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Relationship between coherence length and output power of a multi-longitudinal-mode Ar^+ laser

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Abstract. The relationship between the temporal coherence (i.e., the coherence length) and the output power of a multi-longitudinal-mode Ar^+ laser operated near the threshold current is studied. The experimental measurements show that the coherence length is a hyperbola as a function of the output power. A simple model of the multimode equal-amplitude power spectrum is employed in the theoretical analysis. Good agreement between theory and experiment is obtained. It is shown that for a multi-longitudinal-mode Ar^+ laser, the product of coherence length and the output power is almost a constant.

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As laser applications continue to expand to include areas such as the manufacture of wide-angle large-scene holograms, the holographic display of large depth-of-field engineering structures, and the air-flow field in wind tunnels, the coherent measurements in large-area nondestructive evaluation of defects and the uniformity of large crystalline materials etc., the simultaneous requirements of good temporal coherence and high output power of a laser-light source are more demanded [1–5].

It is known that, though the output power of an Ar^+ laser is far higher than that of a He-Ne laser, its coherence length is much shorter, usually 3–5 cm. In order to improve the temporal coherence and obtain high output power of the Ar^+ laser at the same time, scientists often use the technique of mode selection. When the laser is working in a single-or double-longitudinal-mode configuration, the coherence length can be enhanced several times. However, due to the complexity and high costs of the technique, it is generally difficult to be implemented in the laboratory.

There are many review papers concerning theoretical analyses and experimental measurements on the connection between temporal coherence and other parameters of the longitudinal modes, such as the mode number, the frequency width, Doppler width and the relative intensity of a multi-longitudinal-mode gas laser [6–9]. In order to expand the areas of the applications of the Ar^+ laser in holographic displays, interferometry etc., we measured the coherence length of an Ar^+ laser as a function of its output power. A simple theory was employed to fit the experimental data. Good agreement between the theory and the experiments was obtained. The experimental measurements and the theoretical analyses show: (*i*) the temporal coherence of a multi-longitudinal-mode Ar^+ laser can be enhanced significantly if its output power is reduced properly, and (*ii*) the product of the coherence length and the output power of a multi-longitudinal-mode Ar^+ laser is almost constant.

1 Experimental setup and results

1.1 Experimental setup

The experimental apparatus to study the temporal coherence of a gas laser is shown in Fig. 1, where L_1, L_2 and L_3 are lenses, P_1 and P_2 are diaphragms, BS is a beam splitter, M_1 and M_2 are the fixed and movable mirrors of a Twyman-Green interferometer, respectively,

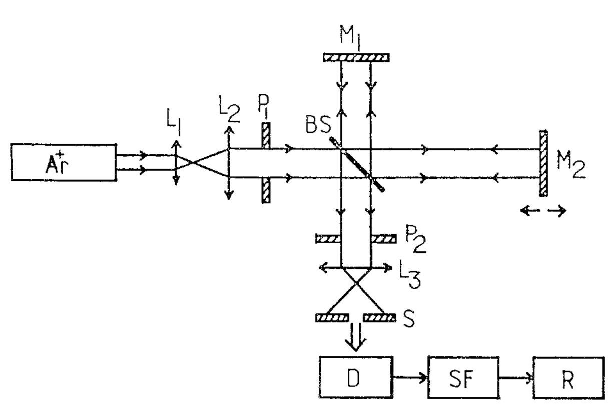

Fig. 1. Experimental setup for temporal coherence measurements

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S is a pinhole ($\varphi < 1$ mm), D, SF and R are photoelectric detector, amplifier and recorder, respectively.

The method of measuring the temporal coherence of a gas laser consists of setting the optical path difference at some value Δl_i, fine-adjusting the interferometer-fixed mirror M_1, and measuring a pair of values I_{max} and I_{min}, we then find the fringe visibility:

$$V_n(\Delta l_i) = \frac{I_{max} - I_{min}}{I_{max} + I_{min}} \quad (1)$$

and the average visibility

$$V(\Delta l_i) = \frac{1}{M} \sum_{n=1}^{M} V_n(\Delta l_i), \quad (2)$$

where n is the number of the nearest pair of I_{max} and I_{min}.

The mirror M_2 is then moved to another path difference Δl_{i+1}, and the above steps are repeated. From this we obtain $V(\Delta l_{i+1})$, ... If Δl is then gradually increased from zero, we can obtain a curve $V(\Delta l) \propto \Delta l$. It can be proved that the fringe contrast $V(\Delta l)$ is connected to the temporal coherence $|\gamma(\Delta l)|$ by [10]

$$|\gamma(\Delta l)| = \frac{1+R}{2\sqrt{R}\cos\theta} V(\Delta l), \quad (3)$$

in which θ is the angle between the polarization directions of the two light beams and R is the optical intensity ratio (I_1/I_2) of the two beams emitted from the pinhole. Due to the divergence of the Gaussian beam and the motion of the movable mirror M_2, the intensity ratio R

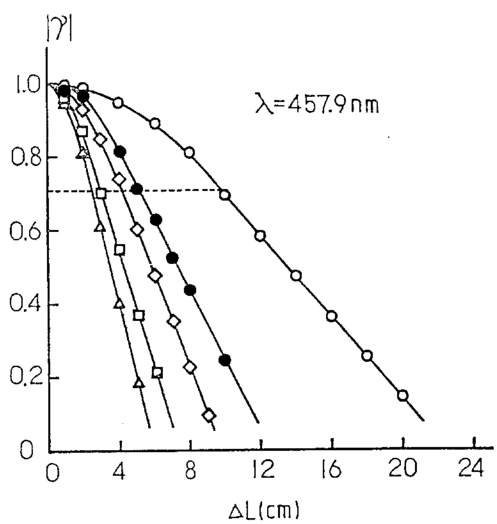

Fig. 2. Temporal coherence as a function of the optical path difference for $\lambda = 457.9$ nm. Experimental data ○: $P = 0.12$ W, $\Delta L_H = 19.8$ cm; ●: $P = 0.25$ W, $\Delta L_H = 10.0$ cm; ◇: $P = 0.30$ W, $\Delta L_H = 8.6$ cm; □: $P = 0.485$ W, $\Delta L_H = 5.7$ cm; △: $P = 0.60$ W, $\Delta L_H = 4.8$ cm; —: theory (14)

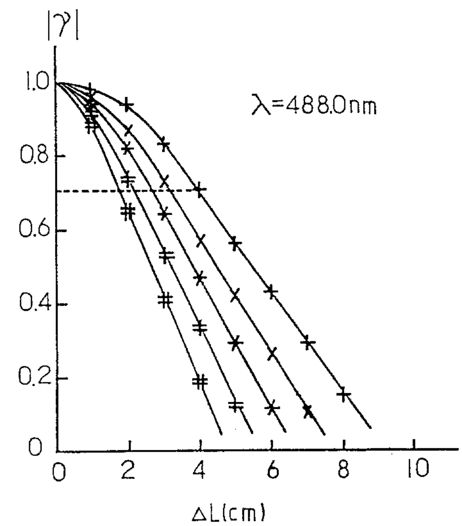

Fig. 3. Temporal coherence as a function of the optical path difference for $\lambda = 488.0$ nm. Experimental data +: $P = 0.8$ W, $\Delta L_H = 6.2$ cm; ×: $P = 1.0$ W, $\Delta L_H = 5.1$ cm; ∗: $P = 1.2$ W, $\Delta L_H = 4.4$ cm; ≠: $P = 1.4$ W, $\Delta L_H = 3.9$ cm; #: $P = 1.6$ W, $\Delta L_H = 3.5$ cm; —: theory (14)

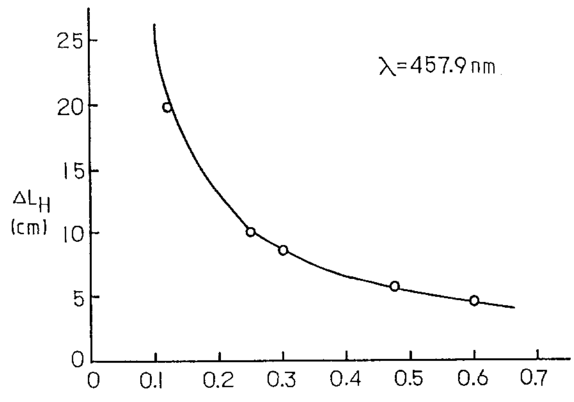

Fig. 4. Relationship between the coherence length and output power for $\lambda = 457.9$ nm. ○: experimental data; —: theoretical predictions from $P \Delta L_H = 2.6$ W cm

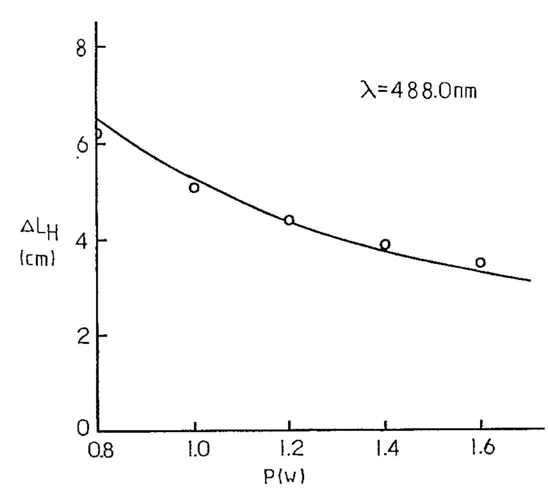

Fig. 5. Relationship between the coherence length and output power for $\lambda = 488.0$ nm. ○: experimental data; —: theoretical predictions from $P \Delta L_H = 5.26$ W cm

is actually a function of the optical path difference Δl, which is denoted as $R(\Delta l)$. In the experiment, we have to ensure that the two light-spot centers are kept coincident in the plane of the pinhole after the movable mirror M_2 is shifted. Furthermore, we must determine $R(\Delta l)$ at the same time as we measure $V(\Delta l)$. Then, we obtain a curve $|\gamma(\Delta l)| \propto \Delta l$ by a point-by-point calculation and normalization according to (3).

Following the definition in [11], when the norm of the temporal coherence $\gamma(\Delta l)$ reduced to $\sqrt{2}/2$, the optical path difference is Δl_c, i.e.,

$$|\gamma(\Delta l_c)| := \sqrt{2}/2 \qquad (4)$$

then the corresponding coherence length is

$$\Delta L_H = 2\Delta l_c. \qquad (5)$$

If we measure the corresponding curve of the temporal coherence (i.e., the curve of $|\gamma(\Delta l)| \propto \Delta l$) at different output powers, then we can get an experimental curve of the coherence length as a function of the output power for a multi-longitudinal-mode Ar^+ laser (i.e., the curve of $\Delta L_H \propto P$) according to the definitions of (4) and (5).

1.2 Experimental results

Since the 457.9 nm and 488.0 nm spectral lines of an Ar^+ laser are frequently used in holography and interferometry, we carried out our experiments on temporal coherence using the above two laser lines. The light source was an Ar^+-laser model 171 with cavity length 1.7 m from Spectra Physics. The output power was changed through the adjustment of the working current of the laser. The experimental results are shown in Figs. 2–5. The curves of the temporal coherence for the wavelength $\lambda = 457.9$ nm and $\lambda = 488.0$ nm are shown in Figs. 2 and 3, respectively. The corresponding curves of the coherence length as a function of the output power are show in Figs. 4 and 5. The solid curves are hyperbolas obtained from the theoretical prediction (17) and are given by

$$P\Delta L_H = \begin{cases} 2.60 \text{ Wcm} \\ 5.26 \text{ Wcm.} \end{cases} \qquad (6)$$

From Figs. 4 and 5, it is clear that, as the output power of the Ar^+ laser is reduced, the temporal coherence is improved considerably and the coherence length is enhanced several times. However, the product of the coherence length and the output power is almost constant.

2 Theoretical analysis

Since the Ar^+ laser we studied was operated under the conditions of low pressure and long cavity length ($L = 1.71$ m), and near threshold current ($I_{th} = 16$ A when $\lambda = 488.0$ nm), the number N of longitudinal modes was quite large and the Doppler width $\Delta\nu_D$ was very wide. Therefore, the model with an equal-amplitude power spectrum proposed by Smith [7] can be employed to describe the structure of the frequency spectrum of the longitudinal modes. When the Ar^+ laser is operated near threshold current, the output power spectrum is stable. Between two adjacent oscillating modes, the separation $\Delta\nu'_q$ contains several cavity mode spaces $\Delta\nu_q$, and the cavity modes do not oscillate [12] within a separation $\Delta\nu'_q$. Assuming the transverse mode of the laser output is the TEM_{00} mode, the frequency separation of longitudinal modes participated in real oscillation is $\Delta\nu'_q\ (=k\Delta\nu_q)$ and each longitudinal mode has a Gaussian line shape $g_d(\nu)$ with equal width $\delta\nu_d$, the power spectral density of a laser-light source with N longitudinal modes in the oscillation band width is given by

$$W_N(\nu) = \left\{ \sum_{n=-(N-1)/2}^{(N-1)/2} \delta[\nu - (\nu_0 + n\Delta\nu'_q)] \right\} \otimes g_d(\nu), \qquad (7)$$

where ν_0 is the laser central frequency, $n = 0, \pm 1, \pm 2, \ldots \pm(N-1)/2$ when N is odd, and $n = \pm 1/2, \pm 3/2, \ldots, \pm(N-1)/2$ when N is even, and

$$\Delta\nu'_q = k\Delta\nu_q \doteq kc/2L, \qquad (8)$$

$$g_d(\nu) = \frac{2}{\delta\nu_d}\sqrt{\frac{\ln 2}{\pi}} \exp\left[-4\ln 2\left(\frac{\nu}{\delta\nu_d}\right)^2\right], \qquad (9)$$

and $\otimes$ denotes the convolution of the two functions, and k can be taken as 2, 3, ..., but is a constant for the given Ar^+ laser and operation condition, and it is the ratio of the separation of actual oscillating mode and cavity mode, i.e., $k = \Delta\nu'_q/\Delta\nu_q$.

Following the Wiener-Khintchine theorem and the definition of the complex temporal coherence function

$$\gamma_N(\tau) = \Gamma_N(\tau)/\Gamma_N(0), \qquad (10)$$

$$\Gamma_N(\tau) = \int_{-\infty}^{+\infty} W_N(\nu) \exp(-i2\pi\nu\tau)\, d\nu, \qquad (11)$$

and the convolution theorem, we obtain the temporal coherence of a multi-longitudinal-mode laser from the laser model with equal-amplitude and equal-frequency width

$$\gamma_N(\tau) = \frac{1}{N} \exp(-i2\pi\nu_0\tau - a^2\tau^2) \sum_{m=1}^{N} \exp\left\{ i[2m - (N+1)] \frac{k\pi c\tau}{2L} \right\}, \qquad (12)$$

where $a = \pi\delta\nu_d/2\sqrt{\ln 2}$. If the optical path difference $\Delta l (= c\tau)$ is used as an independent variable, the norm of the temporal coherence is

$$|\gamma_N(\Delta l)| = \frac{1}{N}\exp\left[-\left(\frac{a}{c}\right)^2 \Delta l^2\right] \times \left|\sum_{m=1}^{N} \exp\left\{i[2m-(N+1)]\frac{k\pi\Delta l}{2L}\right\}\right| = \exp\left[-\left(\frac{a}{c}\right)^2 \Delta l^2\right] \frac{\sin(kN\pi\Delta l/2L)}{N\sin(\pi\Delta l/2L)}. \quad (13)$$

Since the range of the optical path difference in our experiments is quite small (about 0–30 cm), (13) can be approximated further:

$$|\gamma(\Delta l)| = \frac{\sin(kN\pi\Delta l/2L)}{N\sin(\pi\Delta l/2L)}. \quad (14)$$

Since the Ar^+ laser we employed was operated at low pressure and near threshold current, the homegeneous broadening became quite large, and the frequency width δv_h of the hole burning on the gain curve is even larger than the frequency separation Δv_q of two modes of the resonance cavity. When the Ar^+ laser is operated with strong threshold current and low air pressure, the width of homogeneous broadening becomes quite large. It is possible that the width of Lamb dip (i.e., the width of hole burning) on the gain curve is comparable to the cavity-mode separation. However, the longitudinal-mode separation $\Delta v'_q$ of the actual oscillating modes is much large than Δv_q, so the hole on the gain curve is independent for each oscillating longitudinal mode. From the simple model with equal-amplitude power spectrum, we obtain the total output power of a multi-longitudinal-mode Ar^+ laser

$$P(N) = \sum_{i=1}^{N} P_i = NP_0, \quad (15)$$

where $P_i = P_0$ is the output power for each mode.

According to the definitions (4) and (5), the coherence length ΔL_H for different numbers N of longitudinal modes is given by the numerical computations of (14). The curve of ΔL_H vs N is shown in Fig. 6. Since the output power $P(N)$ of the multi-longitudinal-mode laser is proportional to the mode number N, the abscissa of Fig. 6 can be changed to the laser output power $P(N)$. From Fig. 6, it is easy to see that for Smith's model of an equal-amplitude power spectrum, the product of the coherence length ΔL_H and the mode number N of a multi-longitudinal-mode laser is almost a constant for $N \geqslant 10$, i.e.,

$$N\Delta L_H = 303 \text{ cm}. \quad (16)$$

Putting (15) into (16), the theoretical expression of the coherence length and the output power of a multi-longitudinal-mode laser is given by

$$P(N)\Delta L_H = 303\, P_0 \text{ mW cm}. \quad (17)$$

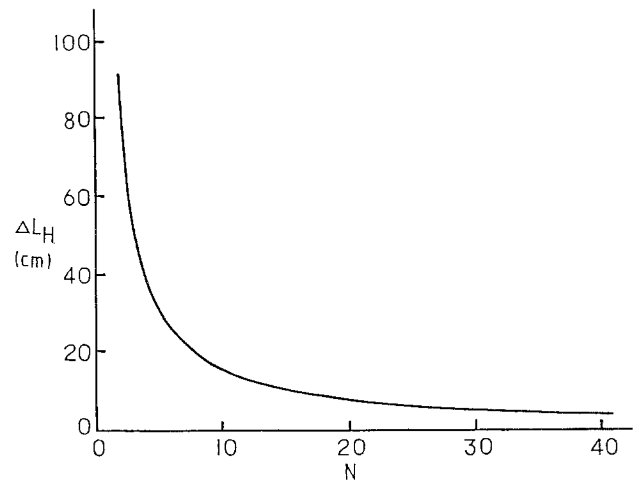

Fig. 6. Coherence length as a function of mode number

When the output power P_0 for each mode is set equal to 8.58 mW for $\lambda = 457.9$ nm and 17.36 mW for $\lambda = 488.0$ nm, (17) is exactly (6).

From (15), (16) and (17), we can see that the temporal coherence of an Ar^+ laser is improved considerably when the output power is reduced. This is because the number of longitudinal modes participating in laser oscillation is reduced. The constant product of the coherence length and output power mainly results because the output power of a laser is almost proportional to the number of oscillating longitudinal modes.

3 Discussion and conclusion

The temporal coherence of an Ar^+ laser has been investigated experimentally and a simple model has been employed in the theoretical analysis. Good agreement between theory and experiment has been obtained. However, there are several interesting remarks and conclusions.

1. For a given Ar^+ laser spectral line, the coherence length decreases monotonically as the output power increases. The absolute value of the rate of change of the coherence length becomes gradually smaller as the output power becomes progressively larger.

2. Within a certain power range, the coherence length is inversely proportional to the output power, i.e.,

$$P\Delta L_H = C, \quad (18)$$

in which C is a constant related to the wavelength λ and the output-power interval ΔP.

3. From Smith's model with an equal-amplitude power spectrum, the product of "coherence length and output power" is a constant for a multi-longitudinal-mode laser when the mode number N is quite large ($N \geqslant 10$).

4. Within a certain power interval (i.e., a certain range of longitudinal mode numbers), the model of an equal-amplitude power spectrum can well explain the experimental measurements on the connection between the coherence length and the output power of a multi-

longitudinal-mode Ar^+ laser. However, when the output power is extremely low or high (i.e., the number of longitudinal modes is extremely small or large), large deviations between theory and experiment occur. This is because the height of the amplitude for each oscillating mode is actually not the same. Though the theoretical value of $P \Delta L_H$ from the model with an equal-amplitude power spectrum agrees well with the measured data within a certain power interval, the N appearing in (14, 15) should be interpreted as the equivalent number of longitudinal modes but not the actual number of oscillating modes N' (usually, $N' \geqslant N$). Hence, the frequency spectrum of the longitudinal modes in an Ar^+ laser is modulated by the Gaussian-gain curve.

5. When the operating current of the Ar^+ laser is reduced, the coherence length is enhanced by many times as the output power is progressively decreased. For example, when the output power of the 457.9 nm line is decreased by a factor of five, the coherence length increases about four times. Moreover, when $P = 120$ mW, $\Delta L_H = 20$ cm, its coherence length is equivalent to that of a He-Ne laser with cavity length $L = 1.5$ m, but its output power is twice as large as that of the He-Ne laser. Furthermore, though the output power of the 488.0 nm line of the Ar^+ laser is five times higher than that of the 457.9 nm line, the sensitization efficiency of the 457.9 nm line is five more than that of the 488.0 nm line when photoetching film or dichromated gelatin is used as the holographic recording medium. The product of the output power and the sensitization efficiency of the two spectral lines is similar, but the coherence length of the 457.9 nm line is three to six times longer than that of the 488.0 nm line. Therefore, when its output power is reduced properly, a multi-longitudinal-mode Ar^+ laser without mode selection still shows considerable promise for applications related to large field-of-view holograms, large-area non-destructive evaluation of defects and large-scene coherent measurements.

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