*Applied Scientific Research* **51:** 507–512, 1993.
*F.T.M. Nieuwstadt (ed.), Advances in Turbulence IV.*



# Interaction of Isotropic Turbulence with a Normal Shock Wave

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**Abstract.** The evolution of incompressible and compressible isotropic 2-d turbulent fields interacting with a normal shock wave up to Mach numbers of 2.4 was investigated by means of direct numerical simulation using an ENO scheme. A comparison of statistics with linear analysis results is presented. Vorticity amplification in the DNS agrees well with the linear theory. Energy spectra are enhanced more in the small scales than in the large scales for incoming incompressible turbulence. The amplification rate for initially compressible turbulence is comparatively small.

**Key words:** shock/turbulence – linear analysis – ENO

## 1. Introduction

Proper turbulence modeling of shock-boundary layer interaction is a difficult yet unsolved problem of aerodynamics and its premise is a profound understanding of the physics of compressible turbulence. An idealized case worthwhile studying is the interaction of isotropic turbulence with a shock wave. Kovasznay (1953) showed that one can - up to first order - decompose a compressible field into an acoustic, an entropy and a vorticity (or shear) mode. Linear analyses have been performed by Ribner (1955), McKenzie (1968) and others for the interaction of a discontinuity with single modes. Application of linear analysis to the interaction of a shock wave with isotropic, incompressible turbulence has been reported by Lee et al. (1991). By means of direct numerical simulation of 3-d compressible isotropic as well as sheared turbulence, recently conducted by Blaisdell et al. (1991) and Sarkar et al. (1992), it was found that there are significant effects of compressibility depending on the initial conditions. Experiments of grid turbulence/shock interaction by Keller et al. (1990) show an enhancement of density fluctuations and a length scale increase through the interaction.

## 2. Results

### 2.1. Basic Mechanisms, Linear analysis

When an initially normal shock interacts with a turbulent flow there are several mechanisms that are responsible for the change of the turbulent field. These are mean compression, shock deformation, shock unsteadiness, the intensification of vorticity fluctuations and the production of small scales. Linear analysis of a shock wave interacting with a turbulent field is conducted here in the mathematical framework of McKenzie et al. (1968) in a shock-fixed coordinate system. Effects of shock front curvature and unsteadiness are incorporated in this approach. Results will be compared with DNS data and added in graphs.

### 2.2. Numerical Simulation

The numerical procedure and general parameters of the flow are given in the appendix.

**Mean compression.** Figure 1 illustrates the vorticity field at time $t = 1.70$ for cases n6400712 and n6401424. While the same inflow conditions for the velocity field are used in both cases, the mean sound speed differs. One can clearly make out the 1d compression effect which leads to an intensification of the vorticity fluctuations through the $\omega_i u_{j,j}$-term in the $\omega_i$-transport equation close to the shock. Further downstream the flow reflects the tendency to return towards isotropy (case n6401424 in the following figure).

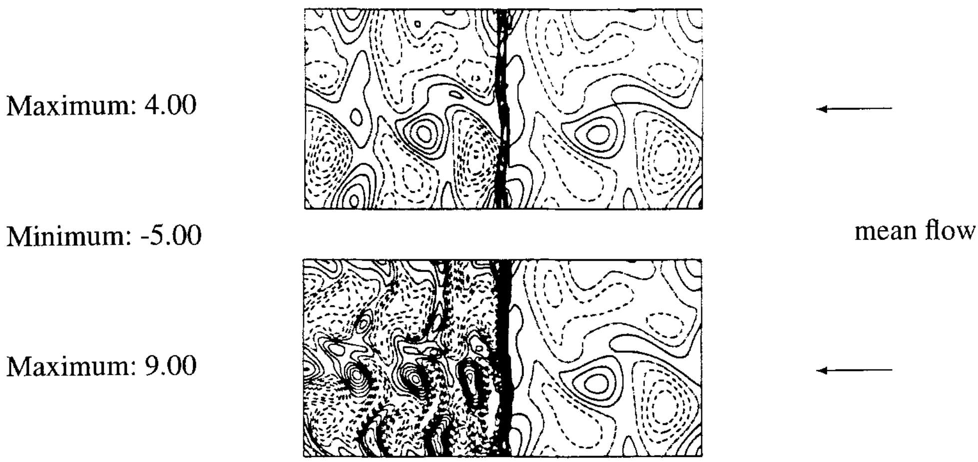


Fig. 1. Vorticity contours of cases n6400712 (top) and n6401424 (bottom). Flow is from right to left. Superimposed are contours of maximum dilatation.

**Shock unsteadiness.** Shock oscillation period and deviation of the shock front from its mean position in units of the initial Taylor microscale in the x-direction $(\lambda_\perp = \overline{u_1^2}/\overline{u_{1,1}^2})$ are given in table I. Shock deformation is not only a function of the Taylor microscales (Lee et al., 1992), but also of the amount of compressibility of the flow.

**Intensification of rms-values.** Figure 2 contains a comparison of linear prediction of vorticity enhancement with DNS data. It can be seen that for the quasi-incompressible initial data (n6400712, n6401424) the vorticity amplification is well

TABLE I

Approximate shock oscillation period $\tau_{\text{shock}}$ and maximum deviation from mean position.

| Case | n6400712 | n6401424 | n6400712c |
|---|---|---|---|
| $\tau_{\text{shock}}$ | 0.40 | 0.40 | 0.40 |
| $(x - \bar{x}_{\text{shock}})/\lambda_{\perp}$ | 1/7 | 1/5 | 1/2 |

predicted by linear theory. For the compressible inflow conditions (n6400712c) linear analysis clearly disagrees with the DNS data. This is to be expected, since the flow upstream is not - as assumed in the linear analysis - a pure shear flow, but contains pressure and entropy fluctuations.

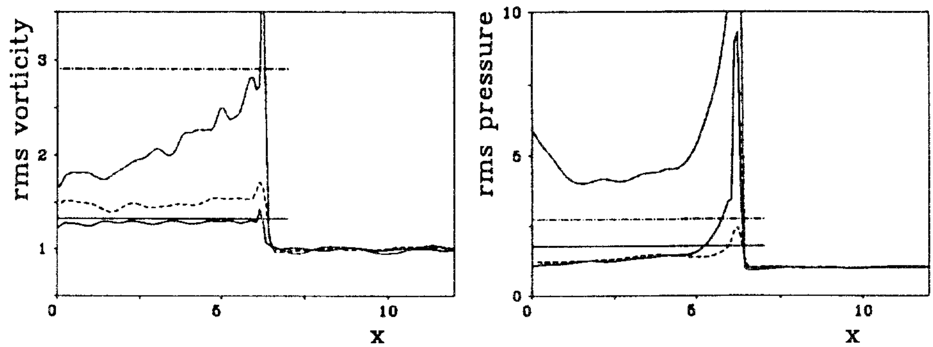


Fig. 2. Rms vorticity (left) and rms pressure amplification (right) for the 3 cases n6400712 (solid), n6401424 (dashed-dotted) and n6400712c (dashed). Straight lines denote predictions of linear analysis.

Figure 2 also shows the amplification rates of the pressure rms values. The enhancement of case n6401424 is largest, as expected, but the values of the two Ma=1.20 cases differ right behind the shock. The compressibility in the inflow conditions leads to a weaker amplification of the pressure fluctuations.

**Energy spectra.** 1-d TKE spectra in figure 3 before and after the shock illustrate that there is an energy increase mainly in the small scales, which is consistent with linear analysis (Lee et al, 1992). The strong amplification on the shock (dashed lines) is due to intermittency effects in the shock unsteadiness as Debieve et al. (1986) pointed out. There is no amplification of energy in the compressible flow case n6400712c.

**Length scales.** The small scale enhancement is consistent with computations of the two Taylor microscales (not shown), that are both reduced by 11 % with respect to their preshock values in the n6400712 case. A reduction in length scales cannot be found in the compressible case n6400712c. In the higher Mach number run n6401424 the microscales are both reduced by almost 40 %.

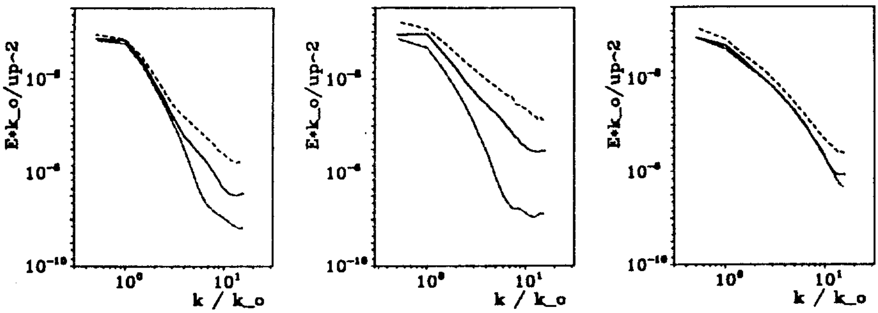


Fig. 3. Normalized 1-d energy spectra before (dashed-dotted lines), after the interaction (solid lines) and on the shock (dashed lines) for cases n6400712(left) and n6401424 (middle) and n6400712c (right).

## 3. Conclusions

From the present results we conclude that the statistical modeling of shock induced turbulence amplification is a subtle task in which special care has to be taken of compressibility effects of the preshock conditions. They do have a strong influence on the behaviour of turbulence passing through the shock in terms of amplification and energy spectra changes, leaving length scales unchanged. Length scales using incompressible inflow data are reduced. Simulation of 3d shock/turbulence interaction is underway.

## Acknowledgements

This work was supported by the German Research Society (DFG). We also gratefully acknowledge the support of the Leibniz- Computing-Center of the Bavarian Academy of Sciences. We shouldn't forget to thank F. Unger for his great graphics package *Polly*.

## Appendix

## A. Parameters, numerical method and boundary conditions

The governing equations are the conservation laws for mass, momentum and total energy along with the equation of state for a thermally and calorically ideal gas ($\gamma = 1.40$). Dimensionless viscosity has a power law dependence on temperature of $\mu(T) \sim T^{0.76}$, the Prandtl number is constant (Pr$= 0.70$). Initial Reynoldsnumber $Re_{\lambda_o}$, based on the Taylor microscale for the shock normal direction is 16 in all cases. The turbulent Mach number is defined by $\mathrm{M}_{\mathrm{t,o}} = u_{\mathrm{rms,o}}/\bar{c}_{\mathrm{o}}$ with $u_{\mathrm{rms,o}} = (u_{\mathrm{i}}' u_{\mathrm{i}}')_o^{1/2}$ and $\bar{c}_o$ being the initial mean speed of sound. For nondimensionalization of space , velocity, time, pressure, density and temperature we use $L = 5\lambda_{\perp}/4$, $u_{\mathrm{rms,o}}$, $L/u_{\mathrm{rms,o}}$, $\bar{\rho}_{\mathrm{o}}$, $\bar{\rho}_{\mathrm{o}} u_{\mathrm{rms,o}}^2$, $\bar{T}_{\mathrm{o}}$ respectively.

TABLE II

Computed test cases and their parameters. Index 'c' refers to the compressible, 'o' to the initial and 's' to shock conditions.

| Case | n6400712 | n6401424 | n6400712c |
|---|---|---|---|
| $M_s$ | 1.20 | 2.40 | 1.20 |
| $M_{t,o}$ | 0.07 | 0.14 | 0.07 |
| $\rho_{rms,o} = T_{rms,o}$ | 0.01 | 0.01 | 0.20 |
| $\chi_o$ | 0.05 | 0.05 | 0.50 |

Statistical averages are gathered by assuming the direction parallel to the shock to be homogeneous. We ran $N$ different realizations of the flow by saving decaying compressible turbulent fields at every time step and using them as inflow conditions. Periodic boundary conditions apply to the y-direction and non-reflecting outflow conditions to the left boundary. The grid is equidistant with 128x64 points in a $4\pi$x$2\pi$ domain. Initial conditions are set by applying the algorithm of Erlebacher et al. (1990) and energy spectra of the type $E(k) = k^4 \cdot \exp(-2k^2/k_o^2)$ with $k_o = 1.80$ are used for all initial fluctuations. The velocity field is split into a compressible (irrotational) and incompressible (divergence-free) component and the ratio $\chi$ of the compressible kinetic energy to the total kinetic energy in the field is specified.

The numerical scheme is a 'method of lines' ENO scheme based on the original paper of Harten et al. (1987) with a TVD time integration by Shu et al. (1989). We use a scheme of formally third order in space and second order in time, although higher order is generally possible. >From many test runs (Hannappel, 1991) we conclude, that ENO schemes are well suited to study turbulence with shocks, even with strong shocks, keeping the high order of accuracy up to discontinuities.

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