BRIEF COMMUNICATIONS AND LETTERS TO THE EDITOR

FOCUSING PROPERTIES OF A SEMIBOUNDED PLASMA LINEAR LAYER IN THE CASE OF AN INTERNAL SOURCE

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In [1–4] in the case of an external source the author investigated salient features of the refraction and focusing of spherical waves in

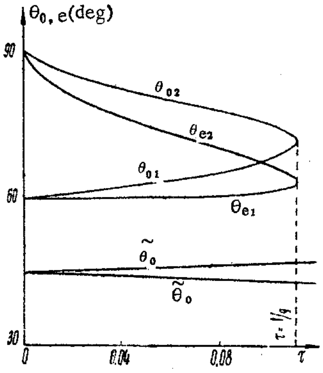

Fig. 1. The quantities $\tilde{\theta}_{0,e}$, $\theta_{01,2}$, and $\theta_{e1,2}$ as functions of τ for rays passing through singular points of the caustic.

inhomogeneous media, namely the failure of the local-field principle and the occurrence at the caustic of nodes, cuspidal points, knots, or isolated "caustic foci."

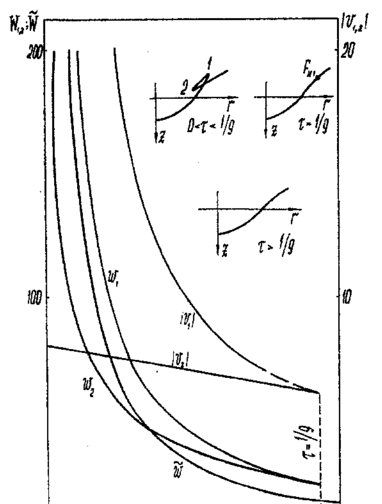

Fig. 2. The quantity $\tilde{w} = \tilde{r}/z_0$, the coordinates $w_{1,2} = r_{1,2}/z_0$ and $v_{12} = z_{12}/z_0$ of singular points of the caustic, and the shape of the caustic as functions of τ (of the value a of the ε-gradient for z_0 = const).

The present communication considers a semi-infinite plasma linear layer

$$\varepsilon(\omega, z) = 1 - a(\omega) z \qquad \left(z > 0, \quad a = \frac{4\pi e^2}{m\omega^2}\frac{dN}{dz} > 0\right) \tag{1}$$

in the case of an internal source ($z = z_0 > 0$ and $r = 0$ are the dipole cylindrical coordinates).

As is well known, the refraction of spherical waves in an unbounded inhomogeneous medium with $\varepsilon(z) = 1 - az (-\infty < z < \infty)$ is accompanied by "ideal" focusing of the field and the formation of a nonsingular caustic. It can be shown that a plane boundary of separation at $z = 0$ imposes on the field an additional focusing effect, as a result of which singular points can arise at the caustic in free space as analyzed below.

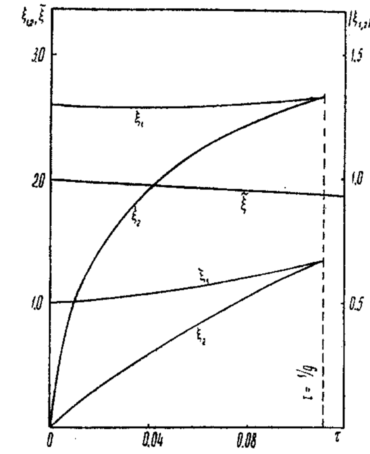

Fig. 3. The quantity $\tilde{\xi} = a\tilde{r}$ and the coordinates $\xi_{1,2} = ar_{1,2}$ and $\zeta_{1,2} = az_{1,2}$ of the singular points of the caustic as functions of τ (of the source coordinate z_0 for a = const).

In the linear layer (1) the ray paths are parabolas [1, 5]

$$z = z_t - \frac{a}{4\alpha^2}(r - r_t)^2, \tag{2}$$

where $\alpha = \sin\theta_e = (1 - \tau)^{1/2}\sin\theta_0$, $z_t = (1 - \alpha^2)/a$, $r_t = [(1 - \tau)/a] \cdot \sin(2\theta_0)$, $\tau = az_0$, θ_0 is the angle between the direction of the wave

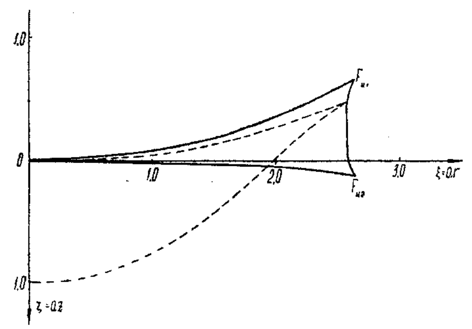

Fig. 4. Locus of the cuspidal points of the caustic for a variation of the parameter τ (the dashed curve represents the caustic for $z_0 = 0$).

normal and the positive direction of the z axis (the ray's angle of emergence from the source), θ_e is the ray's angle of emergence from the inhomogeneous medium, and z_t and r_t are the coordinates of the ray's turning point.

The equation of the refraction caustic of the family of rays (2) has the form

$$v = \frac{1}{\tau} - \frac{\tau}{4(1-\tau)} w^2, \tag{3}$$

where $v = z/z_0$ and $w = r/z_0$. According to (3), the refraction caustic is the surface of a paraboloid of revolution intersecting the plasma boundary $z = 0$ along a circle of radius $\tilde{r} = 2(1 - \tau/a)^{1/2}$. The refraction caustic (3) is nonsingular and has the physical meaning of a smooth refraction-shadow boundary. From (2) and (3) we can obtain θ_0 for rays passing through the points of intersection of the caustic (3) with the boundary $z = 0$

$$\tilde{\theta}_0 = \arcsin [(2-\tau)^{-1/2}]. \tag{4}$$

A considerably more complicated ray picture and a more complicated caustic shape are obtained in the free half-space $z < 0$. The caustic equations for $z < 0$ have the form [4]

$$z = \cos^2 \theta_e \frac{dr_e}{d\theta_e},$$

$$r = r_e - \cos \theta_e \sin \theta_e \frac{dr_e}{d\theta_e}, \tag{5}$$

whence, for the linear law (1), we obtain

$$v = 2\tau^{-1} \cos^2 \theta_e \{\cos(2\theta_e) +$$

$$+ \cos\theta_e [\cos(2\theta_e) - \tau] (\cos^2\theta_e - \tau)^{-1/2}\},$$

$$w = \tau^{-1} \sin(2\theta_e) [1 - \cos(2\theta_e)] +$$

$$+ 2\tau^{-1} \sin\theta_e \{\cos^2\theta_e [1 + \tau -$$

$$- \cos(2\theta_e)] - \tau\} (\cos^2\theta_e - \tau)^{-1/2}. \tag{6}$$

As can be seen from (5), the caustic in free space is formed only by rays with $\tilde{\theta}_0 < \theta_0 < \pi/2$, since only for them is $dr_e/d\theta_e < 0$. As $\theta_0 \to \pi/2$, we have $dr_e/d\theta_e \to \infty$ and the caustic (6) recedes asymptotically to infinity. The asymptote equation has the form

$$w = (2 - v)\tau^{-1/2}(1-\tau)^{1/2}.$$

It is important that the external caustic (6) can have cuspidal points at which the caustic reverses its direction and can form a knot. Rays passing through cuspidal points of the caustic give $dv/d\theta_e = 0$ ($ds/d\theta_e = 0$), whence

$$\theta_{e_{1,2}} = \arccos\left[\frac{3 + 14\tau - 9\tau^2}{8(3-2\tau)} \pm \right.$$

$$\left. \pm \frac{3(1-\tau)}{8(3-2\tau)} (1-\tau)^{1/2}(1-9\tau)^{1/2}\right]^{1/2}. \tag{7}$$

The coordinates r and z of cuspidal points are determined by Eqs. (6) and (7). Calculation results shown in Figs. 1–8 enable us to follow the evolution of the ray picture and the caustic for a variation of the gradient a (Fig. 2) and of the source coordinate z_0 (Fig. 3).

As can be seen from Figs. 1–3, the meridional cross section of the caustic (6) will have two cuspidal points of the first kind only if $\tau < 1/9$; a knot occurs on the caustic lying entirely at $z < 0$. In this case the shadow-region boundary passes along distinct branches of the singular caustic emerging from the "caustic node." As $\tau \to (1/9)-0$, the knot on the meridional cross section of the caustic contracts to a "caustic focus" F_{k1} (Figs. 2, 4) with coordinates $\xi_{F_{k1}} \equiv ar_{F_{k1}} = 8/3$ and $\zeta_{F_{k1}} \equiv az_{F_{k1}} = -2/3$, which is more "luminous" than all the remaining points lying either on the caustic or outside it. If $\tau > 1/9$, the external caustic and hence also the joint caustic become nonsingular. We recall [1] that in the case of an external source a "caustic focus" occurred at $t = -\tau = 2/3$ at a point F_{k2} having coordinates $\xi_{F_{k2}} = \xi_{F_{k1}} = 8/3$ and $\zeta_{F_{k2}} = \tau_{F_{k1}} = 1/9$ (Fig. 4).

Figure 4 shows the curves $0F_{k1}$, $F_{k1}F_{k2}$, and $0F_{k2}$ on which cuspidal points of the caustic can be found for various values of the parameter $\tau \leq 0$. Results of Figs. 3 and 4, and Fig. 5 of [1], show the possibility of extending the ray reciprocity theorem formulated by Babich in [6] to the case of singular caustics having nodes and cuspidal points or isolated "caustic foci." For one and the same type of source the nature of the ray-field singularity must be retained at reciprocal points. The extended ray-reciprocity theorem enables us to analyze easily the case of an internal source in the inhomogeneous media considered in [1–4] for an external source.

Summing up results of the present paper and of [1], we note that for the linear semi-infinite layer (1), the caustic of a spherical wave will be nonsingular only for $\tau > 1/9$ (an internal source) and $\tau < -2/3$ (an external source). A singular caustic is formed if $-2/3 \leq \tau \leq 1/9$.

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