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Monte-Carlo calculations of LIDAR returns: Procedure and results

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Abstract. The procedure of the Florence group for calculation of LIDAR return from clouds is briefly outlined. The results of the particular case chosen for a comparison with other groups are presented.

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In order to calculate the contribution of multiple scattering to LIDAR returns from clouds or fogs, a Monte Carlo procedure is used. It is a version of a more general Monte Carlo procedure dealing with light propagation in turbid media.

The procedure allows us to take into account radiation polarization and can be employed to deal with homogeneous and inhomogeneous media. According to the situations, suitable versions of the procedure are employed.

The cloud properties are represented by an extinction coefficient σ, a scattering coefficient σ_s, a phase function (or a scattering matrix when polarization is considered), all of them being, in general, dependent on the position in the turbid medium.

Since LIDAR systems for atmospheric research present very small Field Of View (FOV) apertures, the case of a plane stratification normal to the (monostatic) LIDAR beam and field of view axis is considered for the medium properties.

The basic procedure is of the semianalytic type. The code employed builds a series of photon trajectories within the medium: the trajectories are formed by rectilinear tracts with bending points at scattering points. Statistical laws are followed to determine the trajectories.

As is typical for semianalytic forms of the Monte Carlo procedure for this type of problem, given the scattering points, from each of them the probability of a photon reaching the receiver within the FOV solid angle without any intervening further scattering are calculated. The sum of these probabilities, sorted in intervals of time corresponding to chosen intervals of distances run, and normalized to the number of photons used in the procedure, provides the calculated average LIDAR returns for these time intervals. In this way LIDAR returns are obtained as a sum of the contributions of different orders of scattering, which is useful both for detailed information and for the possibility offered of employing scaling formulas. The latter allow one to use the data pertaining to certain values of the extinction coefficient, single scattering albedo, and cloud-LIDAR distance to obtain results pertaining to different values of these parameters, without repeating the calculations.

The scaling relationships, in the form presented in Appendix A, can be employed to the case of a homogeneous cloud like that of the case chosen for the comparison with other groups' results.

The data pertaining to the comparison case will be shown in Sect. 2. In the next section we shall briefly outline some features of our general procedure for dealing with LIDAR returns.

1 Characteristics of the general procedure

1.1 Polarization

Polarization is easily introduced into the Monte Carlo procedures. One has to use Stokes vectors associated to each photon trajectory, and rotation and scattering matrices when the deviation of a photon direction at a scattering point has to be evaluated. The procedure considers modified Stokes vectors, whose first two elements represent the two components of power perpendicular and parallel to the scattering plane (plane of the two photon directions, before and after the scattering point).

Along a photon trajectory, at each scattering point, one has to rotate the incident Stokes vector by means of a rotation matrix [1], in order to have it correctly oriented.

The scattering angle is selected according to a statistics determined by the phase function, which, in this case, is proportional to the mean of the two first elements of the scattering matrix. The azimuthal angle, determining the scattering plane, is selected by means of a probability law connected to the relative values of these first elements. More details of the procedure can be found in [2, 3]. The scaling relationships of Appendix A are valid for both components of received power: parallelly and perpendicularly polarized with respect to a linearly polarized emitted pulse.

1.2 Inhomogeneous cloud

FOV angles in LIDAR sounding of clouds are generally small, on the order of mrad or at most a few tens of mrad. The volume of the cloud contributing to the return has therefore a lateral width of a few meters or tens of meters for a cloud whose distance from the LIDAR is of the order of km. A simple cloud model which can be of interest is that with an extinction coefficient with plane stratification normal to the beam axis (z-axis). Using a Monte Carlo procedure, and assuming a function $\sigma(z)$ to calculate the lengths of the rectilinear tracts of a photon trajectory, one needs a predetermined table containing values of the integral

$$I(z) = \int_0^z \sigma(z')\, dz',$$

where $z=0$ corresponds to the position of the near boundary of the cloud. Given the coordinate z' of a scattering point, the coordinate z of the subsequent scattering point is

$$I(z) = I(z') - \ln(1-R)\cos\gamma,$$

where R is a random number between 0 and 1 and γ is the angle between the photon direction and the z-axis. The other coordinates are determined consequently.

1.3 Continuous variable phase function

As in the case of the extinction coefficient, the small FOV of LIDAR sounding a cloud allows one to limit oneself to considering the case of a plane stratification normal to the FOV axis: z-axis (origin at the cloud border).

The following situation is considered with a continuous variation of the particulate phase function. The extension to the case of a continuous varying scattering matrix, to deal with polarization can be made following the same procedure.

As is predicted by theoretical models and by measurements [4, 5] the radii distribution of water droplets in a tropospheric cloud changes, generally, toward larger radii if one moves inside from the upper or lower border, at least in the first few tens of meters.

A simple model of this variation is presently used for our codes. Two models of the size distributions are considered and a different combination of them is taken at the different cloud depths.

The length of the rectilinear tracts of a photon trajectory are determined by the code according to the procedure indicated above. Given the position of the scattering point, the scattering angle is chosen according to the statistics relevant to one or the other phase function. The decision about which of them is used follows the probability law relevant to their relative contribution to the scattering coefficient, variable with the position. The latter choice is thus made according to a random number extracted.

The use of a combination of more than two size distributions is possible, at the expense of a larger memory occupation for the relevant phase functions.

If one wants to consider polarization, the formalism of Stokes vectors and scattering matrices can be used. This necessitates a further larger occupation of memory, but the procedure for taking into account medium inhomogeneity is essentially the same.

1.4 Variance reduction

Currently, two simple procedures can be used in our codes to reduce the statistical spreading of the Monte Carlo data.

The first one takes into account the scaling relationship allowing one to obtain data for different extinction coefficients, if one has the data for one value of this parameters (A2). Thus, the contributions to LIDAR returns of each order of scattering is calculated at a different value of the optical depth τ of the cloud layer considered. Generally τ is taken nearly equal to the scattering order. Then, the returns are scaled to the desired common value of τ. This procedure proved useful to reduce the number of photon trajectories necessary to obtain the required convergence of the data.

The second procedure, which has been directly derived from that introduced by Platt (6), consists in using a fictitious phase function distorted in order to increase the probability of photons undergoing scattering with angles in the backward direction. As in [6], we use a phase function which is obtained by leaving the part of the original phase function $P(\vartheta)$ unchanged between $\vartheta=0$ and $\pi/2$ (ϑ scattering angle). In the backward part $P(\vartheta)$ is taken equal to $P(\pi-\vartheta)$. A consequent renormalization of this function, symmetrical about $\pi/2$, is then made. It is necessary to apply suitable weights to photons at each scattering event to correct the differences resulting from the different occurrence of the scattering angles. The weight is the ratio between the value of the true phase function at the considered scattering angle and the fictitious phase function at the same angle. For each trajectory a photon carries the product of the weights pertaining to the preceding scattering events, changing along the trajectory.

Tests on the two procedures showed that their relative efficiency depends, apart from other factors, greatly on the order of scattering considered. Generally, for orders larger than about 6 or 7, the first procedure is more effective.

2 Comparison with other results

As has been decided for a comparison of the procedures of the different groups, the case of a homogeneous layer with a size distribution equal to that of a C1 model cloud at a distance of 1000 m from a coaxial LIDAR has been considered.

Other parameters: wavelength $\lambda = 1.064$ µm, extinction coefficient $\sigma = 0.01725\ \mathrm{m}^{-1}$; albedo = 1; emitted pulse: linearly polarized; requested distance resolution 6 m; receiver (FOV): $\alpha = 10$ mrad; and 1 mrad (full width), emitted beam divergence 1 mrad.

For this case, since we decided to take into account polarization, given the size distribution, all the four elements of the scattering matrix were calculated at 1000 values of the scattering angle, so as to allow us to use a linear interpolation for a sufficient accuracy of calculating $P(\theta)$ for a given θ. ($P(\theta)$ = average of the two first diagonal elements of the scattering matrix.)

An examination of the calculated $P(\theta)$ with that provided for the various groups showed some small differences on the order of a few percent. Further analysis, however, showed that the differences in the calculated data is not larger than a few percent. Since some preliminary comparisons showed that the differences between the results of the various groups were even much larger than that, it was realized that the small differences in the phase functions were not such as to disturb the comparison.

Figures 1 to 3 present the numerical results. They were obtained by using 3.2 million trajectories in the Monte Carlo procedure.

Figure 1 refers to the case $\alpha = 10$ mrad. The four curves represent, respectively, a) the ratio of double-scattering to single-scattering contribution to LIDAR return; b) summed contributions of 2nd, 3rd, 4th orders divided by first order contribution; c) ratio of summed contributions from 2nd to 6th orders to first scattering; d) ratio of summed contributions of 2nd to 8th orders to first scattering.

Extension of calculation to higher orders did not give results substantially different from those shown by curve d) for depths up to ≈ 250 m. For larger depths our data have strong fluctuations.

Figure 2 presents the results obtained for $\alpha = 1$ mrad. The four curves represent ratios analogous to those of Fig. 1. However, in this case of much smaller FOV, four orders of scattering were found to be almost completely sufficient to cause the returns, up to $z = 250$ m.

Figure 3 presents the calculated part of the LIDAR return which is polarized perpendicularly to the polarization of the emitted pulse divided by the part with parallel polarization. The two curves refer to FOV = 10 and 1 mrad. In order to perform these calculations it was assumed that the Stokes vectors of the received photons could simply be added element by element, since scattered photons are assumed to arrive with random phases. This is a fundamental assumption for the Monte Carlo procedure which implements the transport scheme.

Since several orders of scattering are involved, the results of Figs. 1–3 are very sensitive to a variation of σ,

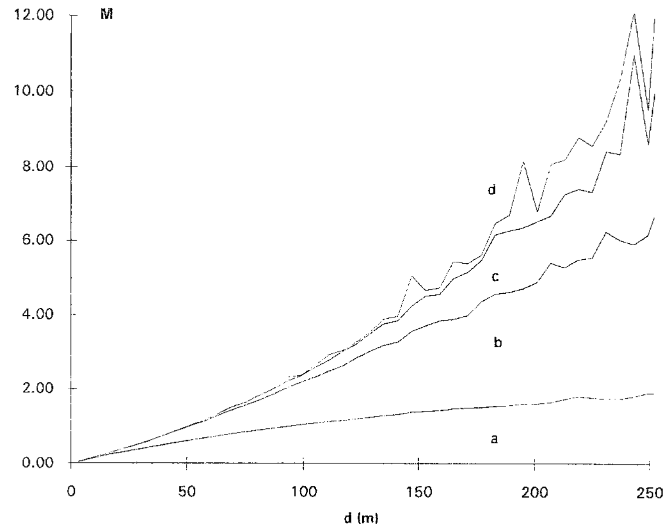

Fig. 1. Ratio (M) of multiple-scattering contribution to single-scattering contribution to LIDAR return, plotted vs penetration depth, C1 model cloud, H = 1000 m, wavelength 1.064 µm, FOV = 10 mrad (full width). *Curves a, b, c, d:* contributions to multiple scattering summed up to second, fourth, sixth, eight order of scattering, respectively

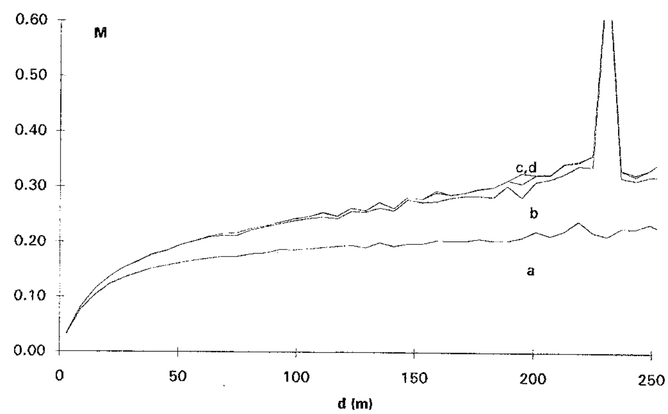

Fig. 2. Same as Fig. 1 For FOV = 1 mrad (full width)

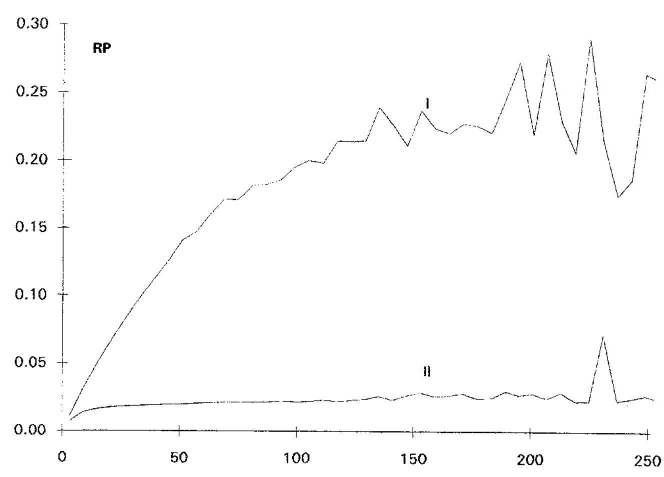

Fig. 3. Part of lidar returns polarized (RP) perpendicularly to the linearly polarized emitted pulse divided by the part with parallel polarization, plotted vs penetration depth (*I*: Fov = 10 mrad. *II*: FOV = 1 mrad)

this can be understood since relationship A2 gives the following dependence of the ratio S_k/S_1 on σ:

$$\frac{S_k(\sigma',t)}{S_1(\sigma',t)}=\frac{S_k(\sigma,t)}{S_1(\sigma,t)}\left(\frac{\sigma'}{\sigma}\right)^{k-1},$$

where S_k indicates the contribution of the *k-th* order of scattering to the LIDAR return.

One can note this dependence if one compares the data of Figs. 1–3 to the corresponding figures obtained with $\sigma=0.02\ \mathrm{m}^{-1}$ for the same type of cloud and the same geometrical situation, presented in [7]. For instance, at the depth of 200 m and at $\alpha=\ \approx 10$ mrad, the ratio multiple to single scattering was found to be ≈ 11 at $\sigma=0.02\ \mathrm{m}^{-1}$, instead of the value ≈ 7 shown in Fig. 1, and the ratio of the two polarization components was ≈ 0.33 instead of ≈ 0.25, which is the value shown in Fig. 3.

Appendix A

Scaling relationships

The following scaling relationships can easily be derived for the case of a homogeneous cloud (for derivation see [8]). Let us indicate as $S_k(H,\sigma,t)$ the contribution of the *k-th* order of scattering to LIDAR returns at time t from the cloud at distance H, with extinction and scattering coefficient σ (albedo = 1). If H is changed to H' one has for the contribution S_k'

$$S_k'(H',\sigma,t') = S_k(H,\sigma,t)\,m^{k-3}\exp\{-2\sigma[(ct/2-H)(H'/H-1)]\},$$

$$t'=mt,\qquad m=H'/H. \tag{A1}$$

A second simple relationship can be used when the extinction and scattering coefficient are changed to σ', while H is kept unchanged. One has now:

$$S_k'(H,\sigma',t) = S_k(H,\sigma,t)\,n^k\exp[-2\sigma(n-1)(ct/2-H)]$$

$$n=\sigma'/\sigma \tag{A2}$$

A scaling relationship for dealing with the variation of both σ and H can be obtained by combining (2) and (3).

Appendix B

Comparison with an analytic formula for double scattering

We show a comparison of the results relevant to the second order of scattering with those obtained by using an integration formula. The latter was initially derived for calculating the ratio of the contributions of double and single scattering to received power from homogeneous fogs or clouds [8]. It was then extended to the consideration of polarization [9].

In [8], for a LIDAR of FOV aperture α (full width), sounding a homogeneous cloud placed at a distance H from the LIDAR apparatus, the ratio D21 of doubly scattered to singly scattered return power was given as (see (1–5) of [8]; some expressions have been condensed and two misprints corrected):

$$\mathrm{D21}=\frac{\sigma(ct)^2}{P(\pi)}\pi\int_0^{\alpha/2}\alpha'\,d\alpha'\int_H^b F(x,t,\alpha')\,P(\theta_1)\,P(\theta_2)\,dx, \tag{B1}$$

where $P(\theta)$ is the phase function,

$$b=ct\frac{(1-2H/ct)\cos(\alpha/2)}{2[1-(H/ct)(1+\cos(\alpha/2)]},$$

$$F(x,t,\alpha')=\frac{2(ct)^{-2}}{1+[(x/ct)^2-(x/ct)\cos\alpha']2(1+\cos\alpha')/\cos^2\alpha'},$$

and θ_1, θ_2 are the angles of first and second scattering,

$$\theta_2=\pi-\theta_1+\alpha',$$

$$\pi-\theta_1 =\cos^{-1}\left(\frac{[1-(x/ct)(1+1/\cos\alpha')]^2-(x/ct)^2(1-\cos^2\alpha')/\cos^2\alpha'}{[1-(x/ct)(1+1/\cos\alpha')]^2+(x/ct)^2(1-\cos^2\alpha')/\cos^2\alpha'}\right).$$

To deal with the polarization effect, the scattering matrix is used in place of the phase function. In the case of a coaxial LIDAR, the symmetry around the beam axis allows one to make a direct integration over the azimutal angle φ. We assume a linearly polarized emitted pulse and indicate the ratios of double scattering to single-scattering contributions to the returns as D21P and D21N, for parallel polarization and cross polarization, respectively. Taking into account the form of the rotation matrix [1] one obtains [9]:

$$\mathrm{D21P}=\frac{\sigma(ct)^2}{2P(\pi)}\int_0^{\alpha/2}\alpha'\,d\alpha'\int_H^b F(x,t,\alpha')\,F_1(P_1,P_2,P_3,P_4)\,dx,$$

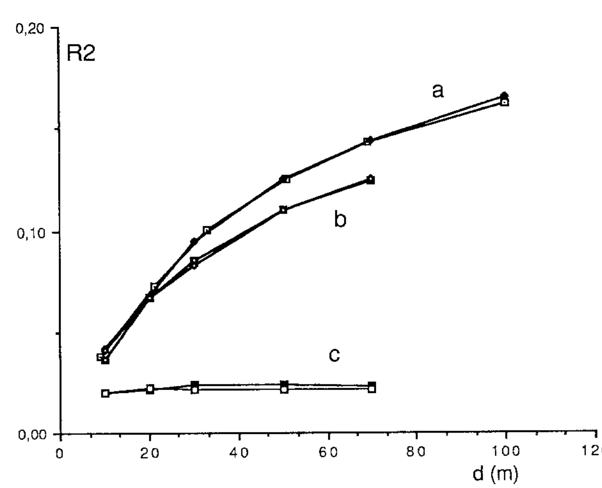

Fig. 4. R2 = D21N/(D21P + 1) (B2) calculated for the case of Figs. 1–3 with an extinction coefficient = 0.02 m^{-1}. *a* FOV 20 mrad; *b* FOV 10 mrad; *c* FOV 1 mrad (full widths). *Squares:* R2 from the equations of Appendix 2; *black marks:* from the Monte Carlo procedure

D21N

$$= \frac{\sigma(ct)^2}{2P(\pi)} \int_0^{\alpha/2} \alpha' \, d\alpha' \int_H^b F(x, t, \alpha') \, F_2(P_1, P_2, P_3, P_4) \, dx, \quad \text{(B2)}$$

with:

$$F_1(P_1, P_2, P_3, P_4) = \frac{\pi}{4} \{3[P_1(\theta_1) \, P_1(\theta_2) + P_2(\theta_1) \, P_2(\theta_2)] - 2[P_3(\theta_1) \, P_3(\theta_2) - P_4(\theta_1) \, P_4(\theta_2)]\},$$

$$F_2(P_1, P_2, P_3, P_4) = \frac{\pi}{4} \{P_1(\theta_1) \, P_1(\theta_2) + P_2(\theta_1) \, P_2(\theta_2) + 2[P_3(\theta_1) \, P_3(\theta_2) - P_4(\theta_1) \, P_4(\theta_2)]\}.$$

Figure 4 shows the ratio D21N/(D21P+1) (indicated as R2 in the figure), which is the ratio of the two polarization components of LIDAR returns, when only double and single scattering are considered. The ratio calculated by using the relationships of Appendix B is shown versus the cloud depth for three values of the FOV aperture and compared with Monte Carlo results. For the figure, the extinction (pure scattering) coefficient has been taken as $0.02 \, m^{-1}$.

The agreement of the results of the two procedures is considered to be a good check of our Monte Carlo procedure to deal with polarization.

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