta})/2 \tag{1.5}$$

The continuity equation of the particle medium can be conveniently written down in the Lagrangian coordinates x_0, y_0 [12] (here, x_0, y_0 are the initial coordinates of an individualized point of the particle medium), from which for $y_0 = 1$ we obtain a relation for determining the particle concentration along the trajectory:

$$\left(\frac{x}{x_0}\right)^j n_s(x_0, t) \frac{\partial y(x_0, t)}{\partial t} \frac{\partial x(x_0, t)}{\partial x_0} + 1 = 0 \tag{1.6}$$

The derivatives $\partial y/\partial t$ and $\partial x/\partial x_0$ can be found from (1.5). It can be seen that the particle concentration in the region of the flow depends only on y. We determine the behavior of n_s at small y. It follows from (1.5) that $t \to \infty$ as $y \to 0$, and $y \sim (d-1)\exp(-ct)/(d-c)$. From (1.6) we obtain in the limit

$$\frac{1}{n_s(t)} \sim \frac{c(d-1)}{d-c}\left(\frac{b}{a+b}\right)^{1+j} \exp[t(2^j a - c)]$$

Eliminating t, we obtain at small y the relation

$$\frac{1}{n_s(y)} \sim c\left(\frac{b}{a+b}\right)^{1+j}\left(\frac{d-1}{d-c}\right)^{2^j a/c} y^p, \quad p=1-2^j a/c$$

Since $\beta > 2^j 4$, we obtain from (1.5) in the case of plane symmetry $0 \leqslant p < 2 - \sqrt{2}$ and in the case of axial symmetry $0 \leqslant p < 3 - \sqrt{6}$. Therefore, the singularity of the particle concentration on the surface of the body is integrable, and with increasing β the singularity becomes weaker, i.e., p decreases. To determine the order of the singularity at points of the plane y = 0, we find the function N(r) introduced in (1.2):

$$N(r) \sim kr^\gamma, \ \gamma=2+2^j a/c, \quad k=\frac{2\pi}{2^j a(2+2^j a/c)}\left(\frac{a+b}{b}\right)^{1+j}\left(\frac{d-c}{d-1}\right)^{2\ a/c}$$

In the case of plane symmetry, we have the inequality $1 + \sqrt{2} < \gamma < 3$; in the case of axial symmetry $\sqrt{6} < \gamma < 3$.

C. We consider the singularity of the concentration that develops on the rotation axis in the case of motion of particles in the velocity field of a viscous carrier phase given by the Burgers solution [13], which in dimensionless form in a cylindrical coordinate system has the form

$$u=-\rho, \ v=\Gamma_\infty[1-\exp(-\rho^2/2)]/\rho, \ w=2z \tag{1.7}$$

Here, u, v, w are, respectively, the ρ, φ, z components of the velocity of the carrier phase, and Γ_∞ is a constant proportional to the dimensionless circulation of the velocity. As characteristic scales for making the quantities dimensionless we have chosen for the coordinates $L = (\nu/A)^{1/2}$, for the velocity components AL (here, ν is the kinematic viscosity, $A = du'/d\rho'$ ($\rho' = 0$), and the prime denotes the dimensional quantities). The Burgers solution can always be represented in the form (1.7) by choosing the above scales to go over to the dimensionless form. In [6], Zheleva and Stulov studied a self-similar solution of the particle equations of motion (1.1) of the form (the scale of the velocity components is AL)

$$u_s=f_s(\rho^2)/\rho, \ v_s=\Gamma_s(\rho^2)/\rho, \ w_s=zg_s(\rho^2) \tag{1.8}$$

Substituting (1.8) in Eqs. (1.1) written down in cylindrical coordinates for parameter values $\Gamma_s(\infty)=\Gamma_\infty$, $\beta>\Gamma_\infty^2/4$ we can obtain [6] a solution in the neighborhood of the z axis that in our notation is

$$\begin{gathered} u_s=-a\rho, \ v_s=b\rho, \ b=[(\beta/2)(4-\beta+((4-\beta)^2+4\Gamma_\infty^2)^{1/2})]^{1/2}/2 \\ a=\beta(1-\Gamma_\infty/2b)/2, \ g_s(0)=\beta[-1+(1+8/\beta)^{1/2}]/2 \end{gathered} \tag{1.9}$$

As the z axis is approached, the particle concentration increases unboundedly [6]. We determine the order of the singularity of the particle concentration on the z axis. From the continuity equation for the particle medium in the cylindrical coordinates in the limit $\rho \to 0$ we obtain the equation for the leading term in the expansion of $n_s(\rho)$:

$$\rho\frac{d}{d\rho}[\ln(\rho n_s)]=\frac{c}{a}-1, \quad c=g_s(0)$$

Hence

$$\ln(\rho n_s)=(c/a-1)\ln\rho+B \tag{1.10}$$

Here, B is a constant whose value must be found by solving the problem in the entire region of the motion.

From (1.10) in the limit $\rho \to 0$ we obtain

$$n_s \sim B/\rho^p, \ p=2-c/a$$

For $\beta > \Gamma_\infty^2/4$ we obtain from (1.9) $a/c>0$, i.e., the singularity of the concentration on the rotation axis is integrable. In the case of negligibly small swirling of the flow ($\Gamma_\infty = 0$), the relations (1.9) simplify appreciably and become

$$b=0, \ a=[1-(1-4/\beta)^{1/2}]\beta/2$$

In this case, a linear solution of the form $u_s = -a\rho$ exists for $\beta > 4$. Calculating the integral of n_s over the sphere of radius r with center on the rotation axis of the flow, we obtain

$$N(r) \sim \frac{B\pi^{3/2}\Gamma(1-p/2)}{(3-p)\Gamma(3/2-p/2)} r^{\gamma}, \quad \gamma = 1 + c/a$$

Here, Γ is the gamma function.

When there is no rotation ($\Gamma_\infty = 0$) we obtain for $\beta > 4$ an inequality for γ:

$$\sqrt{3} < \gamma < 3$$

With increasing β, the singularity becomes weaker, i.e., $\gamma \to 3$. Motions of dusty gases similar to the one considered above can occur in separator devices, cyclone apparatuses, and also in large-scale vortex phenomena in the atmosphere (tornados, water spouts).

2. Example of a Nonintegrable Singularity of the Particle Concentration

The nature of the singularity is determined by the prehistory of the motion of the particles up to the points of infinite concentration. Under some conditions, nonintegrable singularities can develop.

We consider the distribution of the particle concentration near the wall in the two-phase boundary layer on a flat plate [4]. Suppose that for $x \geqslant 0$, $y \geqslant 0$ the velocity field of the carrier phase in dimensionless form is $u(x, y) = y$, $v = 0$. On $x = 0$, the particles have constant concentration $n_s = 1$ and velocity $u_s = 1$, $v_s = 0$. We choose the length scale L on the basis of the condition $\beta = 1$; then Eqs. (1.1) take the form

$$u_s \frac{\partial u_s}{\partial x} = y - u_s, \quad \frac{\partial n_s u_s}{\partial x} = 0 \tag{2.1}$$

The solution of (2.1) is

$$x = 1 - u_s - y(\ln|y - u_s| - \ln|y - 1|), \; n_s = 1/u_s$$

For $x > 1$ when $y \to 0$ we have $y \ln|y - u_s| \to -c$, where c is a positive constant. Therefore, for $x > 1$, $y \to 0$ we have $u_s \sim y$, $n_s \sim 1/y$, i.e., the singularity of the concentration on the surface of the plate is not integrable.

3. Calculation of the Distance Between the Particles at Points of Infinite Concentration

The unbounded growth of the particle concentration in the examples considered above indicates a decreasing distance between the particles near the singularities. In the case of nonintegrable singularities, any finite volume containing points of infinite concentration contain infinitely many particles, and this means that there is an unlimited decrease in the distance between them. The assumption that there is no interaction between the particles, the basis of the model (1.1), cannot be satisfied in this case. The model must be augmented by taking into account the interaction of the particles or introducing a discontinuity surface of "sheet" type.

In the case of integrable singularities of the concentration, the distance between the particles does not decrease to zero. The coordinates of the particles are not known exactly, the distance between the particles is a random variable, and to determine it the methods of probability theory must be used.

Suppose that a disperse mixture moves in the volume V. Suppose that at the time t_0 we obtain from the continuity equation of the particle medium the concentration distribution $n'_s(x)$. Here and in what follows, the prime identifies the dimensional quantities. Following [9], we determine the distance between the particles as follows. Suppose a test particle arrives at the point x_0. We take the random variable l', which is equal to the distance from the test particle to the particle nearest to it, the distance between the particles at the point x_0. We construct the distribution function $F_l(r')$ of the random variable l'. We denote the $P\{A\}$ the probability of the event A. By definition [14]

$$F_l(r') = P\{l' \leqslant r'\} = 1 - P\{l' > r'\}$$

Here, $P\{l' > r'\}$ is the probability that in the sphere $S(x_0, r')$ there is no other particle except the test particle. In introducing the continuous medium of noninteracting particles, we assumed the particles to be independent, i.e., (the coordinates of the particles to be independent random variables). As a result, for the random variable η, the number of particles (except the test one) in $S(x_0, r')$, the binomial distribution holds [14], the Poisson approximation of which for a large total number of particles in V has the form

$$P\{\eta = k\} = \frac{N^k}{k!}\exp(-N), \qquad k = 0, 1, 2\ldots, \qquad N(r') = \int\limits_S n_s' \, dx'$$

Hence, for k = 0 we obtain

$$P\{l' > r'\} = P\{\eta = 0\} = \exp[-N(r')]$$

The distribution function of the distance between the particles takes the form

$$F_l(r') = 1 - \exp[-N(r')] \tag{3.1}$$

It is convenient to use the distribution function (3.1) to determine the limits of applicability of the model of noninteracting particles at the points of integrable singularities of the concentration. If it is borne in mind that the particles interact only in collisions, then the condition for absence of interaction at the point x_0 is

$$P\{l' \leqslant 2\sigma\} = 1 - \exp[-N(2\sigma)] \ll 1$$

This is equivalent to the condition

$$N(2\sigma) \ll 1 \tag{3.2}$$

Introducing dimensionless variables and using the condition $\sigma/L \ll 1$, we write (3.2) in the form

$$kn_{s0}'L^3(2\sigma/L)^{\gamma} \ll 1 \tag{3.3}$$

Here, n'_{s0} is the characteristic value of the number concentration of the particles far from the points of singularity of the concentration, and k and γ are determined in (1.2). It is convenient to introduce the characteristic value of the volume concentration of the particles: $\alpha_{s0} = n'_{s0}4\pi\sigma^3/3$; then (3.3) becomes

$$C\alpha_{s0}(L/\sigma)^{3-\gamma} \ll 1, \quad C = 2^{\gamma}3k/4\pi \tag{3.4}$$

We show that the condition (3.4) is satisfied in the examples considered in Sec. 1 of integrable singularities of the concentration in a wide range of the determining parameters. In example A, for all $t \geqslant \frac{1}{2}$ the quantity k is bounded: $k < 8\pi\sqrt{2}/5$. In example B, for j = 0 and 1, respectively, we certainly have the inequalities

$$k < 2\sqrt{2}\pi/(1+\sqrt{2}), \quad k < \pi/3$$

Therefore, in these examples the quantity C in (3.4) can be assumed to be a quantity of order unity in almost the entire range $t \geqslant \frac{1}{2}$ (A), $\beta > 2^j 4$ (B). Typical for gas suspensions is the case $\alpha_{s0} \lesssim 10^{-4}$, and therefore in examples A and B for $L/\sigma \lesssim 10^4$ (for example, L = 100 cm, $\sigma = 10^{-2}$ cm) the expression on the left-hand of (3.4) is $\lesssim 10^{-2}$. In example C for $\Gamma_\infty = 0$ there may be a stronger type of singularity ($\sqrt{3} < \gamma < 1 + \sqrt{2}$), and therefore the model of noninteracting particles remains valid in a narrower range of the parameters.

It is of interest to express the mean value $M(l')$ and the dispersion $D(l')$ of the distance between the particles at the points of the singularities of the concentration in terms of the characteristics k and γ of the singularity. In accordance with the definition of [14],

$$M(l')=\int_0^\infty r'\frac{dF_l}{dr'}dr'=\int_0^\infty \exp[-N(r')]dr', \quad D(l')=\int_0^\infty r'^2\frac{dF_l}{dr'}dr'-M^2(l')=2\int_0^\infty r'\exp[-N(r')]dr'-M^2(l') \quad (3.5)$$

In this definition it is assumed that with increasing r' the number of particles in the sphere $S(x_0, r')$ increases so rapidly that the integrals in (3.5) exist. If in (3.5) we introduce the dimensionless variables $r=r'/L$, $n_s=n_s'/n_{s0}'$, then the arguments of the exponentials in (3.5) become $[-N(r)/\varepsilon]$, where $\varepsilon=1/n_{s0}'L^3$ is a parameter that is small for most problems associated with the motion of gas suspensions. If for small r we have $N(r) \sim kr^\gamma$, then in the limit $\varepsilon \to 0$ the leading terms of the integrals in (3.5) can be found by Laplace's method [15] and we can obtain the expressions (to small terms of higher order in ε)

$$\frac{M(l')}{L}=\frac{\Gamma(1/\gamma)}{\gamma k^{1/\gamma}}\varepsilon^{1/\gamma}, \quad \frac{D(l')}{L^2}=\left(\frac{M(l')}{L}\right)^2\left[\frac{2\gamma\Gamma(2/\gamma)}{\Gamma^2(1/\gamma)}-1\right] \quad (3.6)$$

Here, Γ is the gamma function, and k and γ are determined in (1.2). It is convenient to express $\sigma/M(l')$ and $\sqrt{D(l')}/M(l')$ in terms of α_{s0}, σ/L, γ, k. From (3.5), we obtain

$$\frac{\sigma}{M(l')}=\frac{\gamma}{\Gamma(1/\gamma)}\left[\frac{3k}{4\pi}\alpha_{s0}\left(\frac{L}{\sigma}\right)^{3-\gamma}\right]^{1/\gamma}, \quad \frac{\sqrt{D(l')}}{M(l')}=\left[\frac{2\gamma\Gamma(2/\gamma)}{\Gamma^2(1/\gamma)}-1\right]^{1/2}$$

In the region of parameter values for which $M(l')$ at the points of the singularities of the particle concentration becomes of the order of the interaction range of the particles, it is necessary to modify the model of the particle medium by taking into account the interactions of the particles or introducing a discontinuity surface of "sheet" type [8]. If the unbounded growth of the particle concentrations is due to the intersection of their trajectories and the development of nonuniqueness of the velocity field (example A), the question of the collisions of particles moving along intersecting trajectories must be considered separately.

Note also that a condition of the type (3.4) in the regions of unbounded growth of the concentration is at least a sufficient condition of stability of the formations of enhanced particle concentration, since with decreasing l' collisions and interactions of the particles can lead to the development of unsteady phenomena in the flow region.

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INVESTIGATION OF TURBULENT VAPOR—AIR JETS IN THE PRESENCE OF CONDENSATION AND THE INJECTION OF FOREIGN PARTICLES

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A study is made of flow in turbulent jets when there is condensation of the water vapor contained in them. A necessary condition for condensation in vapor—air jets is formulated. Relations are obtained for the regime of equilibrium condensation. An experimental investigation was made of the local characteristics of an isobaric turbulent vapor jet exhausting into air at rest when condensation develops in the jet and foreign condensation nuclei (smoke particles) and charged particles (ions produced in a corona discharge) are introduced into the flow. Measurements were made of the local characteristics of the condensed disperse phase — the Sauter diameter d_{32} of the drops and their volume concentration c_s — using the optical method of an integrating diaphragm. It is shown that d_{32} and c_s increase downstream in the main section of the jet. Specific features of temperature measurements using an end-type microthermocouple were established. Quantitative data were obtained about the influence on the condensation of the thermal conditions and the presence of the foreign particles. The conditions under which there is an intensification of the condensation in vapor—air jets in the presence of ions were determined.

1. Basic Equations. Necessary Condition of Condensation

Processes of condensation of water vapor contained in an adiabatically expanding medium have been investigated for many decades, and the results obtained in this direction have been presented systematically in many monographs (see, for example, [1-3]). However, the mechanism of occurrence of regions of vapor supersaturation in turbulent jet flows has certain specific features, and the questions of condensation in vapor—air jets have by no means been fully studied. Among the earlier publications on vapor—air jets we mention the monograph [3], which gives data on the distributions of the temperature and the velocity in jets and notes the influence of the pulsatory nature of the motion on the development of the condensation. The paper [4] determined the flow regions in vapor—air jets in which condensation can occur, estimated the possible amount of the condensate, and gave the distributions of the temperature and the particle diameters in a number of sections.

The present paper is devoted to analysis of the general features of flows in vapor—air jets, the experimental determination of the characteristics of the condensed disperse phase, and elucidation of the possibilities of controlling condensation by introducing foreign particles into the flow.

We consider a two-phase medium consisting of a gaseous phase (air and water vapor) and a condensed disperse phase (water droplets). We shall identify the parameters

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