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Mixing in Horizontally Heterogeneous Flows

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Abstract. An experimental study of mixing across density interfaces produced by laterally heterogeneous turbulence is presented in this paper. The turbulence is generated by a flow or air bubles rising through a density interface produced by brine and fresh water. The mixing efficiency, η, of the process is measured comparing the increase in potential energy with the available kinetic energy. We find that there is a decrease in the global mixing efficiency of the process with the length of the tank, the shape of $\eta(Ri)$ depends also on the air flow producing the turbulence, showing a geometrical limit to the ammount of kinetic energy which may be used for mixing.

Key words: stratified flows – mixing efficiency – bubble-driven flows

1. Introduction

Mixing across stratified fluids in geophysical situations, often takes place in horizontally heterogeneous processes, where the available kinetic energy is confined to a small region of the fluid, In these situations, vertical mixing is afected by the horizontal transport of mass. Important mixing proceses are associated with the formation of fronts, such as sea and mountain brezes, tidal stirring, etc. In most cases, and due to the intermittency of energetic processes, mixing is localized in time and space.

There are conflicting views on the shape of the mixing efficiency versus Richardson number curve for homogeneous flows, some authors, [1][2], have found a decrease in mixing efficiency at high Richardson numbers while others [3], find a monotonic relationship between mixing efficiency and Richardson number.

2. Description of the experiment

The experimental apparatus is shown in figure 1. Similar experiments using grid generated turbulence have been reported by [4]. The experiments used brine and fresh water in order to form a density interface.

The density interface and the turbulence are characterized by means of a local Richardson number $Ri = \frac{g\Delta\rho\ell}{\rho u'^2}$, where g is gravity, $\Delta\rho$ the density step, ρ the reference density. The integral lengthscale of the turbulence, ℓ, and the r.m.s. turbulent velocity u', are related to the air flow Q, which induces horizontally heterogeneous mixing across the interface.

After filling up to 15 cm of the box with brine of a preset density, another layer of 15 cm of fresh water was carefully poured onto a floating sponge in order to avoid

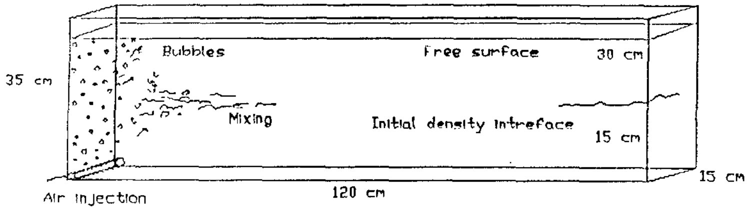

Fig. 1. Description of the experimental apparatus.

diffusing the interface. Micro suction-conductivity probes, described in [2], were used. The air flow injected varied between $Q = 3.0cm^3/s$ and $Q = 35cm^3/s$.

The mixing efficiency is defined as the relation between the increase in potential energy of the fluid, ΔPE and the kinetic energy that produces the mixing, ΔKE. $\eta = \Delta PE/\Delta KE$. Initially there is a sharp density interface and after a time t_m the whole column of fluid is well mixed. The increase in potential energy in the mixing process is

$$\Delta PE = gS \int_0^H [\rho_{final}(z) - \rho_{initial}(z)]\, z\, dz, \tag{1}$$

which for total mixing gives $\Delta PE = \frac{1}{2} gS\Delta\rho h^2$, where h is the height of both the brine layer and initial fresh layer. S is the surface of the tank's base and $H = 2h$ the fluid level.

The kinetic energy available for the mixing process is estimated as the energy disipated by the drag on an air bubble times the number of bubbles. We follow the same procedure as in [5], based in [6], obtaining

$$\Delta KE = \frac{1}{2}\, \rho C_d \pi (\frac{d}{2})^2 V^2 \frac{V}{2d} N t_m L \ , \tag{2}$$

where C_d is the drag coefficient of a single bubble of diameter d rising at a terminal speed V, N is the number of holes producing bubbles. Using the definition of Ri, we obtain that $\eta = K\ Ri\ t_m^{-1}$, with K function of the experimental parameters.

3. Experimental results

As soon as the air was injected, a line of bubbles rose, reaching their terminal velocity, the bubbles entrained dense fluid into the light fluid layer and produced vigorous mixing at the interface. The convective motion induced by the air flow could be observed clearly by injecting dye near the side where mixing is produced. After a time, t_m, the interface vanished and the fluid was well mixed.

Mixing time increases as Ri increases, and when a critical value of the initial Richardson number is reached ($Ri > 1$) the growth is exponential as seen in figure 2 a). Due to the lateraly non-uniform turbulence, the local Richardson number increases as we move away from the source of turbulent kinetic energy. Different

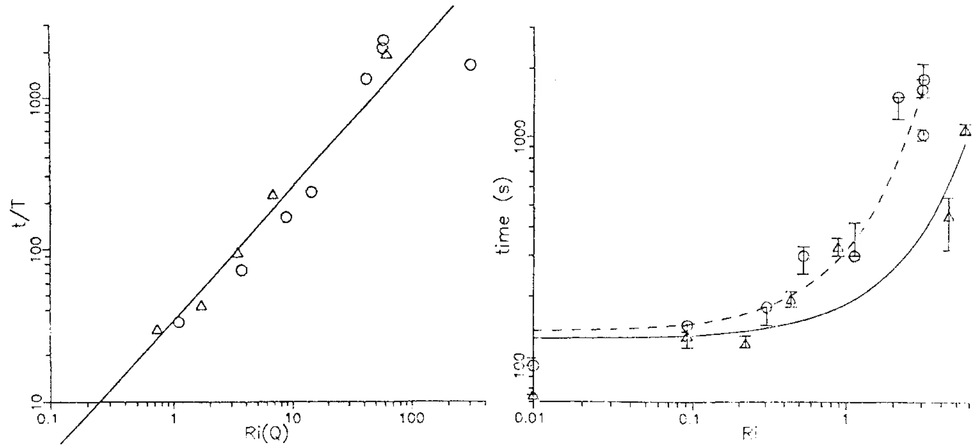

Fig. 2. Mixing time vs. Richardson number.a) Using Ri, based on the rise velocity of the bubbles, $\triangle$ indicates Experiments with $Q > 10$ and $\bigcirc$ indicates $Q < 10$. b) Plot of $t/T(Ri_Q)$, using Ri_Q, based on the air flow.

behaviours of η versus Ri were observed when using different air flow rates, Q. In order to compare experiments with different air flow rates, a Richardson number depending on the air flow was also used $Ri_Q = \frac{g\Delta\rho\ell}{\rho f(Q)}$, where $f(Q) = CQ^{2/3}$, and C depends on the bubble size, this Richardson number takes into account the increase in local shear produced by the bubbles generated by a larger air flux mixing the fluid. Figure 2 b), shows a non-dimensional power fit of $t_m/T(Ri_Q)$, where $T = \sqrt{\frac{H\rho}{g\Delta\rho}}$. The data can be expressed as $\frac{t_m}{T} = k_1\ Ri^n$, giving a mixing efficiency of $\eta = k_2\ Ri^{1-n}$. The value of n may be larger than 1 for large Ri, showing agreement with [1][2].

Point density measurements were made at the centerplane during the mixing process. An example of the two dimensional plots showing the density distribution for an experiment with $Ri = 3.04$ and $Q = 3.1 cm^3 s^{-1}$ is shown in figure 3. Figure 4 a) shows the relationship between the mixing efficiency and the nondimensional length of the tank for similar initial Ri. The mixing efficiency decreases markedly at $L/\ell \approx 60$ in agreement with [4]. The crosses indicate the experimental results of [5].

4. Density fluctuations and mixing

From point conductivity probes placed at selected positions in the tank, density fluctuations were measured. Shown in figure 4 b) is a non dimensional plot of the evolution of the density fluctuations, ρ' at a point 15 cm (h) from the mixing side of the tank. The time is non-dimensionalized with T and the r.m.s. density fluctuations with the initial density difference across the interface, $\Delta\rho$. There is an exponencial decrease of the r.m.s. density fluctuations which varies with Ri.

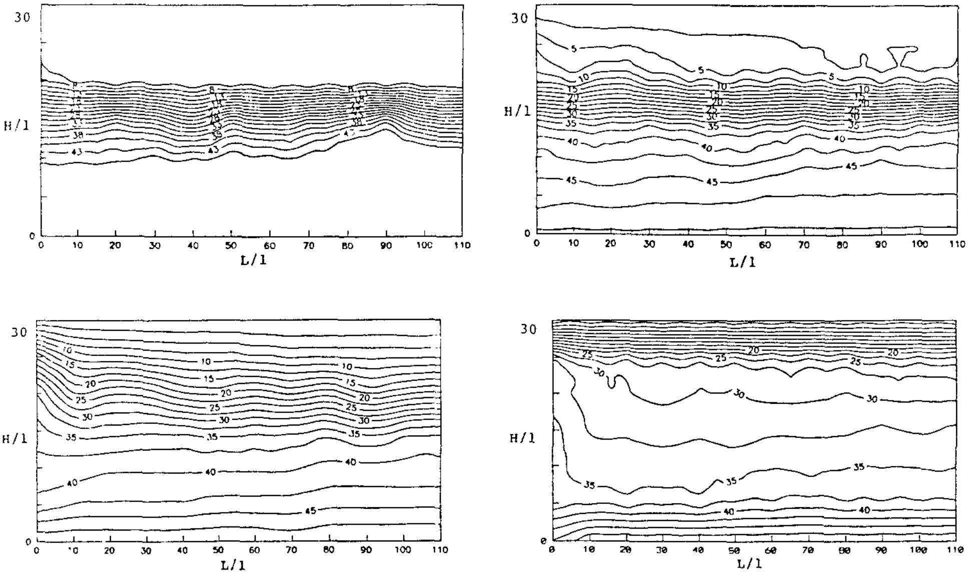

Fig. 3. Two dimensional plots of the density at the centerplane as the two layers mix, times are a) 1 min, b) 5 min, c) 10 min, d) 20 min, from the begining of the experiment.

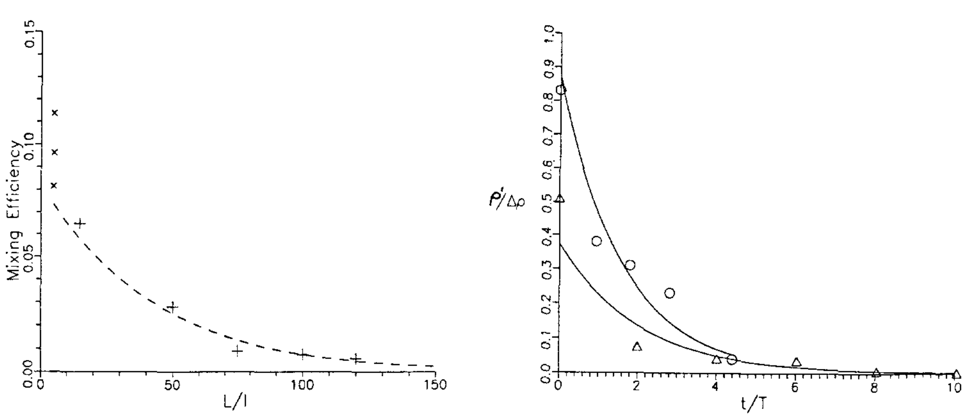

Fig. 4. a) Mixing efficiency vs. length of the tank. b)Density fluctuations vs. non dimensional time, $\triangle$ indicates a high Ri experiment and $\bigcirc$ a low Ri.

Transport equations for velocity and density fluctuations in homogenous turbulence for a zero mean flow can be written as

$$\frac{\partial u'^2/2}{\partial t} + \overline{w' \frac{\partial u'^2/2}{\partial z}} = -\frac{g}{\rho}\overline{\rho' w'} - \epsilon, \tag{3}$$

$$\frac{\partial \rho'^2/2}{\partial t} + \overline{w' \frac{\partial \rho'^2/2}{\partial z}} = -\overline{\rho' w'} \frac{\partial \overline{\rho}}{\partial z} - \chi \ , \tag{4}$$

$$\frac{\partial \overline{\rho' w'}}{\partial t} + \overline{w' \frac{\partial \rho' w'}{\partial z}} = -\frac{g}{\rho}\overline{\rho'^2} - \overline{w'^2}\frac{\partial \overline{\rho}}{\partial z} - \frac{1}{\rho}\overline{\rho' \frac{\partial p'}{\partial x}} - (\kappa + \nu)\overline{\frac{\partial w'}{\partial x_j}\frac{\partial \rho'}{\partial x_j}}, \tag{5}$$

The production of turbulence is due to the velocity fluctuations induced by the bubbles. The terms ϵ and χ are respectively the viscous and diffusive dissipation terms. $-\frac{g}{\rho}\ \overline{\rho'\ w'}$ is the buoyant dissipation of turbulence, and indicates vertical mixing. For stationary flow, using the definition of the scales $L_o = (\epsilon/N^3)^{1/2}$ (Ozmidov), $L_b = w'/N$ (buoyancy) and $L_t = \frac{\rho'}{\frac{\partial \rho}{\partial z}}$, we can write

$$\frac{w'}{\epsilon}\frac{\partial u'^2/2}{\partial z} = -\frac{1}{L_o^2} L_b L_t \frac{\overline{\rho' w'}}{\rho'\ w'} - 1 \ , \tag{6}$$

The interaction between these three scales for different types of instabilties at different ranges of the Richardson number will control the amount of turbulent energy that produces mixing. Note that $\frac{\overline{L_b\ L_t}}{L_o^2}$, can be considered as a flux Richardson number $Rf = \frac{g}{\rho}\frac{\overline{w'\rho'}}{\epsilon}$ or a mixing efficiency, L_t represents a typical vertical distance travelled by fluid particles before either returning to their equilibrium level or mixing. As seen in figure 4, L_t decreases with time, but not exponentially, as the thickness of the interface grows as seen in figure 3. L_t does not vary much horizontally, indicating that there might be significant transport by means of lateral intrusions. The mixing efficiency for bubble generated turbulence can be expressed as $\eta \propto Ri^{1-n(Ri,Q)}$ with $n(Ri, Q)$ a variable exponent found from the time a two layer stratified system takes to mix. Different shapes of the $\eta(Ri)$ curve are possible as noted in [7] for zero-mean flows. A possible explanation is that for very high Ri, there is more time when the interface can support internal waves of higher buoyancy frequency, thus dissipating more energy which could otherwise produce mixing.

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