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Photorefractive four-wave mixing in a high-contrast regime

W. Królikowski

Laser Physics Centre, Research School of Physical Science and Engineering, The Australian National University, Canberra ACT 0200, Australia
(Fax: +61-62/249-0029, E-mail: WZK111@rsphy1.anu.edu.au)

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Abstract. The effect of high contrast in photorefractive four-wave mixing is studied using a recently proposed empirical formula for the grating amplitude. An analytical solution to the coupled-wave equation is obtained and its properties are discussed in the case of a double-phase-conjugate mirror and semilinear phase-conjugate mirror.

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Photorefractive four-Wave Mixing (4-WM) has been shown to offer many interesting applications such as beam amplification, optical signal processing, and phase conjugation [1]. The photorefractive process involves the formation of a refractive-index grating in an electrooptic material through a spatially modulated light distribution produced by the interference of interacting optical beams. The theoretical description of this phenomenon is based mainly on Kukhtarev's et al. model of the photorefractive effect [2]. One of the major points of this model is the assumption of low contrast of the light modulation. As a result, the amplitude of the formed refractive-index grating is proportional to the modulation depth. The approximation of low contrast is not always justified in experiments. As a matter of fact, in order to achieve highest efficiency of the wave-mixing process, contrast of the interference fringes is often close to unity. This has been found to lead to discrepancies between theory and results in wave-mixing experiments. In order to account for experimentally observed nonlinear dependence between grating amplitude and modulation depth, some modification of the Kukhtarev's model has been proposed [3, 4]. However, the empirical relation between grating amplitude and the contrast, introduced in [3] while correctly describing experimental results, does not allow for analytical treatment of even the simplest two-beam coupling process. Recently, an alternative empirical formula has been proposed by Kwak et al. [5]. This model not only exhibits proper behavior for large contrast but, additionally, the equations describing two-wave mixing could be solved analytically.

In this communication we apply the same model to the 4-WM process. We show that corresponding coupled-wave equations can be completely integrated. We use the exact solutions to illustrate the influence of high light-intensity contrast on performance of some commonly used self-pumped photorefractive devices including the double-phase-conjugate mirror and the semilinear phase-conjugate mirror.

We consider typical 4-WM geometry when the photorefractive material placed between plane $z=0$ and $z=d$ is illuminated by two counter-propagating pumps (A_1 and A_2) and signal beam A_4. The phase-conjugated beam (A_3) propagates in opposite direction to the signal. The interaction of beams is described by a system of coupled-wave equations, which, for transmission geometry, read [6]

$$\frac{dA_1}{dz} = \gamma Q A_4,$$

$$\frac{dA_2^*}{dz} = -\gamma Q A_3^*,$$

$$\frac{dA_3}{dz} = \gamma Q A_2,$$

$$\frac{dA_4^*}{dz} = -\gamma Q A_1^*, \qquad (1)$$

where Q represents the amplitude of the grating and γ is the coupling constant. In the standard theory of the photorefractive effect, $Q=m$, where m is proportional to the contrast of the interference fringes $m = (A_1 A_4^* + A_2^* A_3)/I_0$ and I_0 is the total light intensity. The model proposed in [5] is given by the following relation

$$Q = \frac{m}{1+bm}, \tag{2}$$

where b is an adjustable (positive) parameter. For low contrast, (2) reproduces results of the standard model ($Q=m$), while for m increasing it gives a smaller growth of the grating amplitude.

We will consider here the most common situation in the photorefractive process, namely a $\pi/2$ phase shift between refractive-index grating and interference pattern. Then, all amplitudes and coupling constants can be treated as real numbers. For this case, the coupled equations (1) can be analytically integrated using the procedure described in [9]. It can be shown that the output amplitudes of the interacting waves can be expressed as

$$\begin{aligned} A_1(z=d) &= A_{10}\cos\varphi_d - A_{40}\sin\varphi_d, \\ A_4(z=d) &= A_{40}\cos\varphi_d + A_{10}\sin\varphi_d, \\ A_3(z=0) &= -A_{2d}\sin\varphi_d, \\ A_2(z=0) &= A_{2d}\cos\varphi_d, \end{aligned} \tag{3}$$

where we assumed that the input amplitude of the phase-conjugate wave is zero at the crystal face $z=d$. The initial amplitudes of pump and signal beams are denoted by A_{10}, A_{2d} and A_{40}, respectively. An auxiliary function φ_d, which is defined as $\varphi_d = \gamma\int_0^d Q dz$, is found from the following expression

$$\frac{I_0}{(a_1^2+a_2^2)^{1/2}}\log\left(\frac{\tan(\varphi_d-\beta/2)}{\tan\beta/2}\right) = (2\gamma d - b\varphi_d), \tag{4}$$

where

$$\begin{aligned} a_1 &= [I_{2d}\cos 2\varphi_d + (I_{10}-I_{40})]/2, \\ a_2 &= I_{2d}/2\sin 2\varphi_d - \sqrt{I_{10}I_{40}}, \\ \tan\beta/2 &= a_2/a_1. \end{aligned} \tag{5}$$

The intensity of the phase-conjugate wave is given by the simple expression $I_3(0)=I_{2d}\sin^2\varphi_d$ and, for a given set of boundary conditions, can be found once the algebraic equation (4) for φ_d is solved. Interestingly, the nonlinear dependence of the grating on contrast appears in (4) only through the term $(b\varphi_d)$ on its right-hand side. This significantly simplifies analysis (graphical, in particular) of any 4-WM geometry.

As an example of application of solutions (3, 4), we will consider here two commonly used photorefractive devices, namely the Double-Phase-Conjugate Mirror (DPCM) [7] and the Semi-Linear Phase-Conjugate Mirror (SLPCM) [8]. In the one-dimensional theory of 4-WM both devices are oscillators. Phase conjugation takes place when the coupling strength (γd) exceeds some threshold level. In order to analyze both devices, solutions, (3) and (4) have to be accompanied by appriopriate boundary conditions. For DPCM one has $A_{10}=0$, while for SLPCM $A_{10}=0$, and $A_{2d}=MA_1(z=d)$, where M denotes the reflectivity of the external mirror (Fig. 1). It

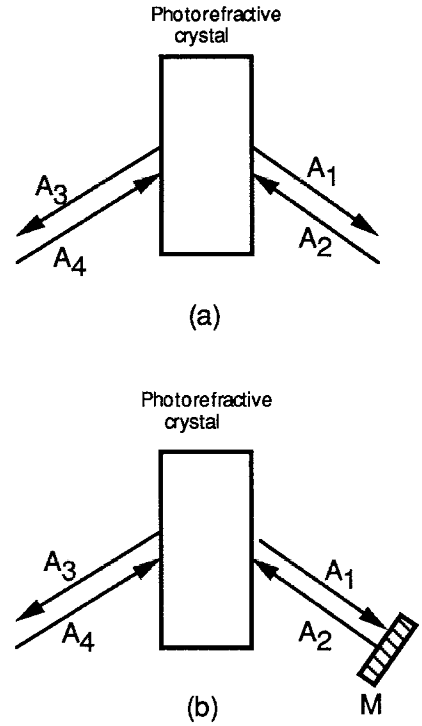

Fig. 1a, b. Interaction geometry for the DPCM (**a**) and SLPCM (**b**)

is known that the DPCM is a soft-threshold device – the amplitude of the generated phase-conjugate beams is zero at the threshold and increases with coupling strength. Therefore, it is clear from (4) that the threshold condition for the nonlinear response (2) will be exactly the same as for the linear one, as then $\varphi_d=0$. However, the nonlinearity will affect the phase-conjugate reflectivity above the threshold. In Fig. 2a, we show the dependence of the phase-conjugate reflectivity $R=I_3(0)/I_{40}$ as a function of coupling strength, for a few values of the parameter b. Clearly, R decreases with larger b but is always starts at the same value of $(\gamma d)_0$. These graphs exhibit qualitatively the same characteristic features as those obtained in numerical studies of large-signal effects in the ring phase-conjugate mirror, which is also a soft-threshold device [10]. Figure 2b displays standard characteristic of the DPCM, namely the output phase-conjugate reflectivity as a function of input signal ratio (I_{40}/I_{20}). Again, all graphs display the same threshold independently of b. Generally, however, the nonlinearity of the grating-formation process results in deterioration of the efficiency of the phase conjugation. Importantly, not only the intensity of the phase-conjugate waves decreases but also conditions for their optimal generation changes.

The semilinear phase conjugate mirror on the other hand, is a hard-threshold device, i.e., the intensity of the phase-conjugate output is non-zero at the threshold. Therefore, because of the term $(b\varphi_d)$, the non-zero parameter b will change the oscillation conditions. Since b is positive, the threshold will obviously increase. This is evident from Fig. 3a, where we plot the phase-conjugate

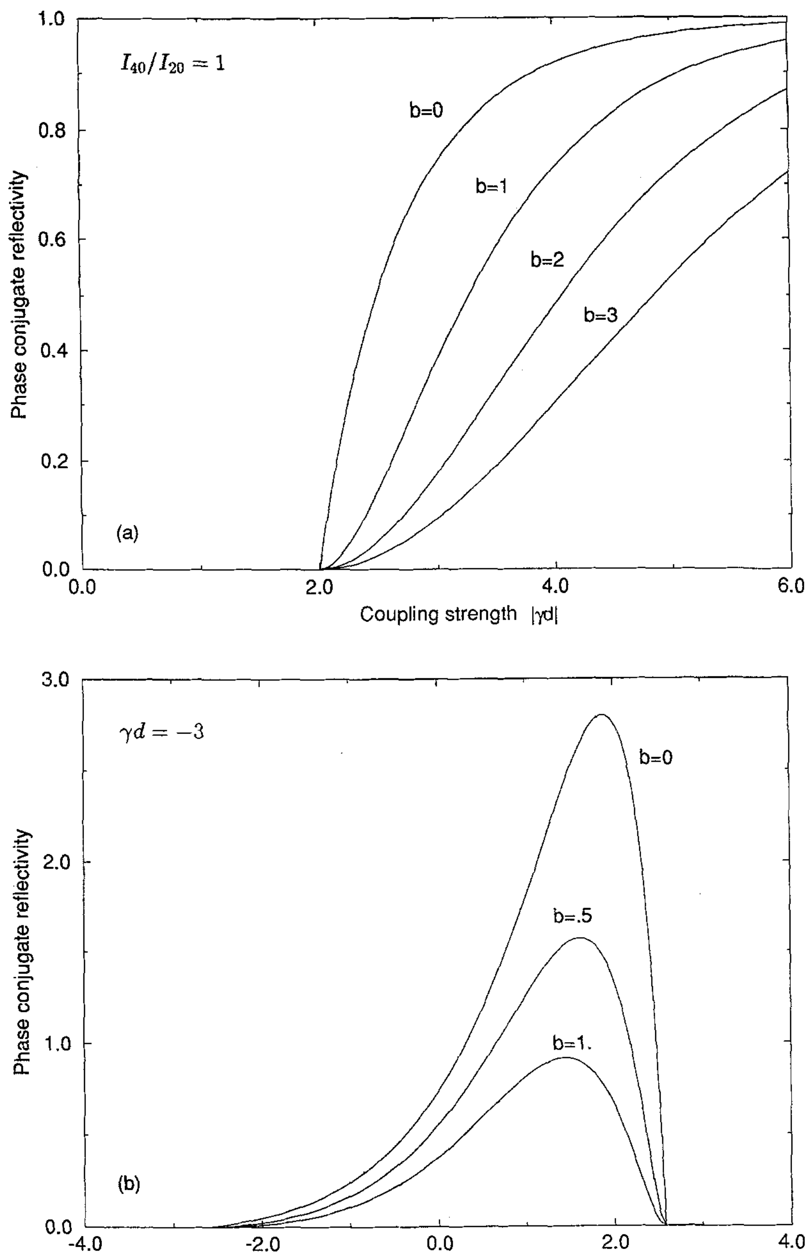

Fig. 2a, b. Phase-conjugate reflectivity of the DPCM as a function of the coupling strength (**a**) and input-beam ratio (**b**)

reflectivity as a function of coupling strength. With b increasing, the threshold increases as well and, at the same time, the intensity of the phase-conjugate beam at the threshold drops. The same effect is also seen in Fig. 3b, where we plot the output phase-conjugate reflectivity vs reflectivity of the external mirror. It has been shown recently that the SLPCM actually cannot start by itself but requires some level of external seeding, which decreases with coupling [11]. Therefore, the nonlinear dependence of grating amplitude on contrast will make the start of SLPCM even more difficult by increasing the required seeding intensity.

In a similar way, one can analyze an arbitrary geometry of four-wave interaction. Because of the form of (4), one can look at the term $(\gamma d - b\varphi_d)$ as an effective coupling strength. It can be shown that φ_d has the same sign as γ. Consequently, any contribution coming from $b \neq 0$ will always have an effect similar to decreasing the coupling. This is understandable, since the model described by (2) gives a lower grating amplitude compared to the standard photorefractive process with the same contrast. This, in turn, leads to weaker energy transfer between the interacting beams. It is worth mentioning that when the relation between fringe contrast and amplitude of the grating is nonlinear but does not decrease with contrast approaching unity (as considered in [12]), the efficiency of the phase-conjugation process may be higher than in that described by Kukhtarev's model.

In summary, we discussed the effect of nonlinear dependence of the refractive-grating amplitude on the contrast of the interference pattern in photorefractive 4-WM. We solved exactly the coupled-wave equations

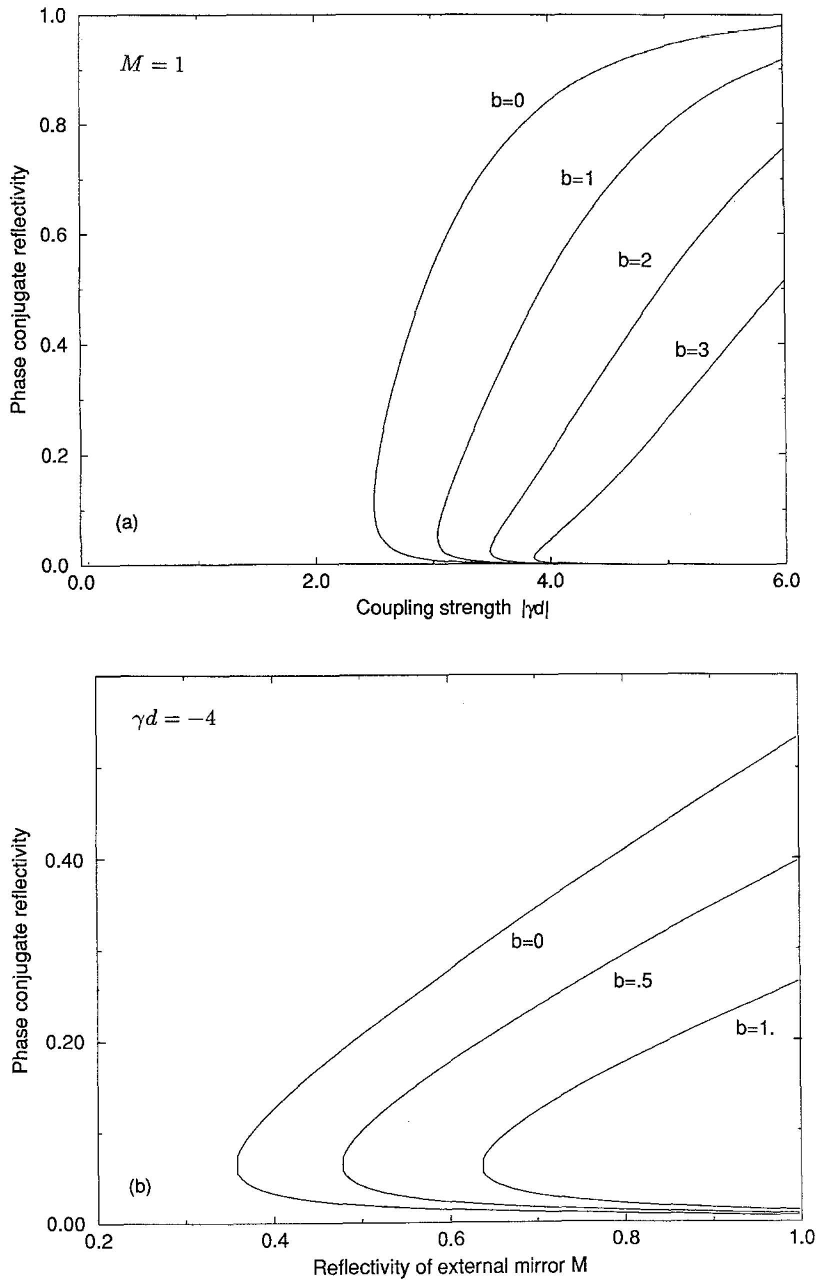

Fig. 3a, b. Phase-conjugate reflectivity of the SLPCM a function of the coupling strength (**a**) and reflectivity of the external mirror (**b**)

for the model function describing the nonlinear relation between grating amplitude and fringe contrast and applied the solutions to the DPCM and the SLPCM. We showed that, generally, the efficiency of the phase-conjugation process is deteriorated. Additionally, threshold conditions for hard-threshold photorefractive oscillators increases.

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