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Principles of Passive Mode Locking

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Abstract. The ultrashort pulse-forming properties of lasers are reviewed in terms of the master equation time-domain description of mode locking. The pulse shortening strengths and steady-state operating characteristics of a broad range of modern experimental systems are discussed within the framework of the classic slow-saturable-absorber and fast-saturable-absorber analytical models.

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In a general sense, one may describe the formation of ultrashort pulses in a laser in terms of either the frequency domain or the time domain. The term mode locking comes from the frequency-domain description. Any steady-state output of the laser consists of a superposition of frequencies (modes) separated by $\Delta f = c/2nL$, where c is the speed of light, L is the length of the laser resonator and n is the effective group-velocity index. In the time domain such an output is periodic, with period $1/\Delta f$ corresponding to the round-trip time of the light in the resonator. Mode locking implies that the relative phases of the modes are held fixed with respect to each other. With proper relative phases, the resulting periodic temporal output is a train of pulses. To produce short pulses, one has to lock modes over a wide frequency range. One's ability to make even shorter pulses is ultimately limited by the overall laser bandwidth and the uneveness in mode spacing caused by variations of the effective index with frequency.

In a practical sense, one wants to be able to describe analytically how the mode locking occurs and what pulse characteristics are produced. Then the frequency-domain approach can become intractable. Ultrashort pulses utilize a very large number of modes; and the mode-coupling mechanism that produces the locking is generally nonlinear and not so easily described in the frequency domain. A review of early frequency-domain analyses may be found in Smith et al. [1]. Most of our current analytical understanding of mode locking comes from time-domain theory.

1 Time-Domain Propagation and Filtering

Before considering the more complicated, nonlinear and time-varying, modulations used in mode locking, let us first define the necessary time-domain descriptions of linear propagation and filtering effects caused by elements in any laser [2]. The two most important of these are: (a) pulse broadening by bandwidth limiting in the laser, and (b) pulse broadening and chirping by Group-Velocity Dispersion (GVD).

In the frequency domain, the gain changes an incident spectral amplitude $E(\omega)$ into

$$E'(\omega) = \left[1 + g\left(1 - \frac{(\omega-\omega_0)^2}{\omega_g^2}\right)\right] E(\omega) \tag{1}$$

if we assume that the gain is small and that deviation from the center of the gain is small compared to the gain bandwidth. Then, by Fourier transformation we find the output pulse in terms of the input

$$E'(t) = \left[1 + g\left(1 + \frac{1}{\omega_g^2}\frac{d^2}{dt^2}\right)\right] E(t). \tag{2}$$

Similarly, passage through an element with GVD (a quadratic phase variation with frequency) produces

$$E'(\omega) = [1 - \mathrm{i}D(\omega-\omega_0)^2]E(\omega), \tag{3}$$

where, again, we assume a small change over the frequency range. The new pulse in the time domain is broadened and chirped. It becomes

$$E'(t) = \left(1 + \mathrm{i}D\frac{d^2}{dt^2}\right) E(t). \tag{4}$$

Thus, we have two dispersion operators for use in the time domain, one real (band limiting) and one imaginary frequency dispersive). Next we consider the types of modulation that shorten pulses and produce modelocking.

2 Active Mode Locking

As an analytical starting point, we consider active mode locking. First demonstrated thirty years ago [3], it remains a reliable method for the generation of short pulses when passive methods are not applicable or when phase locking to an electronic signal is desired. Time-domain theory describing how it works is now well-established [2, 4, 5]. An amplitude modulator inside the laser is used to provide transmission gating at the cavity roundtrip frequency. A pulse, arriving at maximum transmission, is passed with relatively low loss. The shorter it is the less loss it will experience. Other light in the laser is suppressed.

If we consider the pulse $E(t)$ to be characterized near its peak by a parabolic curvature in time (true of Gaussian or sech pulse shapes), then, for $t \ll \tau$,

$$E(t) = E_0\left(1 - \frac{t^2}{\tau^2}\right). \tag{5}$$

Upon passing through the amplitude modulator which has a transmission

$T = 1 - m(1 - \cos\omega_m t) \cong 1 - m(\omega_m t)^2/2,$

the shape of the pulse peak becomes

$$E(t) = E_0\left(1 - \frac{t^2}{\tau'^2}\right), \tag{6}$$

where

$$\frac{1}{\tau'^2} = \frac{1}{\tau^2} + \frac{m\omega_m^2}{2}. \tag{7}$$

The Pulse-Shortening Rate (PSR), $\Delta\tau/\tau$ per pass, produced by the modulator, is therefore

$$\frac{\Delta\tau}{\tau} = \frac{m\omega_m^2\tau^2}{4}, \tag{8}$$

where we have defined $\Delta\tau$ as a decrease in pulse width. As the pulse gets shorter, the effectiveness of the modulator (i.e. its PSR) decreases significantly. This is a key difference between active mode locking and passive mode locking.

If we restrict ourselves to gain media with long relaxation times (solid-state lasers), we can neglect time variations of the gain during the pulse and define g as the steady-state saturated gain. Figure 1 illustrates how the active loss modulation creates a window of gain for the peak of the pulse under this condition. In the laser, in the steady state, the changes produced by all of the internal elements must cancel for each roundtrip. For simplicity, and since active mode locking seldom produces pulses short enough for it to be a factor, we neglect GVD. Then, we can write for active mode locking

$$\left[g\left(1 + \frac{1}{\omega_g^2}\frac{d^2}{dt^2}\right) - l - m(1 - \cos\omega_m t)\right]E(t) = 0, \tag{9}$$

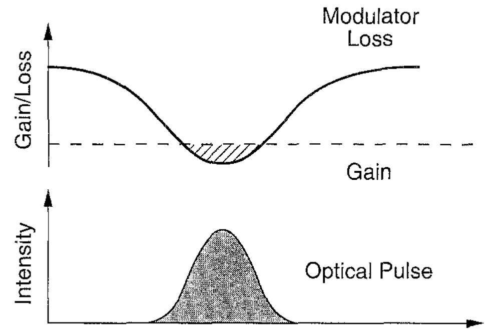

Fig. 1. Pulse-shaping loss dynamics for active mode locking

where l is defined as the roundtrip linear loss, and we have assumed for simplicity that the modulator frequency is exactly equal to the roundtrip frequency of the laser. This is a Mathieu equation [6] and has periodic solutions.

If we approximate the modulation as parabolic, as done above, (9) becomes the Schroedinger equation for a particle in a parabolic well. The solution [2, 4] is a Gaussian

$$E(t) = E_0 \exp\left(-\frac{t^2}{\tau^2}\right) \tag{10}$$

with

$$\tau = \sqrt{\frac{1}{\omega_g\omega_m}}\left(\frac{8g}{m}\right)^{1/4}. \tag{11}$$

Thus, the pulsewidth varies inversely with the square root of the gain bandwidth and the modulation frequency. We note that this steady-state pulsewidth represents the exact balancing of the pulse-shortening rate of (8) and the pulse-spreading rate produced by the gain bandwidth filter: $\Delta\tau/\tau = 2g/\omega_g^2\tau^2$. Including GVD would make τ^2 complex, increasing its magnitude and adding a chirp to the pulse. The effect of modulation frequency detuning can also be balanced with a per-roundtrip timing delay expressed by the operator $t_D(d/dt)$ [2, 4].

The predictions of this analytical theory have been found to work quite well for solid-state lasers such as Nd:YAG [5]. But, care should be taken when trying to apply them to actively modulated semiconductor lasers, in which fast gain dynamics play a role. There, the model of the synchronously-pumped laser [7] should be more appropriate since the effective modulation frequency is increased by the pulse's self-saturation of the gain. Stability problems [8, 9] then also tend to complicate ultrashort pulse generation.

3 Passive Mode Locking

Ever since the first picosecond pulses were generated with only the help of a passive, saturable-absorber element in the laser [10], passive mode locking has been the means for generating the shortest pulses. It led to the first subpicosecond pulses [11] and can now be used to produce pulses on the order of 10 fs [12, 13]. In one sense passive mode locking is similar in all systems: the pulse inside the laser self-modulates itself, more rapidly than would be possible with any active modulation. On the other hand, the type of element in which this occurs can be quite different from one system to another. For purposes of analysis, they fall into two classes: fast saturable absorbers and slow. First we discuss the latter.

4 Slow Saturable Absorbers

A slow saturable absorber is a lossy element that becomes more transparent with increasing light intensity but cannot recover its absorption on the timescale of an ultrashort pulse. It favors pulse generation over cw radiation, but cannot do much shortening on a timescale shorter than its own recovery time. Early studies of passively mode-locked flashlamp-pumped dye lasers [14] revealed, however, that picosecond pulses were in fact being generated with dye saturable-absorber elements that recovered on a nanosecond timescale. An insightful rate equation analysis by New [15] suggested that this was made possible by dynamic saturation of the gain. The absorber would preferentially absorb the leading edge of the pulse; gain depletion would cause loss on the trailing edge. Both loss and gain recover in time for the next roundtrip. Each time, the wings of the pulse experience loss while the peak of the pulse receives gain. The modulation dynamics of this process are illustrated in Fig. 2.

When the pulse is much shorter than the recovery time of either the absorber loss (l_a) or gain (g), simple rate-equation analysis, in the temporal vicinity of the pulse, yields:

$$l_a(t) = l_i \exp\left(-\sigma_a \int_{-\infty}^{t} |E(t)|^2\,dt\right) \tag{12}$$

and

$$g(t) = g_i \exp\left(-\sigma_g \int_{-\infty}^{t} |E(t)|^2\,dt\right), \tag{13}$$

where σ_a and σ_g are the effective absorber and gain cross-sections, respectively, and l_i and g_i are the initial (partially recovered) values of the loss and gain just before the arrival of the pulse. Two conditions are immediately apparent for successful mode locking: $l_i > g_i$, and $\sigma_a > \sigma_g$. The first, which ensures loss for the pulse leading edge, generally requires the absorber to recover more rapidly than the gain (since, for start-up, the fully recovered value of g must be greater than that of l). The second allows the center of the pulse to see net gain. Two experimental techniques used to help achieve this second condition are tighter focussing in the absorber than in the gain (which actually changes the relative energy densities, but the effect is the same), and Colliding Pulse Mode locking (CPM) [16], which does the same and increases the effective σ_a by a factor between 2 and 3 [17]. More detailed conditions for steady-state pulse stability, which depend upon recovery rates and cavity roundtrip time, have been given by New [18] and Haus [19].

Concerning pulse shortening by these slow-absorber/gain dynamics (see Fig. 2) we note that the depth (m_s) of the Self-Amplitude Modulation (SAM) depends only on the pulse energy and is therefore independent of pulsewidth. But the speed (curvature) of the modulation is determined by the pulse duration since the window of net gain narrows proportionally. Within the parabolic approximation we have used above, such a modulation function can be written $T = 1 - m_s(t^2/\tau^2)$. The negative change in pulsewidth per pass is obtained from $1/\tau'^2 = 1/\tau^2 + m_s/\tau^2$, which gives

$$\frac{\Delta\tau}{\tau} = \frac{m_s}{2} = \text{const.} \tag{14}$$

That is, the PSR due to this shaping effect does not lose its strength as the pulse gets shorter.

The first analytical description of steady-state pulse parameters was obtained by Haus [19]. In the steady state both gain and loss vary on the timescale of the pulse, so

$$\left[g(t) - l_a(t) - l_0 + i\psi + \frac{l_0}{\omega_g^2}\frac{d^2}{dt^2} + t_D\frac{d}{dt}\right]E(t) = 0, \tag{15}$$

where we have included the timing shift t_D and the optical phase change needed for closure. The amplitude of the gain bandwidth-limiting operator, proportional to the net average saturated gain, is most easily approximated here by the nonsaturable loss l_0. If both $g(t)$ and $l_a(t)$ are, in turn, expanded only to second order in $\int_{-\infty}^{t} |E(t)|^2\,dt$, the closed form solution is

$$E(t) = E_0 \operatorname{sech}\left(\frac{t}{\tau}\right), \tag{16}$$

where

$$\tau = \left(\frac{1}{\omega_g}\right)\left(\frac{4}{\sigma_a W}\right)\left(\frac{l_0}{l_i}\right)^{1/2} \tag{17}$$

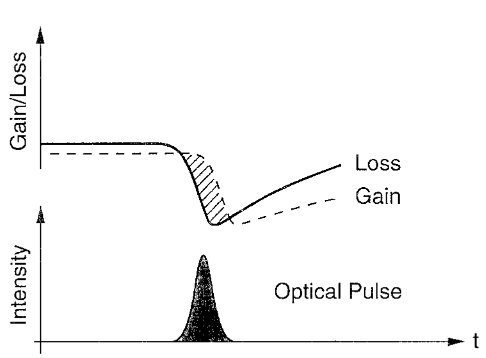

Fig. 2. Pulse-shaping gain and loss dynamics for slow-saturable-absorber mode locking

Here $W = \int_{-\infty}^{\infty} |E(t)|^2 \, dt$ is the total pulse photon density, and l_0/l_i is the ratio of DC loss to saturable absorber loss. Plugging this solution back into (15) yields equations for E_0, t_D and ψ.

The pulsewidth is proportional to the inverse of the gain bandwidth, which we note is expected from the balance between the constant PSR of slow-absorber mode locking and the $2l_0/\tau^2\omega_g^2$ spreading due to bandwidth limiting. The result tells us that the PSR is given by $l_i(\sigma_a W)^2/8$, which is proportional to the amount of saturable absorber and to the pulse energy squared. Although this analytic theory assumes only small changes per element, the results it predicts have been found to compare surprisingly well to those obtained from more complete numerical studies of non-perturbational systems [17, 20, 21]. The PSR for CPM dye-laser systems is estimated by these numerical studies to be in the range of 1–4%. When one compares this with the PSR for synch-pumped systems, estimated to be in the range $10^{-3} - 10^{-5}$ [22], one appreciates the strength of slow-absorber mode locking.

Early experimental work also indicated the presence of chirp in passively mode locked laser pulses [23, 24]. With the introduction of intracavity dispersion compensation by prism pairs in the CPM dye laser [25], it became possible to study the causes of this in more detail. Pulse duration depended not only on the magnitude of the GVD in the laser but on its sign, a clear indication of interplay between GVD and Self-Phase Modulation, SPM. The possible sources of SPM in the individual CPM-laser components were then investigated, evaluated and incorporated into Haus's master equation by Martinez et al. [26]. They obtained [26, 27] an elegant extension of the sech-pulseshape analytical solution:

$$E(t) = E_0 \operatorname{sech}\left(\frac{t}{\tau}\right), \exp\left[\mathrm{i}\beta \ln \operatorname{sech}(t/\tau)\right] \tag{18}$$

which is expressed in terms of a chirp parameter β as well as the pulsewidth τ. The variation of τ with intracavity GVD obtained for the predominant nonlinearities of a CPM laser is illustrated qualitatively in Fig. 3. It is assumed that the SPM is due primarily to a negative index change proportional to the absorption decrease of the saturable absorber. In the master equation this amounts to adding an imaginary term proportional to $l_a(t)$ in (15) and a dispersion operator as per (4). The shortest pulses, and the least chirp, are then found for positive (normal) dispersion as expected for soliton-like balancing of the two effects.

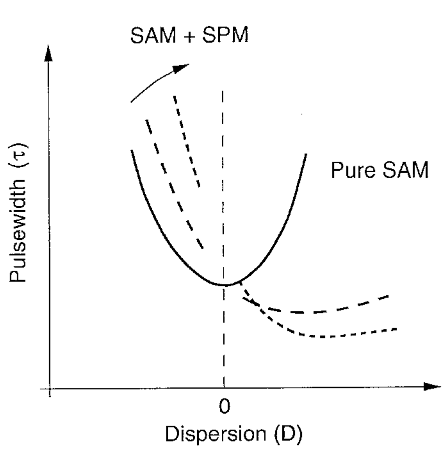

Fig. 3. Pulsewidth vs GVD for different values of dynamic SPM in a passively mode-locked dye laser

The interplay of SPM and GVD can result in pulses that are shorter than would be generated by the SAM alone, but not by a large factor. Martinez [26] estimated the additional shortening to be less than a factor of 2. Perhaps an even more important consequence of this interplay is the potential instability of the steady state. In the presence of SPM, stable oscillation is often not obtainable with zero GVD or with small values of GVD of the wrong sign. This is indicated by the gaps in the dashed curves shown in Fig. 3. Thus, it is especially important that GVD be adjustable. Several authors have studied the complex temporal evolutions of apparent pulseshape that can occur. They have been described in terms of higher-order soliton effects [28, 29] and have been identified as instabilities [30] associated with SPM and bandwidth limiting. An additional, wavelength-stability criterion has also been derived by Chilla et al. [31].

The theory of slow-absorber mode locking was greatly stimulated by development of the cw passively mode-locked dye laser [32]. That laser, in turn, has served as a particularly good model system with which to test the theory, since the response times and optical properties of the individual elements could be determined separately. It produced the first sub-picosecond pulses [11], spawned the CPM technique [16], permitted the adjustable compensation of GVD with prism sequences [25], produced pulses as short as 28 fs [33], and facilitated pulse compression down to durations of 6 fs [34].

The passively mode-locked semiconductor laser [35] has become another increasingly important application of slow-absorber mode locking. Integrated saturable absorbers have been created by defects [35–37] and by inhomogeneous excitation with tandem contacts [38–41]. The effects of SPM play a strong role in semiconductors as do a variety of fast and slow time constants [42] so that structures with optimized GVD and bandwidth control need to be developed. As they are, monolithic semiconductor devices will become ever more reliable and practical ultrashort-pulse sources. Promising results are now being obtained from a variety of monolithic device geometries [41, 43]. Semiconductor CPM lasers have been made to generate sub-picosecond pulses at repetition rates as high as 350 GHz [43, 44].

5 Fast Saturable Absorbers

A fast saturable absorber is an element that responds essentially instantaneously to changes in light intensity. That is, it can recover its initial absorption in a time short compared to the optical pulse duration. Thus, it can

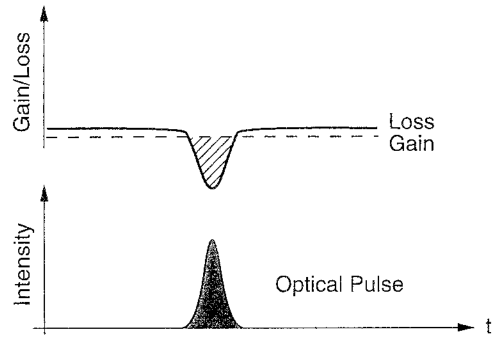

Fig. 4. Pulse-shaping loss dynamics for fast-saturable absorber mode locking

produce pulses in a laser without any help from gain-saturation dynamics. Figure 4 shows how this works in the steady state. The absorber shapes the pulse on both leading and trailing edges and discriminates against background light between pulses. Rate-equation analysis for the loss, in the limit of absorber response $\tau_a \ll \tau$, yields

$$l_a(t) = l_0 - \gamma |E(t)|^2, \tag{19}$$

where $|E(t)|^2$ is the photon flux density, γ is the SAM coefficient, and we have assumed $\gamma |E(t)^2| \ll l_0$. The gain is assumed to be approximately constant during the pulse, and equal to its saturated level determined by the steady-state average power. This is the case for media with small gain cross-sections and long upper-state lifetimes. It is, in a practical sense, the defining condition for the fast-absorber mode locking model.

6 Real Saturable Absorbers

For solid-state lasers with long upper-state lifetimes that prohibit significant dynamic gain saturation within a roundtrip, fast-saturable absorbers are a necessity for passive mode locking. But, when they have fast recovery times, real saturable absorbers can require either ultrashort pulses or pulses with high intensity to produce any sizeable effect. Thus, for many years passive mode locking of solid-state lasers was restricted to transient, flashlamp-pumped systems in which high intensities were achieved rapidly. Theoretical investigations of such systems focussed on the selection and evolution of picosecond pulses from noise fluctuations [45–47]. Successful experimental application required the selection of single pulses during build-up, before higher-order effects took over [48, 49]. Of course, pulse durations could not get any shorter than the real response times of the absorbers [50].

Only recently, with the advent of cw broadband systems has it become possible to mode-lock solid-state lasers passively, in a controlled way. Semiconductor saturable absorbers have been used with success in color-center lasers [51, 52], in coupled-cavity systems [53], in anti-resonant Fabry-Perot devices [54] and in fiber lasers [55, 56]. Real absorbers have the advantage of simplicity; because of their real lifetimes, however, they have not produced the shortest pulses. That has been accomplished with artificial fast-saturable absorbers.

7 Artifical Fast-Saturable Absorbers

The fastest optical nonlinearities are reactive and nonresonant. The index of refraction nonlinearity in glass, for example, has response on the order of a few femtoseconds [57]; and it may be utilized over a wide range of wavelengths. Because they are fast, such nonlinearities are also relatively weak. Their potential applicability to mode locking has been recognized for some time in the context of pulsed systems [58–61], but it was not until the emergence of fiber and high-power cw solid-state lasers that their utility has been fully appreciated. Several recent reviews document this renaissance [62].

Four realizations of artificial saturable absorbers are discussed below. In each case, changes of refractive index with intensity are converted into amplitude modulation. These artificial absorbers have clear advantages over real absorbers in that they need not dissipate power (since they deflect power out of the laser instead of absorbing it) and their operational parameters can be varied experimentally. The effective small-signal losses they introduce and their effective SAM coefficients can be optimized by proper choice of lenses, beam splitters and cavity dimensions.

8 Additive-Pulse Mode Locking

The coupled-cavity configuration of Fig. 5 was investigated first in conjuction with the soliton laser [63] and then discovered to produce mode locking in non-soliton systems as well [64, 65]. Its pulse-shaping properties have been related to fast-absorber theory and described in terms of Additive-Pulse Mode locking (APM) [66–68]. Its applications have included F-center [66, 69], Ti:Sapphire [70, 71], Cr:Forsterite [72], Nd:YAG [73, 74], Nd:YLF [75, 76], and Nd:Glass [77] lasers.

The SAM in such lasers arises from the coherent interference of a pulse in the main laser cavity and with a synchronized pulse returning from the nonlinear auxiliary cavity. The pulse from the auxiliary cavity has experienced self-phase modulation which causes the peak of the pulse to be shifted in phase with respect to the wings. If the wings of the pulses (main cavity and auxiliary cavity) interfere with a relative phase of Ψ_{BIAS} (determined by the relative cavity lengths), then other parts of the pulse will interfere with relative phase $\Psi = \Psi_{BIAS} + \Psi_{NL}$, where Ψ_{NL} is proportional to intensity. Pulse shor-

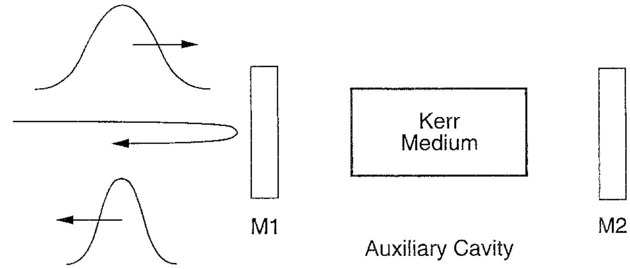

Fig. 5. Pulse shortering by nonlinear coupled-cavity APM

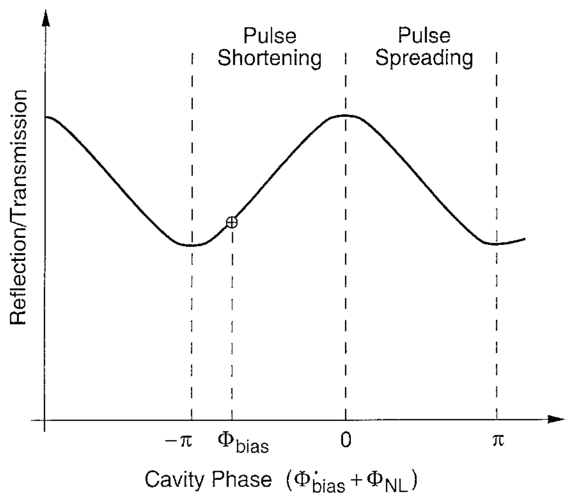

Fig. 6. Effective nonlinear amplitude-modulation characteristics of an APM system. Ψ_{NL} is proportional to intensity

tening will occur if the interference is more positive at higher intensities. The resulting SAM may be thought of as due to a mirror with a nonlinear reflectivity, as illustrated in Fig. 6. The magnitude of its small-signal amplitude reflectivity varies periodically with the difference in relative cavity lengths. It may be approximated by [67]

$$R = r + L(1-r^2)\cos\Psi, \tag{20}$$

where r is the amplitude reflectivity of the coupling mirror and L (assumed $\ll 1$) is a roundtrip multiplicative factor accounting for loss in the auxiliary cavity. Here it should be noted that if there is no loss ($L = 1$) there can be no pulse shortening. If the cavity phase bias is chosen so that an increased phase delay produces increased reflectivity (i.e. positive $(dR/d\Psi)$, then pulse shortening will occur. The SAM coefficient may be written [68, 78]

$$\gamma = -\kappa L(1-r^2)\sin\Psi_{\mathrm{BIAS}}, \tag{21}$$

where $\kappa = \Psi_{\mathrm{NL}}/|E(t)|^2$ is the effective nonlinear coefficient. We note that here, and in the expressions to follow below, κ is proportional to the length of interaction as well as the intrinsic Kerr coefficient, and inversely proportional to the effective mode area.

Two key aspects of APM are apparent from Fig. 6: 1) proper bias requires interferometric stabilization of the two cavities; and 2) the effect tends to saturate with a nonlinear relative phase shift on the order of π.

9 Polarization APM

Intensity dependent polarization rotation [79–81] also provides a mechanism for artificial saturable absorption as shown in Fig. 7. It may be analyzed as an automatically stabilized form of APM since it utilizes the coherent addition of two polarizations in the same cavity. If they undergo different nonlinear phase shifts, their combination can result in an intensity dependent polarization rotation. With a polarizer this is converted into SAM. Bias between the polarizations is usually controlled with

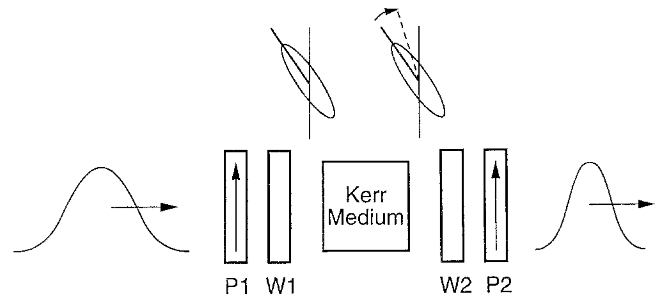

Fig. 7. Pulse shortening by nonlinear polarization rotation, or polarization APM

linearly birefringent phase plates. Characteristically of APM, the amplitude transmission through the polarizer varies periodically with linear plus nonlinear phase difference, as in Fig. 6. In an isotropic medium the two polarization components are most conveniently described in terms of circular polarizations. Then the SAM coefficient may be written [68, 78]

$$\gamma = -\frac{\kappa_c}{4}(2r^2-1)\left(\frac{r}{\sqrt{1-r^2}}\right)\sin 2\Psi_{\mathrm{BIAS}}, \tag{22}$$

where $r/\sqrt{1-r^2}$ is the ratio of the two orthogonal circular polarization amplitudes and κ_c is the nonlinear coefficient for circular polarization. This SAM coefficient is the result of the difference in nonlinear phase modulation between the two modes, so there is always significant common mode SPM as well. Since large SPM tends to cause stability problems, a figure of merit M has been proposed for the comparison of artificial saturable-absorber systems [68]:

$$M = \frac{\gamma}{\delta l_0}. \tag{23}$$

For polarization APM this figure is on the order of one, much smaller than for coupled-cavity systems, but the need for interferometric stabilization is avoided. The principle application of polarization APM has been fiber lasers, with demonstrations in both linear [82–86] and ring [87–92] cavities. Pulses as short as 38 fs [84] and 77 fs [92] have been achieved in neodymium- and erbium-doped systems, respectively.

10 Nonlinear Loop Mirror

The Nonlinear Optical Loop Mirror (NOLM) [93–96] may also serve as an artificial saturable absorber. A non-50/50 coupler, or an asymmetrically placed gain element [97], as shown in Fig. 8, provides greater intensity for one direction around the loop than for the other. The difference in SPM of the two directions produces interference SAM on the output in a manner similar to that of the APM cases discussed above. It has been shown that proper polarization biasing of the loop with

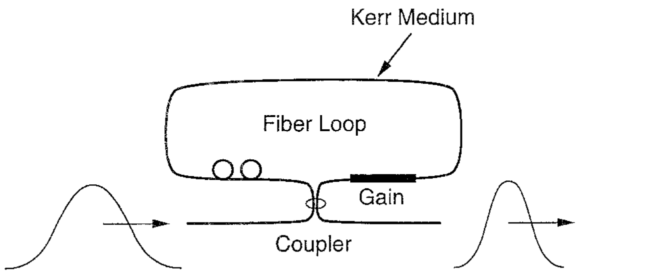

Fig. 8. Pulse shortening by a nonlinear fiber-loop mirror with either an imbalanced coupler or an asymmetrically placed gain element

phase plates, or the polarization-controlling loops shown in Fig. 8, can permit use of such a loop in either transmission or reflection mode [78]. For the example of a Nonlinear Amplifying Loop Mirror (NALM) with a lumped gain element (power gain = G) the perturbation SAM coefficient is [78]

$$\gamma = -\frac{\kappa}{4}(G-1)\sin 2\Psi_{\mathrm{BIAS}}, \tag{24}$$

where κ is the effective Kerr coefficient for each direction. The figure of merit M of this device is also found to be on the order of one [78], so that it has about the same sensitivity to SPM-induced instabilities as a nonlinear polarization rotation device.

Following the first passive mode locking of a fiber laser incorporating a NALM in figure-eight configuration [98, 99] a variety of workers have achieved considerable success in generating sub-picosecond pulses with both NALM [100–103] and NOLM [104–105] based systems. Recently, pulse durations shorter than 100 fs have been achieved [106, 107].

11 Kerr-Lens Mode Locking

Dynamic self-focussing [108] has been used with particular effectiveness for mode locking bulk-element solid-state lasers. Its importance became apparent shortly after the observation of self-mode-locking in a cw Ti:Sapphire laser [109, 110] which revealed different spatial mode parameters under short-pulse operation than under cw. Piche [111] suggested that self-focussing might be responsible. Cavity designs were developed to enhance the lensing effects and utilize them for effective saturable absorber action as illustrated in Fig. 9. The technique was dubbed Kerr-Lens Mode locking (KLM) by Spinelli et al. [112] who demonstrated improved mode locking with an aperture appropriately placed in the cavity. Excellent results were quickly achieved by a variety of groups in both Ti:Sapphire [113–124] and other systems [125–131]. In one of these, a microdot mirror assembly demonstrates that an artificial saturable absorber can be created as a modular unit [121]. Several numerical [111, 132–134] and analytical studies [135–138, 68] provide insight into various characteristics of the process. A relationship between SPM and SAM for KLM has been

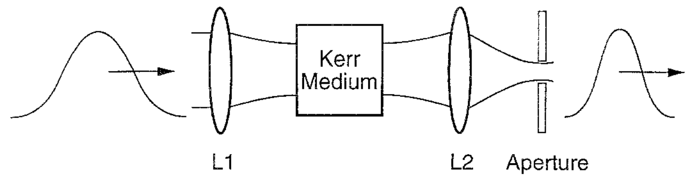

Fig. 9. Pulse shortening by dynamic self-focussing, or KLM

determined analytically [68] and the figure of merit, $M = \gamma/\delta l_0$, under proper design, is found to be about the same as that for polarization-APM and NALM systems. KLM Ti:Sapphire lasers have produced the shortest mode-locked laser pulses to date [12, 13].

12 Pulse-Shortening Rate

Since the amplitude of the pulse-shortening modulation produced by a fast absorber is proportional to the peak intensity of the pulse, it depends inversely on the pulse duration for a given average power in the laser: $m_f = \gamma W/2\tau$. The parabolic temporal curvature of the modulation also becomes stronger as the pulse gets shorter, just as in the slow-absorber case. Together they produce a modulation function that can be written $T = 1-(\gamma W/2\tau)(t^2/\tau^2)$. The negative change in pulsewidth per pass is obtained from $1/\tau'^2 = 1/\tau^2 + \gamma W/2\tau^3$, which gives

$$\frac{\Delta\tau}{\tau} = \frac{\gamma W}{2\tau}. \tag{25}$$

Thus, unlike the case of slow-absorber mode locking in which the PSR remained constant, fast-absorber dynamics produce an ever increasing PSR as the pulse gets shorter. A limit is reached, of course, when the absorber is completely saturated or when the pulse duration ap-

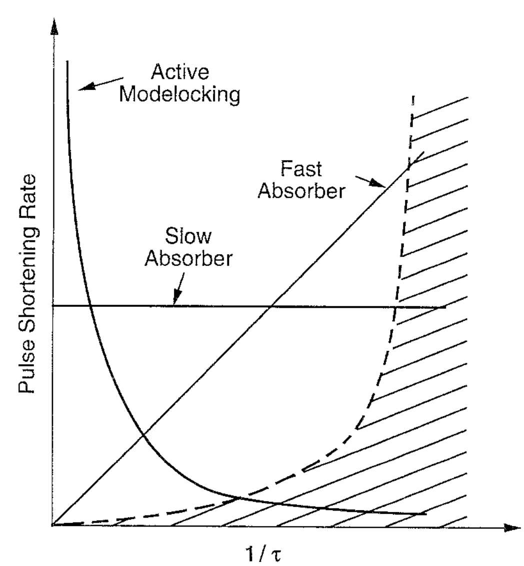

Fig. 10. Pulse-shortening rates for three different mode-locking mechanisms. *Dashed curve* indicates eventual limiting by pulse spreading effects

proaches its response time. The downside of such strong shortening for ultrashort pulses is that the shortening is very weak initially when the pulse is long. We can see this clearly in Fig. 10 where the pulse-shortening rates for the different mode-locking mechanisms are plotted illustratively versus $1/\tau$. This latter aspect has led to problems with self-starting in fast-absorber systems. More about that below; first we consider the steady-state behavior predicted by the master equation [139, 140, 68].

13 The Steady State

For all of the above fast-absorber systems, we may write the master equation for steady-state mode locking as follows,

$$\left[g-l+i\psi+\frac{g}{\omega_g^2}\frac{d^2}{dt^2}+iD\frac{d^2}{dt^2}+t_D\frac{d}{dt}\right.$$

$$\left.+(\gamma-i\delta)|E(t)|^2\right]E(t)=0, \tag{26}$$

where we specifically include δ, the proportionality constant for fast SPM. It is generally related, as discussed above, to the SAM coefficient γ by the nature and design of the fast absorber. Remarkably, the analytical solution for this equation has the same form [26, 140] as that obtained for the slow-absorber case: Equation (18).

Equations for τ, β and W are obtained by plugging (18) into (26). Figure 11a, b show plots of pulsewidth τ and chirp β as a function of GVD for a fixed energy ω and gain bandwidth ω_g but for different values of SPM (δ). Here it is assumed that δ is positive (positive change in index with intensity) as is the case for most femtosecond nonlinearities in solid-state materials. For zero SPM the minimum pulsewidth occurs at zero GVD and has a value of [140]

$$\tau_0=\frac{4g}{\gamma W\omega_g^2} \tag{27}$$

consistent with a simple balance of PSR and bandwidth limiting. As SPM is increased, the point of minimum pulsewidth moves to negative GVD where the chirp is compensated. The GVD value required for $\beta=0$ is given analytically by [140]

$$|D| = (\delta/\gamma)\,(g/\omega_g^2), \tag{28}$$

and at this point again

$$\tau=4|D|/\delta W=\tau_0. \tag{29}$$

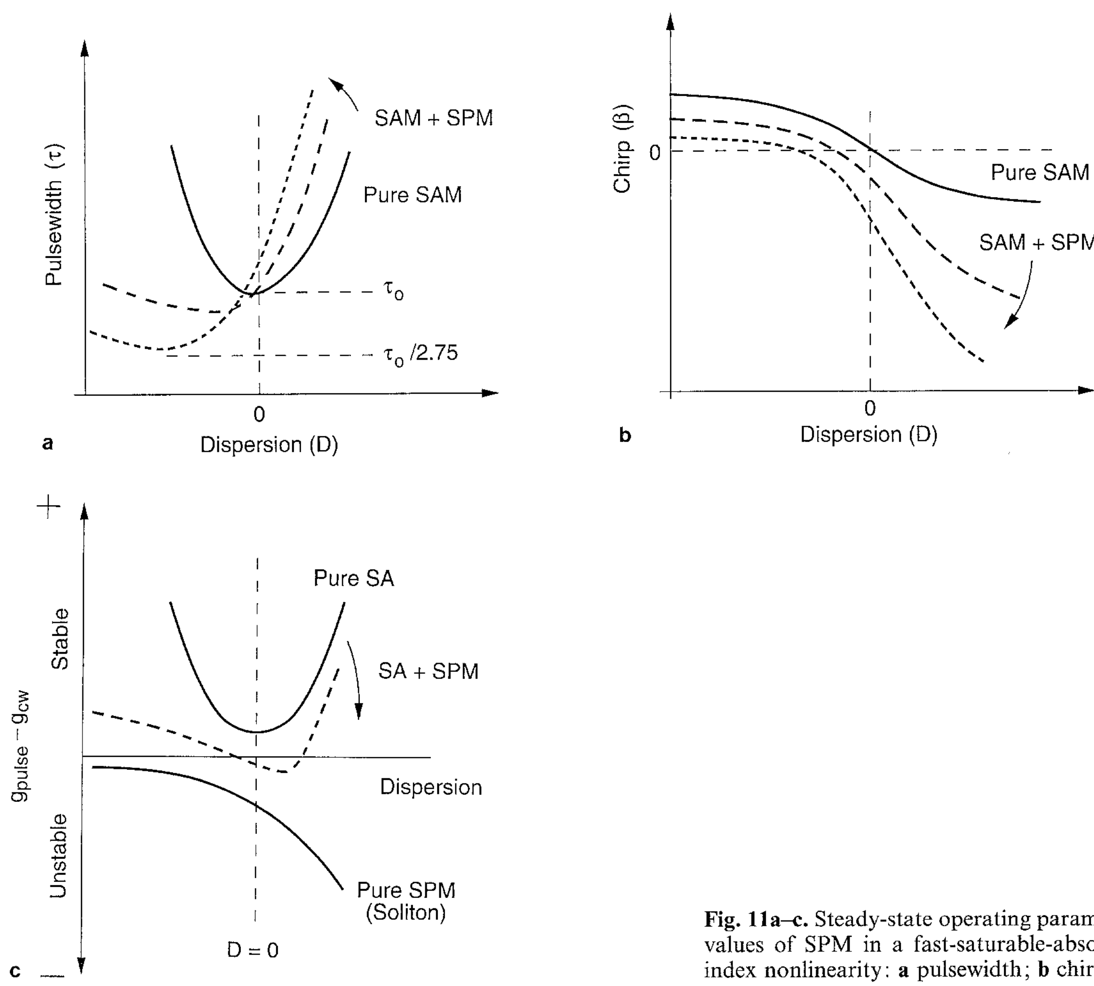

Fig. 11a–c. Steady-state operating parameters vs GVD for different values of SPM in a fast-saturable-absorber system with positive index nonlinearity: **a** pulsewidth; **b** chirp; **c** stability parameter

It is interesting to note that shorter pulses with chirp can be obtained at higher values of $|D|$, but they are not more than a factor of 2.75 shorter than they would be with SAM alone [141].

The solutions thus predicted by the master equation are, as in the slow-absorber case, not always stable. The condition for stability is not just that a steady-state pulseshape be produced but that it has greater gain than competing cw oscillation. To elucidate this condition we plot $g_{\text{pulse}} - g_{\text{cw}}$ in Fig. 11c. It reveals an interesting condition: a pure soliton laser (i.e. one without any amplitude modulation) is always unstable. Because the soliton requires more bandwidth than cw oscillation at the gain peak, the latter will always take over. Even with SAM, SPM can drive the system unstable unless sufficient GVD is introduced at the same time.

14 Higher-Order Effects

The master equation incorporates several approximations that are certainly violated by some practical laser systems. To obtain accurate operating parameters under such conditions, numerical simulations are required [142, 143]. Nevertheless, we can at least discuss some higher-order effects in the context of the analytical model described above.

a) At high powers the saturable absorber saturates fully, so that its modulating effect is no longer linear with intensity. It still provides a window of gain for stability, but this window becomes flatter and extended in time. Soliton shaping can keep the pulse somewhat shorter than the width of the amplitude shaping window, but the steady-state pulse duration will be longer than that predicted by the analytical theory alone. If the peak power of the pulse is limited, for example, by the particular saturation characteristics of an artificial saturable absorber, the single-pulse soliton energy is also limited. In fiber lasers this, along with periodic perturbations, can cause pulse break-up into multiple solitons [144]. One method for increasing the energy obtainable in a single pulse is the recently demonstrated stretched-pulse APM technique [145]. Finally, in soliton systems, when the roundtrip SPM phase shift becomes large, so that the cavity length is no longer much shorter than the soliton period, roundtrip-periodic perturbations produce pulse-width-limiting instabilities and spectral sidebands [146, 147].

b) At very short pulsewidths, Third-Order Dispersion (TOD) can be the dominant factor. In the master-equation formalism it introduces a third-order time-derivative term [148] for which no simple analytical solution has been found. Numerical simulations [148, 149] as well as experimental evaluations [149] indicate that TOD limits pulse shortening and produces asymmetries in shape and spectrum. Narrow-band features observed in the output spectrum also result from TOD phasematching to the continuum [148, 149, 12]. Efforts to characterize and compensate for TOD in femtosecond lasers are crucial for the reduction of pulse durations below current limits [12, 13, 115, 149–152].

c) When the changes produced by individual elements are large, the pulse may have significantly different durations and spectra in different parts of the cavity. The ordering of the elements can then be important, particularly in the case of strong SPM. A laser operating under these conditions has been called a solitary laser [153]. The pulse duration may be approximated by [153, 143]

$$\tau = \frac{3.53|D|}{\delta W} + \alpha \delta W, \tag{30}$$

where α is an empirical constant. The first term, like the master-equation result, is consistent with the prediction of adiabatic soliton shaping.; the second term gives the increase due to the large periodic changes. Equation (30) has shown good correspondence with results obtained with lumped-element solid-state lasers [153] and fiber lasers [142].

15 Self-Starting

Finally, we should note that self-starting is problem inherent to fast-saturable absorber systems. As Fig. 10 illustrates, fast-absorber systems differ significantly from active- and slow-absorber systems in that they have a very slow pulse-shortening-velocity when the pulse is long. In theory, if you wait long enough, a short pulse should develop out of any initial fluctuation. In practice, this does not always happen. One reason can be dynamic gain saturation. When it is significant, only fluctuations shorter than a given duration will develop [154]. Another reason can be competing pulse-dispersing processes. If the pulse does not shorten significantly within a cavity mode coherence time [155], it will be dispersed. This sets a power threshold for self-starting. Mode pulling due to spurious reflections [156] can cause a short coherence time, as can spatial hole burning [157]. It has been shown [158] that unidirectional ring operation, which reduces both spatial hole burning and the effects of multiple reflection, greatly facilitates self-starting. Still, there is not yet any completely satisfying way to ensure starting in all systems. Current methods (other than banging on the table) include tilting plates and moving mirrors [159–161], adding a real saturable absorber either intracavity [113] or in RPM mode [118], synch-pumping [63, 123, 162], regenerative initiation [120], or active modulation [163]. Each method brings with it trade-offs. Which is actually used will depend upon the application.

16 Summary

The generation of ultrashort pulses by passive modelocking is now an advanced and highly sophisticated art. A large number of passively mode-locked lasers has been developed for different wavelength regimes, different power levels, and different applications. All of them have somewhat different components, specific design needs and operating characteristics. The purpose of this paper has not been to discuss all of these systems and to document their characteristics. Instead we have tried to illustrate some of the common principles by which they operate. We classify each of them as either a slow-saturable absorber system or a fast-saturable absorber system. With a master equation, perturbational time-domain analysis we show that these two classes have characteristically different pulse-shortening rates as a function of pulsewidth. This affects their steady-state parameters, their starting properties, and their stability. Although more detailed, numerical analysis is becoming increasingly necessary as these systems are refined and pushed to their limits, the classic analytical analyses of slow-absorber and fast-absorber mode locking continue to provide valuable insight.

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