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Coupling Laser Radiation to a Fabry-Perot Cavity with a Single-Mode Optical Fiber

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Abstract. Three aspects of coupling to Fabry-Perot cavities used in optical frequency standards are discussed: the use of a single-mode optical fiber to maintain coupling stability while improving vibration isolation of the cavity, the required stability of the coupling geometry, and the phase and polarization variations resulting from fiber movement. Optical fiber coupling should be useful when laser linewidths and stabilities at the Hertz level are desired.

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Improvements of the stability of lasers will permit advancements in many areas of physics and metrology [1]. Some transitions of single, cooled, trapped ions [2] offer the potential that lasers stabilized to them will exceed the stability of cesium clocks, which have the best long-term stability to date. Such an ion is only a passive reference and an active frequency generator of high short-term stability is required to probe it. Five such generators are currently under study by various groups: a laser stabilized to a Fabry-Perot cavity [1], a phase-locked chain [3, 4] from a hydrogen maser, and atomic transitions involving many atoms, trapped clouds of ions [5], linear traps [6] and gas lasers [7].

A number of factors that affect the stability of a Fabry-Perot cavity as a frequency discriminator have been published [1, 8, 9]. The stability of the coupling between a laser and a cavity is a major factor controlling overall system stability. This paper deals with some geometrical aspects of coupling stability, related to coupling to a cavity with a single-mode optical fiber to obtain an optical frequency reference with a drift-corrected stability comparable to modern atomic time standards, $u_{ts} = 5 \times 10^{-15}$ for times longer than 1 s [10]. For this purpose, a cavity need not have an absolute length stability of u_{ts}. It is necessary only that its optical length at any given time be predictable to u_{ts} as a function of measurable properties, time and temperature in particular.

The length of a cavity will change in response to external forces. A kinematic mount [11] can ensure that longitudinal stress is very small. The vacuum required (10^{-5} Pa) to ensure that air pressure and air composition variations do not change the optical length of the cavity by more than u_{ts} will also reduce direct acoustic coupling to the cavity. However, in the application being considered, the radiation interacting with the cavity comes from an external source, and a cavity support sufficiently rigid to maintain transverse alignment between the external laser and the cavity will couple mechanical vibrations to the cavity. Hough et al. [11] and others consider mechanical vibrations to be the factor primarily responsible for their observed instability, 8×10^{-13} in Hough's case even with a massive cavity suspended in vacuum and surrounded by a lead-lined enclosure on a modern optical table.

An attractive method of overcoming this limitation is to use a flexible optical fiber to transmit the light between the cavity and the external laser source. This would allow the use of vibration isolation between the cavity and all components outside the vacuum. Vibration isolation systems have been developed for use in scanning microscopes [12] and in gravitational wave detectors [13] to a performance far beyond the 0.01 (40 db) ratio of u_{ts} to the best stabilities observed to date [11, 14], thus offering the possibility of reducing any contribution of seismic vibration of the cavity to observed instability to less than u_{ts}. A single-mode fiber is also useful as a spatial filter to improve locking stability [9]. However, use of such a fiber raises other problems. The transverse beam profile of an optical fiber differs from that within a Fabry-Perot cavity, which will increase coupling between the external laser and unwanted transverse modes of the cavity. Also, changes in length of the fiber will introduce phase errors in the signals to and from the cavity. These potential problems are discussed in the following paragraphs.

1 Mode-Shape Effects

The transverse profile of the field within a single-mode optical fiber differs from that in a cavity, where it is approximately Gaussian [15]. The problem can be treated analytically by expressing the fiber beam profile

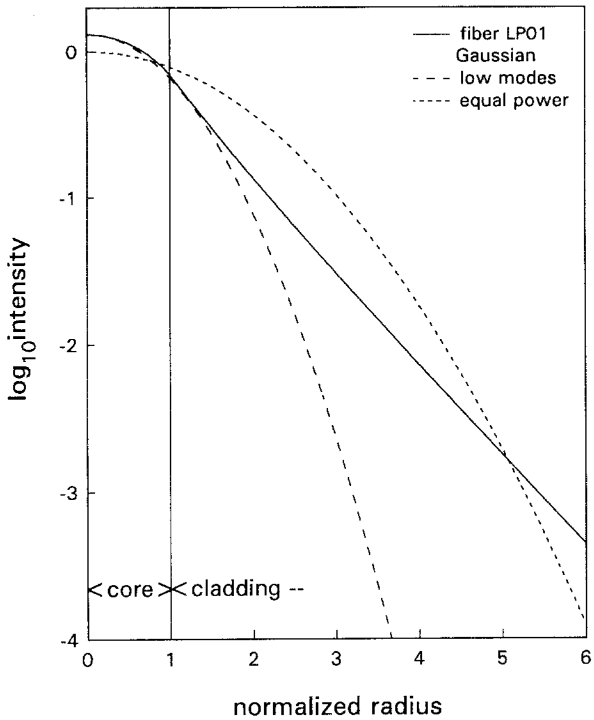

Fig. 1. Calculated near-field profile of a silica fiber, of the Gaussian profile selected for minimum coupling to modes higher than 0,0 and of the Gaussian profile of the same power as the fiber profile

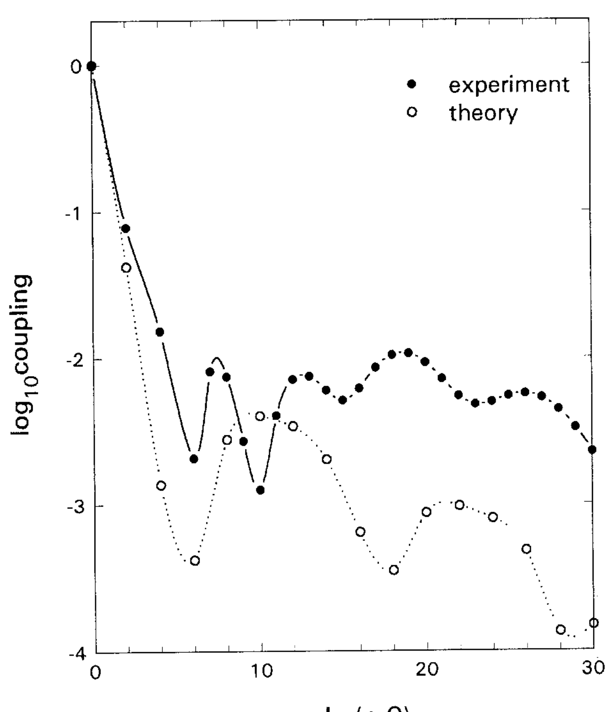

Fig. 2. Gauss-Laguerre decomposition of the calculated near-field profile of a silica fiber and of the measured far-field profile

as a Gauss-Laguerre series, then obtaining the coupling coefficients k between input Gauss-Laguerre modes and cavity Gauss-Laguerre modes $p > 0,0$.

The field distribution across the face of a single-mode fiber is approximately [16]

$$E_r = J_0(ur/a)/J_0(u), \quad 0 \leq r \leq a, \tag{1}$$
$$= K_0(vr/a)/K_0(v), \quad r \geq a, \tag{2}$$

where u and v are wavelength-dependent propagation scalars for the fiber core and cladding respectively, r is the radial distance from the fiber center, a is the radius of the core, J is a Bessel function and K is a modified Bessel function. The parameters u and v may be derived from far-field measurements at two wavelengths [17]. A profile of the output from the fiber we used for preliminary investigations [18], calculated from (1) and (2), is shown in Fig. 1 together with a Gaussian profile having the same power and one having the lowest coupling to high-order modes. Figure 2 shows the Gauss-Laguerre decomposition of this theoretical profile, and of the actual profile we obtained by far-field measurements on that fiber. Although the theoretical profile is cylindrically symmetrical, indicating no coupling to odd-order modes, imperfections in our fiber, particularly of cleaved end faces, result in measurable coupling to all modes above 6.

Over a measurement period, the coupling to all cavity modes of frequency other than the one desired must vary less than

$$u_m = u_{ts} v / v_c, \tag{3}$$

the ratio of the laser stability required to the cavity linewidth v_c, about 10^{-3} for a modern ($Q = 2 \times 10^4$) cavity. The eigenfrequencies of a Fabry-Perot cavity are given by [15]

$$v = c/2d\{q' + 1 + [(m+n+1)/\pi] \cos^{-1}(1 - d/r)\}$$
$$\equiv q v_l + (m+n) v_t, \tag{4}$$

where c is the speed of light, d the mirror spacing, r the mirror radii, q the longitudinal mode number, m and n the transverse mode numbers (either Cartesian or cylindrical), v_l the longitudinal mode spacing and v_t the transverse mode spacing. The ratio d/r should be chosen to minimize the product of the transverse mode comb

$$\Sigma_i k_i L_c(v - i v_t) \tag{5}$$

and the phase-modulation comb arising from the laser modulation for servolocking to the cavity

$$\Sigma^j_{-j} J_j(\beta) B(v - j v_p). \tag{6}$$

Here, k_i is the total coupling to transverse mode frequency i including the coupling due to the fiber, L_c the Fabry-Perot line shape, β the modulation index, v_p the modulation frequency and B the base band line shape of the laser to be locked. A typical pair of combs are shown in Fig. 3.

m and n should be limited to as small a value as possible to prevent an excessive number of terms in (5). This is usually done by an aperture. For a solid spacer of the highest stability, however, a better method is to use two orthogonal wires that may be inserted and adjusted, after contacting of the mirrors to the spacer, via two small holes in the spacer. (One hole is required in any case to evacuate the cavity.) It may be shown that the wires should subtend an angle $2\pi/m$ at the optical axis when their distance from the axis is $\omega_0 \sqrt{m}$, in order to evenly attenuate high-order cylindrical modes. For a

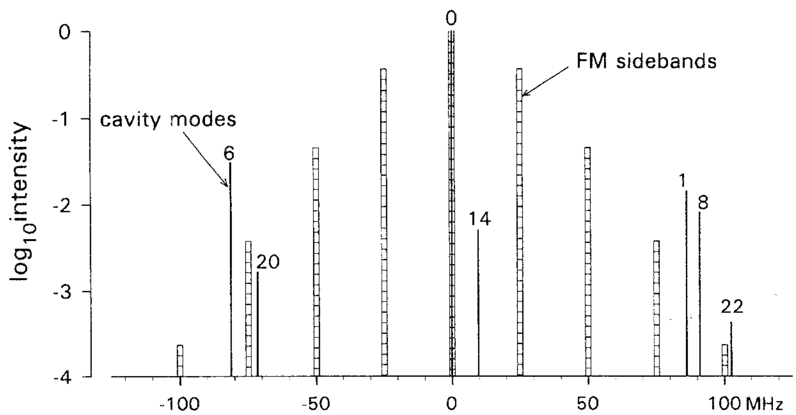

Fig. 3. Calculated overlap of the modulated output from a silica fiber and a cavity, with a modulation frequency of 25 MHz, modulation index 0.5, mirror radius 1.9877 m and mirror spacing 0.2456 m. The widths of the lines have been enlarged for clarity

$Q = 2\times 10^4$ cavity, $m+n$ can be limited to 10 without significant broadening of the TEM_{00} linewidth. The linewidth of a diode laser optically coupled to an external cavity can be as low as 10 kHz [19], so the product of (5) and (6) can be made very small even with the k_i factors due to a fiber.

2 Mode-Alignment Factors

The couplings k between an input Gaussian beam and cavity Cartesian modes $n>0,0$ and cylindrical modes $p>0,0$ due to geometrical perturbations from alignment may be derived from the equations of Bayer-Helms [20],

Factor	Coupling	
radial misalignment Δx,	$k^x_{n,0} = (\Delta x/\omega_0)^n/(n!)^{1/2}$,	(7)
angle misalignment $\Delta \gamma$,	$k^\gamma_{n,0} = (\Delta\gamma\pi\omega_0/\lambda)^n/(n!)^{1/2}$,	(8)
waist size mismatch $\Delta\omega$,	$k^\omega_{2p,0} = (\Delta\omega\sqrt{2}/\omega_0)^{2p}$,	(9)
waist position mismatch Δz,	$k^z_{2p,0} = (\Delta z\lambda/\pi\omega_0^2)^{2p}$;	(10)

from Andrade [21],

aperture of radius $a\omega$ $\quad k^a_{2p,0} \leq a^2 \exp(-a^2)/p$, (11)

where ω_0 is the Gaussian waist radius and λ the wavelength of the light. $k_{2p+1,0}=0$ for cylindrically-symmetric mismatches (9–11).

A numeric example is useful. If the $j=1$ phase-modulation sideband (6) were to coincide with the $i=1$ transverse mode (5), Δx would have to vary less than 0.1 µm, $\Delta\gamma<0.3$ mrad, and $\Delta\omega<2$ µm for the fiber and lens treated as a unit. Even worse, Δx_f for the fiber relative to the coupling lens would have to vary less than 10 nm for the optimum fiber-cavity matching lens. Clearly, such coincidences must be avoided for a cavity to be of use at u_{ts}. The matching of the axial waist is much less critical, Δz being 5 mm and $\Delta z_f=40$ µm, while (11) is satisfied for $a>3$.

While setting up a test cavity to check these calculations, we found that the most useful method of adjusting the input beam and cavity apertures to the required precisions was to scan a tunable laser (we used a piezo-tuned HeNe) over ν_l while viewing the cavity transmission on an oscilloscope synchronized to the tuning waveform. Varying the input beam γ or x from alignment resulted in each transverse mode in turn rising to a maximum then decreasing into oscillations. Even though the ν_l of our cavity exceeds the tuning range of our HeNe, it was possible to identify each mode in turn. We found a simple computer program useful to plot the mode heights h over the HeNe tuning range as the coupling geometry was adjusted, using

$$h_n = e^z[z^n/n! + (1-k)k^{n/2}], \quad (12)$$
$$\nu_n = n\nu_t \bmod \nu_l, \quad (13)$$

where z is the Cartesian mismatch and k the cylindrical mismatch of the input mode and the cavity mode. The term containing $k^{n/2}$ is used only for even modes. The parameters z and k were repeatedly input until the computer plot matched the pattern on the oscilloscope. With this method, the distance between the mode 0,0 peak and a transverse peak of $n\nu_t \approx q\nu_l$ yielded d/r to high precision, permitting the length of our cavity spacer to be chosen so that the product of (5) and (6) will remain small with maximal allowance for long-term spacer-length changes.

The input beam waist radius should be chosen for minimal coupling to high-order transverse modes, not for maximum coupling to the 0,0 mode. This criteria should also take account of coupling arising from lens spherical aberration and coma. It should be noted that, at the focal lengths involved, 5–10 mm, most lenses are very thick, and a thick lens can produce a Gaussian waist different from a thin lens of the same focal length.

This method also permits optimal positioning of cavity waist aperture wires. Attenuation of mode $2p$ by such a wire increases rapidly as a^2 becomes less than $2p$, so a may be determined from the large loss of a high order mode rather than by the broadening of mode 0,0.

3 Effects of Fiber Bending

The path followed by the LP_{01} mode within a fiber is dependent upon the curvature of the fiber [22]. When a fiber is bent in a radius R, the radius $R+\delta R$ followed by the mode exceeds that of the fiber by

$$\delta R \approx 130\ a^2/R \quad (14)$$

for a typical single-mode fiber, where a is the fiber core radius and $R/a > 500$. Variation of the radius of a bend will thus result in a frequency shift of the light transmitted around it, of

$$\delta v = (dl_o/dt)/\lambda, \tag{15}$$

where l_o is the optical length of the fiber. Due to the small size of the fiber core, typically 2 μm, this is a small effect–if the feed to a cavity is a 90° bend of radius 0.1 m and the cavity vibrates vertically ±1 mm at 3 Hz, the resultant linewidth is only 1 mHz at visible frequencies. The small size and large elastic modulus of the fiber also ensure that the physical length of the fiber centroid will change little with bending, unlike the case with coaxial cable where the elastic modulus of the shield is strongly nonlinear. No transverse movement of the output from the fiber will result from bending if the fiber end is held straight for a distance large compared to a, as it will be with any practical coupler.

Another factor should be looked at when optical fiber coupling is being considered. The polarization of light transmitted along a fiber rotates where the fiber is curved. Also, imperfections in real fibers introduce polarization noise–a linearly polarized LP_{01} mode degenerates into random polarization. Although the incident and reflected light from the cavity via the fiber need not be separated by polarization methods (which offer higher efficiency and isolation than nonpolarizing beam splitters), cavity mirrors have birefringence that is significant at the levels discussed here–of the order of 1 kHz [1]. Polarization-preserving fiber must be used when stabilities of u_{ts} are desired.

4 Conclusions

A single-mode optical fiber should offer advantages for coupling a laser to a stable Fabry-Perot cavity. The drawback of coupling to high-order cavity transverse modes due to the non-optimum fiber mode profile should be more than offset by the benefit obtainable from better vibrational isolation of the cavity spacer. Phase variations resulting from fiber movement are likely to be very small.

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