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OPINION LEADERS, INDEPENDENCE, AND CONDORCET'S JURY THEOREM

ABSTRACT. Condorcet's Jury Theorem shows that on a dichotomous choice, individuals who all have the same competence above 0.5, can make collective decisions under majority rule with a competence that approaches 1 as either the size of the group or the individual competence goes up. The theorem assumes that the probability of each voter's being correct is independent of the probability of any other voter being correct. Contrary to several authors, the presence of mutual or common influences such as opinion leaders does not easily rule independence either in or out. Indeed, and this ought to be surprising, *under certain conditions deference to opinion leaders can improve individual competence without violating independence, and so can raise group competence as well.*

Keywords: Condorcet, Jury Theorem, voting, democracy, independence, judgments.

1. INTRODUCTION

Condorcet's Jury Theorem shows that on a dichotomous choice, individuals who all have the same competence (or probability of being correct) above 0.5, can make collective decisions under majority rule with a competence that approaches 1 (infallibility) as either the size of the group or the individual competence goes up.[1] For example, 250 voters at competence of 0.51 have a group competence of 0.62, while a group of 10 000 at the same competence have a group competence of 0.98. The theorem assumes that the chance that voters A and B will both be correct is the probability of A's being correct times the probability of B's being correct.[2] This is only warranted if A's being correct and B's being correct are independent events in the following sense: the probability of A's being correct = the probability of A's being correct *given* that B is correct. If this does not obtain – if, for example, A is sure to do whatever B does – then each could have a 0.5 chance of being correct, while the chance of A's being correct would be 1 given that B is correct. This makes the probability of both being correct 0.5 rather than $0.5*0.5 = 0.25$ as the independence assumption would have it.

Worries arise over whether the existence of common influences such as opinion leaders necessarily undermines the independence of voters. If so, independence will be violated for many voters in democratic political communities, since political parties, national press, and other elements have wide influence. I ignore other objections to the theorem's applicability such as whether the idea of a 'correct' political choice makes sense.[3]

The problem of independence in the democratic application of the Jury Theorem has been thought to be more severe by some than by others. John Rawls, in a brief discussion of the theorem (1971, p. 538), has said that it is "clear that votes of different persons are not independent," because their views will be influenced by the course of discussion, and so "the simpler sorts of probabilistic reasoning [such as those employed by Condorcet's Jury Theorem] do not apply." Similarly, Grofman and Feld (1988, p. 570) claim that the independence involved in the Jury Theorem requires that "each voter is polled about his or her independently reached choice, without group deliberation." On the other hand, Waldron (in Estlund et al., 1989, pp. 1327–1328) argues that "the sort of interaction between voters that would compromise independence would be interaction in which voter X decided in favor of a given option just because voter Y did." "It does not matter, for Condorcet's argument, whether or not individual *competences* are independent of one another .... What matters, for the purposes of independence, is what happens when competence is exercised." Independence is met, he suggests, so long as there is no causal interaction between voters after the time the individual competences are assigned. Contrary to these authors, I will argue below that the presence of mutual or common influence among voters does not necessarily violate the requirement of independence. Indeed, and this ought to be surprising, *under certain conditions deference to opinion leaders can improve individual competence without violating independence, and so can raise group competence as well.*[4]

2. AN ACCESSIBLE PROOF OF CONDORCET'S JURY THEOREM

It is useful to begin with an accessible proof of the Jury Theorem, in order to see just where the issue of independence originates.

A few preliminaries:[5]

$$\sum_{h=m}^{n} (\cdots)$$

means sum the results of the formula for $h = m$, with the same formula for $h = m + 1$, for $h = m + 2$, and so on up to $h = n$. The values of n and m would be given.

Probability is measured in degrees between 0 and 1. 0 means no probability, or impossible, and 1 means absolutely certain. 0.5 means a 50/50 chance.

The probability of 'A and B' is usually less (and never more) than the probability of A or the probability of B. It is less likely that both will occur than that either will. The probability of 'A and B' = probability of A TIMES probability of B.[6] Since probabilities are usually less than 1, this product will be lower than either the probability of A, or the probability of B, unless one or the other equals 1. 'And' calls for multiplication of probabilities, and so yields smaller numbers.

'Or', on the other hand, calls for addition, yielding larger numbers. 'A or B' is usually more likely than A, and more likely than B.[7]

There is more than one way of choosing 2 things from a total of 3. From x, y, and z you could choose xy, or yz, or xz. The general formula for choosing r things from n is

$$n!/r!(n - r)!$$

$n!$ (pronounce 'n factorial') means $n * n - 1 * n - 2 \cdots * 1$. We won't bother to prove this formula here,[8] but notice that it gives the right answer for choosing 2 things from 3:

$$3!/2!(3 - 2)! = 3 * 2 * 1/2 * 1(3 - 2)!$$
$$= 6/2 * 1!$$
$$= 6/2$$
$$= 3$$

The formula for '3 choose 2' is abbreviated as

$$\binom{3}{2}$$

2.1. Proof of Jury Theorem

Consider as a simple example the case of 3 voters, each at 2/3 competence (0.666 . . .). We want to know the likelihood that at least a bare majority will be correct.

n (for number) = 3   (for simplicity we will assume n is odd)

p (for probability) = 2/3

m (for majority) = $(n + 1)/2 = 2$

1. The probability of at least a bare majority being correct, is the probability of the following: exactly m correct, OR exactly $m + 1$ correct, OR exactly $m + 2$ correct, OR . . . exactly n correct.

We'll make h a dummy variable that can take each of these disjunct probability values in turn, and then (since OR is present) we want to SUM all these probabilities for each value of h. That's what the following formula says:

$$\sum_{h=m}^{n} (\text{prob. of exactly } h \text{ voters being correct})$$

(read, 'where h goes from m to n, sum the probabilities of exactly h voters being correct').

2. What is the probability of exactly h voters out of n being correct? While h is less than n there will be more than one way of exactly h voters being correct, as noted in the preliminaries. The probability of exactly h voters being correct, is the probability of any one of these being the case; one way OR another way OR . . . . So we want to add the probabilities of each of the ways this could happen.

Take our example of 3 voters. There are 3 ways that exactly 2 of the 3 could be correct:

x and y correct, z incorrect

y and z correct, x incorrect

z and x correct, y incorrect

The probability of exactly 2 being correct is: probability of the first way

PLUS the probability of the second way PLUS the probability of the third way.

3. Now if each of these ways is equally likely, then the probability of exactly 2 being correct would just be the number of possible ways (or '3 choose 2') TIMES the prob. of a single way.

Let's take the first possible way of exactly 2 being correct: x and y correct, z incorrect. What is the chance of this occurring? It is the conjunction of 3 events, x being correct, AND y being correct, AND z being incorrect. So, assuming that these three probabilities are independent of each other, the probability is the probability of the first TIMES the probability of the second TIMES the probability of the third. *We see here that the Jury Theorem assumes that the probability of each voter being correct is independent of the probability of any other voter's being correct.*

The probability of x being correct is 2/3. That is all we mean by 'competence'. Same for y. The probability of z being incorrect is 1/3 just because the chance of z being correct was 2/3. So the probability of x and y correct, z incorrect is

$$2/3 * 2/3 * 1/3 \quad \text{or}$$

$$p * p * 1 - p$$

Notice that this will be the same for the other two cases as well

y and z correct, x incorrect

z and x correct, y incorrect

because all voters are assumed to have the same competence.[9] Therefore, as noticed above, the probability of exactly 2 being correct = '3 choose 2' TIMES the prob. of a single case of 2 being correct, or

$$\binom{3}{2} * (\text{the prob. of a single case of 2 being correct})$$

4. The probability of a single case of exactly 2 of our voters being correct is $2/3 * 2/3 * 1/3$. Notice the structure of this formula. The number for the individual competence, or p, occurs twice, and the other number is $1 - p$. The probability of a single case of exactly h

being correct is

> p times itself h times, TIMES $1 - p$ times itself the rest of
> the times, or $n - h$ times   or
>
> $$p^h * (1 - p)^{n-h}$$

5. We can see from steps (3) and (4) that the probability of exactly
h voters being correct is equal to

> $\binom{n}{h} *$ (the prob. of a single case of h being correct) $=$
>
> $\binom{n}{h} * p^h * (1 - p)^{n-h}$

6. From steps (1) and (5), the probability of AT LEAST m voters
being correct equals

> $$\sum_{h=m}^{n} \text{(prob. of exactly } h \text{ voters being correct)} =$$
>
> $$\sum_{h=m}^{n} \binom{n}{h} * p^h * (1 - p)^{n-h}$$

So this is the formula for the probability that at least a majority will be
correct, where $n =$ number of voters, $m =$ a bare majority, or $(n + 1)/$
2 (assuming n is odd), $p =$ the competence of every individual.

We can now calculate this for our example of 3 voters and compe-
tence of $2/3$: $n = 3$, $m = 2$, $p = 2/3$

> $$\binom{3}{2} * 2/3^2 * 1/3^1 + \binom{3}{3} * 2/3^3 * 1/3^0 =$$
>
> $$\binom{3}{2} * 4/9 * 1/3 + \binom{3}{3} * 8/27 * 1 =$$
>
> $$\binom{3}{2} * 4/27 + \binom{3}{3} * 8/27 =$$
>
> $$[3!/2!(3 - 2)!] * 4/27 + [3!/3!(3 - 3)!] * 8/27 =$$
>
> $$[6/2] * 4/27 + [6/6] * 8/27 =$$
>
> $$3 * 4/27 + 1 * 8/27 =$$
>
> $$12/27 + 8/27 =$$
>
> $$20/27$$

So while the individual competence is 2/3 or 18/27, the group competence under majority rule is 20/27.

3. IGNORING OPINION LEADERS' VOTES

The question is, how restrictive is the assumption, in step (3) of the proof, that all voters' competences are independent values? In particular, is it violated by the presence of opinion leaders who are followed to some (higher than random) degree by numerous voters? It is not, or so I shall argue.

I will be assuming that opinion leaders do not vote. Admittedly, this is unrealistic, but the advantages of the assumption outweigh the disadvantages. The main advantage is that we can assume homogeneous competence among voters, and still let the voters' competence vary from the opinion leader's . If we assumed that the opinion leaders voted we could only consider the special case where the voters have the same competence as their opinion leader.[10] Another advantage of denying suffrage to opinion leaders is that we can ignore the question of voters' independence from the opinion leader, and concentrate on the question of their independence from each other.

Neither of these advantages would be decisive if the impact of opinion leaders' votes were enormous. However, the Jury Theorem shows that (under appropriate assumptions) group competence increases with the number of voters, not with the ratio of voters to non-voting opinion leaders. Therefore, the impact of ignoring or counting the votes of opinion leaders will be negligible (so long as their competences don't appreciably *decrease* the average competence) even if they make up ninety percent of the population, so long as the population is large enough that the remaining ten percent is a very large group.[11]

To the extent that some of the OL's would have been voters, ignoring their votes amounts to acknowledging the fact that interdependence effectively reduces the number of voters to the number of voting blocks. But ignoring the OL's allows us to consider the question of two voters who defer to the same (non-voting) OL. Is this effectively a voting block for purposes of the Jury Theorem? I will argue that it is not, that this case does not preclude counting both the influenced voters in the Jury Theorem calculations.

4. FIDELITY AND INDEPENDENCE

Two events, a and b, are said to be independent if the probability of a given b is the same as the probability of a considered alone, that is, if

$$\text{prob}(a \mid b) = \text{prob}(a) \, .$$

The kind of event involved in the Jury Theorem is a person's voting correctly or incorrectly.

The issue of independence is sometimes stated as the question whether the several individuals' *votes* are independent events. However, two votes may be independent with respect to one property but not independent with respect to another.[12] The question that matters for the Jury Theorem is whether A's *voting correctly* is independent of B's doing so, whether the probability of A's voting correctly given that B does equals the probability of A's voting correctly considered alone.[13] This context relativity of independence is the key to showing that deference is compatible with independence.

Many find it tempting to think that, in general, the presence of a common influence destroys the independence of two events: that if neither A nor B is independent of C then A is not independent of B. Perhaps this fallacy is what makes it seem as though voters who defer to the same OL could not be independent. It is a fallacy, however. A simple counterexample is where C just is the conjunction of A and B. Neither A nor B is independent of this, but they might yet be independent of each other.

It is also natural, it seems, to think that two highly competent voters' competences could never be independent. However, the fact that A's voting 'yes' makes it quite likely that B votes 'yes' does not bear on the question of independence that concerns us. The probability that B votes correctly was already known to be 0.9. The probability that B votes correctly given that A does is not necessarily any higher. It may be higher, but that is not determined by their high competence, but only by some additional fact such as that B always follows A, or they both always follow some other leader, or they always reason alike, or some such thing.

Consider such a case more closely. Two voters, A and B, faithfully

follow the same opinion leader (OL).[14] (Assume, remember, that OL does not vote.) Does this undermine independence in the way that matters to the Jury Theorem? Independence is certainly not *necessarily* violated. Suppose, for example, that the OL has a competence of 1 (infallibility), and the probability that *A* agrees with OL and the probability that *B* agrees with OL are both also 1. *A* and *B* will always agree, but they remain independent, contrary to our intuitions. The probability of *A*'s being correct given that *B* is correct equals the probability that *A* is correct considered alone, since both equal 1.

The case is similar if the infallible OL is not followed absolutely by *A* and *B*, but each only follows OL with a fidelity of 0.7. The probability of *A*'s being correct given that *B* is correct will be 0.7, the same as the probability of *A*'s being correct considered alone, unless there is a non-random correlation between *A*'s voting yes and *B*'s voting yes (which is a separate issue from the correlation between *A*'s being correct and *B*'s being correct). Unlike the previous case, we cannot say that independence is *guaranteed*, since, for all we know, *A* might be following *B* rather than OL. That is consistent with our assumptions, but would make the probability of *A*'s being correct, given that *B* is, equal to 1, violating independence.

Suppose, now, that the OL is not infallible, say 0.7, but *A* and *B* both follow OL absolutely faithfully. The probability that *A* is correct given that *B* is correct equals 1. Since they both always follow OL, they will always agree with each other. But this is higher than the probability of *A*'s being correct considered alone, which equals 0.7, since this is the competence of one who faithfully follows a 0.7 OL. So, in this case independence will be violated.

What if a fallible OL is followed imperfectly? The answer, of course, depends on whether this results in the voters tending to agree with each other more (or less) often than they would based on their competences alone.

5. AN EXAMPLE OF INDEPENDENCE IN THE PRESENCE OF DEFERENCE

The example that follows describes two voters and an OL with their competences and the degree of voter fidelity to OL. *T* means true or

correct, F means false or incorrect. All choices are dichotomous, e.g., yes/no. A and B are voters, OL is an opinion leader. The eight possible outcomes are listed as ordered triplets, consisting of OL's vote, A's vote, and B's vote, respectively. For example, TTF means OL and A voted correctly, but B did not. Each possible outcome has a certain probability between 0 and 1.

TTT	TTF	TFT	TFF
0.26	0.24	0.24	0.06
FTT	FTF	FFT	FFF
0.10	0	0	0.10

In this example,

OL's competence $= 0.8$

A's and B's competence $= 0.6$

Each voter defers to OL to degree $= 0.6$

And yet A's being correct is independent of B's being correct:

prob(A corr GIVEN B correct) = prob(A correct)

$= (0.26 + 0.10)/(0.26 + 0.24 + 0.10 + 0)$

$= 0.6$

This case proves that (assuming x, y, $z \geqslant 0.5$) higher than random deference to a superior OL is not incompatible with voter independence. The case raises questions about the general structure and limits of such cases, and these are addressed below. First, though, consider a few implications. One implication is, of course, that so long as independence is not violated, the Jury Theorem still applies. The

deference has not invalidated the application of the Jury Theorem, as has often been supposed.

6. IMPROVING GROUP COMPETENCE THROUGH DEFERENCE

Furthermore, higher than random deference to a superior OL will typically increase the voters' competence over the case of a random level of deference. Since the Jury Theorem can still be applied, group competence will likewise be increased.

To see this, consider a "before and after case." The case just described will serve as the "after" case. An appropriate "before" case would have the following characteristics:

1. OL's competence is the same as in the "after" case.
   $x = 0.8$
2. The voters' competence is lower than in the "after" case.
   $y < 0.6$
3. The voters have the same competence as each other.
   $\text{prob}(A \text{ corr.}) = \text{prob}(B \text{ corr.})$
4. The probability of anyone (of OL, A, and B) being correct (or incorrect) is independent of the correctness (or incorrectness) of any other individual or pair being correct (or incorrect). In other words,

   > – Call a "case" an ordered triplet of T's and F's. Call the T's and F's "assignments." Call each member of the triplet a "component." A triplet represents OL's being correct or not, A's being correct or not, and B's being correct or not, respectively.
   > – The probability of each component's assignment in a case is independent of the assignments in the other components.
   > – Therefore, probability of any case obtaining is the product of the probabilities of its components. (Recall multiplication of probabilities gives the probability of the conjunction if and only if the conjuncts are independent.)

The following case meets these conditions (I have inserted in brackets
the more general relationships between the cases and the value of z. I
won't prove these here for reasons of space, since all I need is one
example.) In this example, the value of z is 0.55 (Jamie Dreier helped
me in constructing this example.)

TTT $[0.8z^2]$	TTF $[0.8(z - z^2)]$	TFT $[0.8(z - z^2)]$	TFF $[0.8(1 - 2z + z^2)]$
242	0.198	0.198	0.162
FTT $[0.2z^2]$	FTF $[0.2(z - z^2)]$	FFT $[0.2(z - z^2)]$	FFF $[0.2(1 - 2z + z^2)]$
0.605	0.0495	0.0495	0.405

This proves that it is possible to improve voter competence by way of
deference to a superior opinion leader and therefore increase group
competence without violating independence.

7. IN GENERAL, WHEN IS DEFERENCE COMPATIBLE WITH INDEPENDENCE?[15]

What is behind the examples? What are the general features that
define the class of cases in which deference does not preclude in-
dependence? Recall that $x = $ OL competence, $y = $ level of voter fidelity
to OL, and $z = $ homogeneous voter competence.

It is important to see that the fact that voters have competence z
does not determine their correlation. For example, voters that are all
correct 7 out of 10 times may agree with each other in every case, or
only in considerably fewer cases. If one's being correct is *independent*
of the other's being correct, then they will agree as often as they would
if each randomly chose which 7 out of 10 times to be correct. This is
the level of correlation 'expected' from voters of a given competence.
This correlation is the probability that either A is correct and B is
correct, or A is incorrect and B is incorrect. Assuming A's being
correct is independent of B's being correct, this is

$$(z * z) = ((1 - z) * (1 - z)) =$$

$$z^2 + (1 - z)^2 =$$

$$0.49 + 0.9 =$$

$$0.58$$

Call this level of correlation '*z-random-correlation*'. The surplus correlation that counts as a violation of independence for the Jury Theorem is correlation other than z-random-correlation, even though there could be other correlations given z (namely, when votes are not independent in the relevant respect.)

Turn now to the notion of the correlation 'expected' on the basis of y, the level of fidelity to OL. Again, y does not determine a single level of correlation. Voters who follow OL to degree y could agree with each other all the time, or considerably less often. There is, as with z, a special point in the range of correlations allowed by y, namely *y-random-correlation*. This is the correlation that would occur if it were randomly determined which times to agree with OL. That is, if A's agreeing with OL is independent of B's agreeing with OL, then the probability that A and B will agree with each other is the probability of either A's agreeing with OL and B's agreeing with OL, or A's disagreeing with OL and B's disagreeing with OL. This equals,

$$(y * y) + ((1 - y) * (1 - y)) =$$
$$y^2 + (1 - y)^2$$

If $y = 0.8$, for example, this equals

$$0.64 + 0.4 = 0.68$$

The formula (which is isomorphic to that for z-random-correlation) is not the important thing here. The important point is that there is a salient single level of correlation associated with y that is being referred to in the phrase 'the level of correlation expected from y'. It is true that y allows other correlations, and correlation can even be 1 for any value of y, as we have seen, but that isn't relevant to our question. When the correlation is higher than y-random-correlation, something is going on beyond the voter's fidelity to OL. For example A might be following B. Of course, the fact that A and B agree with OL much of the time does not rule out such higher correlations, but that is not our question. Our question is whether their fidelity of a given level, to an OL of a certain competence, violates independence. This is not the same question as whether independence is or is not violated under those conditions. The specified conditions underdetermine whether independence is violated. Still, where $y \leqslant z$, independence is only

violated (assuming x, y, $z \geqslant 0.5$) if there also happens to be a non-y-random-correlation. Therefore, it is not underdetermined whether the level of fidelity violates independence. It does not.

We may distinguish between independence being *insulated*, from its being *guaranteed*, in the following way. Independence is guaranteed if and only if given the level of y (fidelity to OL) the actual correlation could not differ from the z-random-correlation. However, even where this does not hold, it may be that the *level of fidelity* does not violate independence. Independence is insulated if and only if y-random-correlation would not differ from z-random-correlation, whatever the actual correlation. That is, independence is insulated if and only if it would be guaranteed if the correlation were y-random. The idea is that then, if independence is nonetheless violated, the reason is whatever is causing the non-y-random-correlation rather than the level of y itself. Therefore, we can see that when we ask whether a certain level of fidelity to an OL of a certain competence violates independence, we are asking whether independence is insulated, not whether it is guaranteed.

We cannot know whether independence is violated unless we know whether the actual correlation is higher than z-random-correlation. But *as long as y-random-correlation is not higher than z-random-correlation (which is whenever y is not higher than z under our assumptions) we can say that the fidelity to OL does not itself violate independence (though it might be violated).*[16]

A second complication[17] is to extend this rule for values of x, y, $z <$ 0.5. It should be clear that z-random-correlation, and y-random-correlation increase with the distance of y or z (respectively) from 0.5. The random correlation from voters at 0.1 (either z or y) will be the same as that for 0.9 (distance from $0.5 = 0.4$ in each case). Therefore, we can't say that in general independence is not violated so long as $y \leqslant z$. A counter-example would be $z = 0.1$, $y = 0.8$. Here z is much lower than y, but the z-random-correlation is much higher than the y-random-correlation (it equals the y-random-correlation if $y = 0.9$). So the general rule is that independence is not violated *by the presence of the OL*, independence is *insulated*, so long as y's distance from $0.5 \leqslant z$'s distance from 0.5 or so long as,[18]

$$|y - 0.5| \leqslant |z - 0.5| \,.$$

Suppose it were objected that by concentrating on y-random-correlation, rather than on the whole range of correlations allowed by y, we are assuming what is in question, that the voters are independent. If they aren't independent, the objection would go, their correlation will not be y-random-correlation. The mistake in this objection stems from using the undefined notion of *votes* being independent, a vice warned against earlier. What is in question is whether A's being correct is independent of B's being correct. Nothing is assumed about this by assuming that A's agreeing with OL is independent of B's agreeing with OL. It is entirely possible for the former kind of independence (which is what the Jury Theorem needs) to obtain without the latter, or *vice versa*.[19] By limiting our attention to y-random-correlation, nothing has been assumed about what is at issue, whether A's being correct is independent of B's being correct.

8. COMPETENCE IN LIGHT OF DEFERENCE

How useful is this general rule? Suppose we know that an OL of competence x is followed with fidelity y. We would like to know whether this violates independence. The answer, we have seen, depends on the individual competence, z. So we would have our answer if we could know z given x and y. However, we cannot. For example, suppose OL competence $(x) = 0.7$, and fidelity to OL $(y) = 0.5$. Voters will disagree with OL 5 out of 10 times. But we cannot assume that the voters will be wrong whenever they disagree with OL. They might disagree where OL was wrong, and then they would be right. If one can have 5 disagreements out of 10 answers with a 0.7 OL, as many as 3 of them could be in cases where OL was wrong, making the voter right. Add these 3 correct answers to 5 agreements when OL was right, and we see that the voter could be as competent as 0.8 $(0.3 + 0.5)$. On the other hand, the 5 disagreements could have been all when OL was right, making the voter wrong. That leaves 5 agreements out of which OL would be right only 2 times. This would give the voter a competence of 0.2. It is clear, then, that x and y do not determine z, and so we cannot in general know whether independence is violated just by knowing how good the OL is and how closely voters 'follow' or agree with it.

Let us use the name 'z-min' for the lowest z could be, given x and y, and the name 'z-max' for the highest z could be, given x and y. We can prove that x and y do determine z-min and z-max, and the respective formulae. We cannot know z based on x and y, but we can know all its *possible* values. As we will see later, these will be enough to yield some substantial conclusions about when deference is compatible with independence.

While there is a range of possible competences given x and y, there is, again, a salient single level of correlation that helps us to understand the nature of the range (it also becomes important in its own right later on). There is a single competence given x and y if we assume that voters' agreeing with OL is independent of whether OL is correct. This is equal to the probability that either the OL is correct and the voter agrees with OL, or the OL is incorrect and the voter disagrees with OL. This equals,

$$(x * y) + ((1 - x) * (1 - y))$$

This represents voters' competence as if, supposing a 0.7 fidelity, for example, the voter chose which 7 out of 10 times to agree with the OL randomly. This is a case of blind, but not complete deference; call this level of competence the *Blind Partial Deference Competence*, or BPD competence.[20] Since people may do better or worse than this kind of randomness, BPD competence is not the only possible competence given x and y. There is a range of possible competences.

8.1. How to Determine z-min Given x and y

The least I can be correct is to use all my agreements on OL's errors. That minimizes the fraction of times I would disagree with OL's errors (making me correct). But if my fraction of agreements (y) is less than the fraction of OL's errors ($1 - x$), then I will have to disagree with some $((1 - x) - y)$ errors, making me correct. If, on the other hand my fraction of agreements (y) is greater than the fraction of OL's errors ($1 - x$), then the worst I could do is to agree with all OL's errors. But this leaves more agreements, and there are no more OL errors to agree with, so I must agree with some $(y - (1 - x))$ correct answers.

Only if my fidelity exactly equals the number of OL errors could I have a competence of 0.

$$\text{If } y < (1 - x), \text{ then } z\text{-min} = (1 - x) - y$$

$$\text{If } y > (1 - x), \text{ then } z\text{-min} = y - (1 - x)$$

$$\text{If } y = (1 - x), \text{ then } z\text{-min} = 0$$

This can be summarized as

$$z\text{-min} = \text{the difference between } y \text{ and } (1 - x) \quad \text{or}$$

$$z\text{-min} = |y - (1 - x)|$$

(or, equivalently, $= |(1 - x) - y|$.)

8.2. How to Determine z-max Given x and y

If $y > x$, then I will have to agree sometimes where OL is wrong. The minimum of such times is $(y - x)$. So the best I could be is $1 - (y - x)$.

If $y < x$, then I will have to disagree with OL in some cases where OL is correct. As always, I will have to disagree in at least $(1 - y)$ cases. Of those, $(x - y)$ will be cases where OL is correct but I disagree, so I am incorrect. In the rest $(1 - x)$ I will have correctly disagreed. So I can be no better than $1 - (x - y)$.

If $y = x$, then I can agree exactly when OL is correct, no more, no less, and so I could have a competence of 1.

$$\text{If } y > x, \text{ then } z\text{-max} = 1 - (y - x)$$

$$\text{If } y < x, \text{ then } z\text{-max} = 1 - (x - y)$$

$$\text{If } y = x, \text{ then } z\text{-max} = 1$$

This can be summarized as

$$z\text{-max} = 1 - \text{the difference between } x \text{ and } y, \quad \text{or}$$

$$z\text{-max} = 1 - |x - y|$$

(or, equivalently, $1 - |y - x|$).

Conclusion

$$z\text{-min} = |y - (1 - x)|$$
$$z\text{-max} = 1 - |x - y|$$

As we have seen, independence is insulated (not violated by the deference to OL) if and only if,

$$|y - 0.5| \leqslant |z - 0.5|$$

By knowing x and y, we can know z-min and z-max. We can now use z-min and z-max to tell us in which cases, if any, independence is violated whatever the value of z, and in which cases, if any, independence is insulated or guaranteed whatever the value of z.

8.3. Guaranteed Violation

Independence will be violated whatever the value of z if and only if neither z-min nor z-max has a distance from 0.5 greater than or equal to y's distance from 0.5 (both have a distance less than y's). If either does have a distance from 0.5 greater than or equal to y's, then some values of z will produce expected correlations (z-random-correlations) at least as high as those expected from y (y-random-correlations), and so independence could be met (though it is not guaranteed since not all z's will do this.)

Violation is guaranteed if and only if,

$$|z\text{-min} - 0.5| < |y - 0.5| \quad \text{and} \quad |z\text{-max} - 0.5| < |y - 0.5|$$

8.4. Insulated Independence

By insulated independence I mean only that, given x and y, independence is not violated *by* the presence of the OL, whatever the value of z. As noted earlier, this does not guarantee that there is not some other threat to independence, such as a separate OL, and so indepen-

dence is not guaranteed. Independence will be insulated if and only if
y's distance from 0.5 is less than or equal to the distance from 0.5 of
the closest possible value of z to 0.5. In other words independence is
insulated if and only if y's expected correlation is less than or equal to
the lowest correlating value of z (the z-random-correlation of the
closest z to 0.5).

Call the closest possible value of z to 0.5 given z-min and z-max,
'z-close'. Its value is determined from z-min and z-max as follows:
One and only one of the following will always be true:

(a)   (z-min $\leqslant 0.5 \leqslant z$-max) ,   in which case z-close $= 0.5$

(b)   (z-max < 0.5) ,   in which case z-close $= z$-max

(c)   (z-min > 0.5) ,   in which case z-close $= z$-min

Independence is insulated if and only if,

$$|y - 0.5| \leqslant |z\text{-close} - 0.5|$$

8.5. Guaranteed Independence

We have distinguished insulated independence from guaranteed in-
dependence, and have yet to state conditions under which the latter
obtains. Independence is guaranteed if and only if y-correlation (not
limited to expected or y-random) could not exceed z-random correla-
tion. This will be the case only when $z = 0$ or 1, in which case
z-random correlation $= 1$. For any value of y, actual correlation could
always be one, so the only cases where z-random correlation cannot be
exceeded is where z-random correlation $= 1$.

I conclude that, from x and y alone, it can be determined whether
independence is certainly violated, insulated or guaranteed whatever
the value of z within its possible limits as determined by x and y.

8.6. Z-Contingent Cases

Even in the cases where we can't, on the basis of x and y alone, say
that independence is insulated, we can specify what z would need to be

Fig. 1. *V* means indep. violated whatever *z*. *I* means indep. insulated, whatever *z*. *G* means indep. guaranteed. $\leq n$, or $\geq n$ means indep. violated unless $z \leq n$ (or $\geq$). If there are two such numbers independence can be insulated in either way. Upper right gives *z*-max. Lower right gives *z*-min.

in order for independence to be insulated. We have seen that independence is insulated so long as

$$|y - 0.5| \leq |z - 0.5| .$$

z must be at least as far from 0.5 as *y*. For any value of *y*, there are two ways *z* could meet this, and there are two relevant cases of *y*:

If $y \geq 0.5$, then *z* could either be $\geq y$, or $\leq (1 - y)$, though it may be that both, only one, or neither of these is within the possible range for *z*.

Similarly, if $y \leqslant 0.5$, then z could either be $\leqslant y$, or $\geqslant (1 - y)$, though these might not both (or either) be allowed by the range for z.

These two cases for y overlap if $y = 0.5$ (and in that case independence is insulated). There is no value for z with less expected correlation than that. For any value of y, then, y and $(1 - y)$ will be important thresholds if they are allowed within z's range.

Figure 1 summarizes some calculations for round values of x and y. While, here and throughout, the charts are limited to round values, the formulae used here and defended above pertain to all values.

The following implications are significant (note especially (5)):

1. If OL's competence $= 0$ or 1, then independence is insulated.
2. If fidelity to OL $= 0.5$, then independence is insulated.
3. If $z = 0$ or 1, then independence is guaranteed. This is itself guaranteed if both (a) $x = 0$ or 1, and (b) $y = 0$ or 1.
4. If fidelity $= 1$, then independence is guaranteed to be violated, whatever individual competence, unless (as noted in (1)) OL's competence $= 0$ or 1, in which case independence is guaranteed.
5. *If fidelity $\geqslant 0.5$, then independence is met whenever individual competence $\geqslant$ fidelity to* OL.

> $-$ This is sometimes not a possible value for individual competence given OL's competence and individual fidelity to OL. (In those cases, refer to the chart.)

6. If fidelity $\geqslant 0.5$, then independence is met if individual competence $\leqslant$ (1-fidelity to OL).

> $-$ Since fidelity $\geqslant 0.5$, this will always make individual competence $\leqslant 0.5$, which is worthless to the Jury Theorem.

7. If fidelity $\leqslant 0.5$ ('loyal opposition'), then independence is met whenever individual competence $\geqslant$ (1-fidelity). This amounts to the individual being 'better than the OL is bad'. This will always make individual competence $\geqslant 0.5$, which is useful to the Jury Theorem.

– This is sometimes not a possible value for individual competence given OL's competence and individual fidelity to OL.

9. BLIND PARTIAL DEFERENCE VIOLATES INDEPENDENCE

What is the meaning of the fact that (simplifying now to the cases where x and $y \geqslant 0.5$) independence can be met only if individual competence is at least as high as fidelity to OL?

Part of what this requirement means is that independence requires that the individual 'add some wisdom' to that of the OL in a certain sense. As we have seen, it is not required that the individual have a higher competence than the OL, so that is not the relevant sense; that would not be adding, but rather supplanting, in any case. A fidelity of 0.7 means the voter will agree with the OL in 7 out of 10 cases. There is still the question of which 7 cases will be the cases of agreement. The requirement that competence be higher than fidelity implies (though, as we will see, is not equivalent to) a requirement that the particular 7 agreements out of 10 cases be chosen with better than random accuracy. We are not saying that OL's cannot be followed very closely; under the right circumstances they can be followed as closely as you please.[21]

Given that a voter will agree with OL 7 out of 10 times, there is still the possibility that the particular 7 will be chosen no better than randomly. For example, suppose I closely follow the New York Times editorials, and vote accordingly 7 out of 10 times, but that I regard myself as being able to do better than the Times by knowing when to reject their advice. That is why I disagree with them 3 out of 10 times. But I may be wrong about my abilities to choose when to reject their advice, and I will do no better than if I randomly chose which 7 out of 10 to agree with and which 3 to disagree with. Someone else, on the other hand, may be able to disagree with better than random accuracy. Perhaps this person can detect certain 'blind spots' of the Times, or certain commercial pressures on their opinions. This is the sense in which some wisdom must be added, beyond the OL's competence, and beyond the degree of fidelity to OL, if independence among voters is to be maintained.

This kind of adding of wisdom is implied by the requirement that competence meet or exceed fidelity, because that will always require a competence higher than if the particular 7 of 10 agreements were chosen as if randomly. We have called competence in the random case, competence under Blind Partial Deferrence to OL, or BPD competence. We can calculate this value precisely given OL's competence and the voters' fidelity to OL. To say that the agreements are chosen as if randomly is to say that the voters' voting as OL does is independent (in our usual sense) of OL's being correct. Then, recall, BPD competence, the chance of such a voter being correct, will be the chance that either the OL is correct and the voter agrees, or the OL is incorrect and the voter disagrees:

$$z(\text{under BPD}) = (x * y) + [(1 - x) * (1 - y)] \,.$$

If, as we have been assuming, x and $y \geqslant 0.5$, the value of this BPD competence will be less than y unless $y = 0.5$ or 1, or $x = 1$. We can say then,

> for $0.5 < x$, $y < 1$, the requirement that $z \geqslant y$ for independence to be met, implies a requirement that z be better than it would be under Blind Partial Deference.

The two requirements are not equivalent, however, since even if some wisdom is added in the appropriate sense, it might not be enough. Deference may be better than BPD, but not enough better to meet independence. BPD competences can be added to the previous chart for the cases of round values of x and y, to show how much better than this z must be, as in Figure 2.

The amount of wisdom that must be added (in the previously defined sense) can be gauged by comparing the BPD competence with the z-threshold(s) as in Figure 3. For example, if $x = 0.7$, $y = 0.7$, then BPD competence $= 0.58$, but competence (z) must be $\geqslant 0.7$ if independence is to be preserved. The individual must be sufficiently better than random (in choosing which 7 out of 10 cases to agree with) to increase competence by 0.12.

For x, $y \geqslant 0.5$, and round values of x and y, the most that must be added is 0.24 ($x = 0.6$, $y = 0.8$, BPD $= 0.56$, z-threshold $= 0.8$). It is

Fig. 2. *V* means indep. violated whatever *z*. *I* means indep. not violated, whatever *z*. *G* means indep. guaranteed. $\leq n$, or $\geq n$ means indep. violated unless $z \leq n$ (or $\geq$) ('*z*-threshold'). If there are two such numbers independence can be insulated in either way. Upper right gives *z*-max. Middle right gives *Blind Partial Deference* competence. Lower right gives *z*-min.

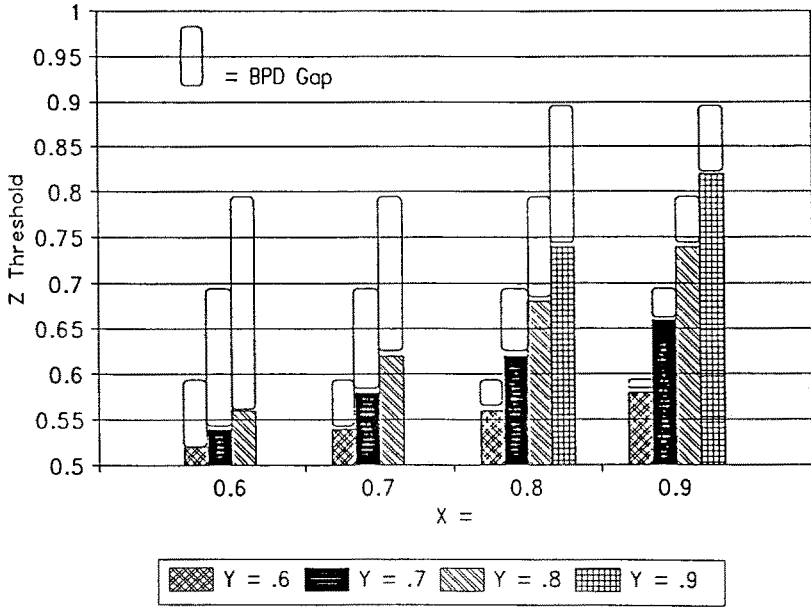| Y	.6	.7	.8	.9
.9	V	V	.16	.08
.8	.24	.18	.12	.06
.7	.16	.12	.08	.04
.6	.08	.06	.04	.02

X→

Fig. 3. BPD gaps.
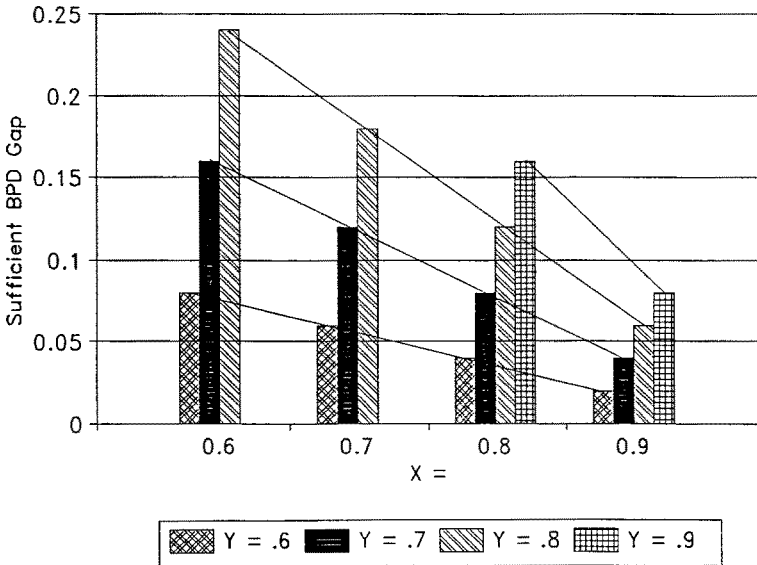
Fig. 4. BPD values and gaps.

Fig. 5. BPD gaps compared.

tempting to conclude that independence only requires a modest increase from the BPD competence, but it is somewhat unclear what should count as 'modest' in this context. Figure 4 gives BPD competences along with the gap between that and the z-threshold. The last graph, Figure 5, isolates the values which, if added to the BPD competence is sufficient to preserve independence.[22] We might call each of these values a Sufficient BPD Gap.

Implications of BPD results

1. The BPD gap goes down as either OL's competence goes up or fidelity goes down (approaching 0.5). (Or, BPD goes up as either OL's competence goes down or fidelity goes up.)

> – Therefore the largest gap obtains for the lowest OL competences with the highest fidelity. (The largest BPD gap seems to occur in $x = 0.58$, $y = 0.79$, BPD gap $= 0.2436$. Making x at all lower, or y at all higher guarantees violation of independence.)
> – The smallest gap obtains for highest OL competence, with the lowest fidelity. (BPD competence never reaches y, so there is always a BPD gap, unless $y = 0.5$. E.g., $y = 0.51$, $x = 0.99$, BPD gap $= 0.0002$.)

2. Blind partial deference never yields independent voters, except in the special cases where independence is insulated or guaranteed ($x = 0$ or 1, or $y = 0.5$).

The general formula for the size of the BPD gap is

$$y - \{(x * y) + [(1 - x) * (1 - y)]\}$$

($=$ fidelity to OL minus BPD competence)

10. DOES ADDING WISDOM INTRODUCE TROUBLING DEPENDENCIES?

Deference does not itself violate independence so long as it is wise rather than blind deference, and wise to a certain degree. The voters

must be better than random at choosing when to defer to the OL and when not to. More precisely, the probability that the OL is correct given that the voter deferred to OL must be greater (to some certain extent) than the unconditional probability that OL is correct (or OL's competence). This requirement raises new questions: Will such wise deference create dependence among the voters at least in those cases where OL is correct? Even though this doesn't logically preclude overall independence, wouldn't such dependence have to be balanced out by a countervailing dependence in those cases where OL is incorrect, and doesn't this seem an unlikely arrangement (even if possible)?

Taking these in reverse order, saving the most important for last, the occurrence of countervailing dependencies of this kind does indeed seem anomalous. Even though it is possible, it is hard to imagine any common mechanism that could bring this about in real voting situations. This should be a problem if adding wisdom entailed voter interdependence when OL was correct. Fortunately, and surprisingly it does not. Voters can add wisdom to the OL in the required way and be independent given OL's correctness, independent given OL's incorrectness, as well as independent overall.

The first thing to notice is that in the example provided above (the "after" case), the voters are not positively dependent, but negatively dependent in those cases where OL is correct. That is, the probability, in those cases where OL is correct, of A being correct given that B is correct is less, not more, than the probability of A's being correct overall in those cases $(0.4 < 0.52)$. This strongly suggests, and it can be proven, that they could have been either positively dependent, negatively dependent, or independent in the cases where OL is correct. In the following example, A and B are independent overall despite higher than random deference to OL (to the same degree as in the earlier example) and A and B are still independent even in those cases where OL is correct, and also in those cases where OL is incorrect.

TTT	TTF	TFT	TFF
0.3125	0.1875	0.1875	0.1125

FTT	FTF	FFT	FFF
0.0475	0.0525	0.0525	0.0475

In this case, as before, x (= OL's competence) = 0.8, y (= level of deference) = 0.6, and z (= voter competence) = 0.6. The voters are independent overall, since,

$$\text{prob}(A \text{ correct} \mid B \text{ correct}) = \text{prob}(A \text{ correct}) = 0.6$$

They are independent in the cases where OL is correct, since,

$$\text{prob}(A \text{ correct} \mid B \text{ correct \& OL correct})$$

$$= \text{prob}(A \text{ correct} \mid \text{OL correct})$$

$$= 0.625$$

Since they are independent when OL is correct, they must be independent in the remainder.

Now, even though this is possible, it may require some structure that is implausible in actual voting procedures. Further work would be required to determine what kinds of voter attitudes and interactions must be posited to allow such a case to occur, and to assess their meaning and plausibility. For now, I conclude only that voter deference to OL's does not necessarily undermine voter independence. It does require "adding wisdom" while deferring, but this is compatible with voter independence even in the set of cases where OL is correct, and the set where OL is incorrect.

11. CONCLUSION

Independence cannot be easily ruled in or out merely by knowing voters are influenced by common opinion leaders. If their deference is complete then independence is violated, and even if it is partial, if it is blind in the discussed sense independence is violated (except in the special cases of insulated or guaranteed independence.) But if it is informed, or better than blind, and partial, then independence may well be met even for realistic values of OL competence, fidelity, and individual competence. In addition, in that case, which amounts to choosing wisely (but imperfectly) when to defer to the OL, it is possible to improve individual competence without violating independence, and so to improve group competence through deference.

It must be kept in mind that in the cases I have discussed, voters are not independent of OL's, but only independent of each other. Inter-dependencies among voters have the same effect as reducing the number of voters who figure in the Jury Theorem calculations. In the limit, it effectively reduces to the number of independent voters.[23] If the deferring voters were not independent of each other the effective number of voters would be reduced in the direction of the number of OL's. In certain kinds of factionalization, the number of OL's is vastly smaller than the real number of voters, and so the loss would be devastating from a Condorcetian point of view. The present argument shows, however, that where sets of voters defer to voting OL's[24] this effectively reduces the number of voters in the direction of *the number of non*-OL's, since the non-OL voters can remain independent of each other. This will be a much smaller reduction when the number of OL's is a small fraction of the number of real voters. For example, consider 10 000 voters divided into 20 OL-led factions. The effective number of voters is $\geq 9\,980$, rather than the ≥ 20 that would result if independence of non-OL voters were violated.

It may turn out that the cases in which independence is compatible with deference are very unlikely or undesirable in real voting situations for some reason. So far, there is no reason to believe this or to deny it. At present, the independence assumption should not be regarded as impossible for large numbers of voters to meet, and so it is not obviously a serious obstacle to the application of the Jury Theorem in democratic theory, whatever other obstacles there may be.[25]

NOTES

[2] It is assumed that the choice is between two alternatives, of which only one is correct, or best, or different in some (any) way that allows us to speak of the probability of choosing that one rather than the other. Here we will speak of correctness, but the Jury Theorem is not limited to this.
[3] I discuss this question in detail, and press a distinct objection to the Jury Theorem, in Estlund (1993).
[4] It is important to distinguish the present topic from the issues involved when independence is assumed not to be met. Shapley and Grofman (1984) consider cases where independence is violated by assumption. They point out that under certain conditions group competence is drastically lower than if voters were independent. They

also consider the optimal scheme for weighting votes under certain conditions of interdependence. Clearly this is a different project from that of the present paper, which asks roughly, When is deference compatible with the kind of independence the Jury Theorem requires?

In another related discussion (Grofman, Owen, and Feld (1983)), Owen presents a result based on the assumption that homogeneous voters follow an opinion leader (OL) (who is assumed to have the same competence as the rest) in some fraction (α) of cases, and vote independently of the OL in the rest. Owen's α is not the same as my y, or fidelity to OL, since in their fraction of independent votes, Owen's voters may still agree with OL. My y is the fraction of all of a voter's agreements with OL. Also, I remove the restriction that OL's competence must be the same as the rest.

Owen considers what happens to group competence when a certain kind of dependence obtains in this special case. He concludes that under certain conditions group competence is drastically reduced. My question, on the other hand, is under what conditions can there be deference to an OL without violating voter independence in the appropriate sense, and so leaving group competence at the level set by the Jury Theorem. Owen subtitles his theorem 'Think for Yourself, John'. This is an unwarranted interpretation of his result. The results below show that in some cases voter competence can be raised by (non-blind, partial) deference without losing the Jury Theorem effects, and so group competence can be raised through deference as well. This all anticipates material that is better explained below. The point here is that neither of the cited discussions, nor any that I know of, takes up the question of when, if ever, deference to opinion leaders or other common influences undermines voter independence from the standpoint of the Jury Theorem.

[5] The theorem is originally proved in Condorcet (1785). A famous explanation and discussion occurs in black (1958).

[6] I am assuming that event A is independent of event B, that is, that the probability of A is no different than the probability of A given that B.

[7] I am assuming that A and B are mutually exclusive: A entails not-B, and B entails not-A.

[8] Any text on statistics or probability will discuss it under 'binomial coefficients'. See, for example, *Theory and Problems of Probability*, Lipshutz (1965).

[9] This is a restrictive assumption, but the basic result is not much changed if we assume that we know only the average individual competence. See Owen *et al.* (1989).

[10] My framework has this advantage over Owen's. See brief discussion in text above.

[11] Suppose, for example, we regard any improvement in group competence over 0.999 as negligible, and suppose that opinion leaders make up 4/5 of the population, and assume individual competence = 0.55. How large must the whole population be in order that the non-opinion leaders alone, have a group competence, under majority rule, of at least 0.999? Since a group of 1000 voters has a group competence of 0.9993 under majority rule (I skip that calculation here), a population of a mere 5000 would be sufficient, even under our unfavorable assumptions, to render the impact of the opinion leaders' votes as negligible.

[12] For example, A's voting in the afternoon might not be independent of B's voting in the afternoon, since they might always go together, but go in the morning half the time. In that case, the probability of A's doing so given that B does = 1, while the probability of A's doing so considered alone = 0.5. Ought we to conclude that A's and B's votes are not independent events? Does this show the Jury Theorem could not apply to them? Clearly not.

[13] There are two different ways we might define A's competencé, the probability that A votes correctly. We could choose to mean either

> = the probability of
> [(A votes yes and yes is true) or
> (A votes not and no is true)]

or we could choose to mean,

> = the probability that (A votes yes given that yes is true)

> = the probability that (A votes no given that no is true).

The latter approach asserts that A is equally competent when the truth is yes as when it is no. This need not be so, and so it is an advantage of the former approach that it does not assert this. It is the former interpretation that is adopted here. Of course, the Jury Theorem itself is absolutely indifferent to the interpretation of p, so long as its value falls between 0 and 1 (inclusive). The interpretation of p, may, however, affect p's value, and in turn the level of group competence. To interpret p is partly to interpret group competence.

Also, we will assume that yes and no are equally likely to be true, even though this too could fail to be the case under certain circumstances.

[14] Following our definition of competence, A's degree of fidelity to an opinion leader can be defined as the probability that either (A says yes and OL says yes) or (A says no and OL says no).

[15] The next two sections are more technical. No central conclusions would be missed by skipping them.

[16] There is no need to ask whether y-random correlation is possible. Independence requires (a) actual correlation = z-random-correlation, which requires (b) actual correlation $>$ y-random-correlation (by BPD gap, see below). The question is when *this* is possible.

[17] The first was the explicit notice of ranges of correlation and defense of the salience of z-random-correlation and y-random-correlation.

[18] '$|n|$' means absolute value of n, or change negatives to positives.

[19] X would have to be $\geqslant 0.4$ for this to be a possible value for z, as shown in the charts discussed later.

For an example of z-independence without y-independence, if $y = 0.6$, y-random-correlation is 0.52. Suppose voters turn out to agree with each other to degree 0.6 (6 out of 10 times), higher than expected from y alone. We could call this a violation of y-independence. But none of this precludes individuals having competence that gives a z-random-correlation of 0.6, namely $z = 0.723607$. Since the actual correlation = z-random-correlation in this case, z-independence, as we might call it, is met. And this is the independence the Jury Theorem requires.

Next, consider an example of y-independence without z-independence – of A's agreeing with OL being independent of B's agreeing with OL, but A's being correct *not* being independent of B's being correct. Such a case would require an actual correlation that equals y-random-correlation, but is greater than z-random-correlation. An example is $y = 0.8$, $z = 0.6$, correlation = 0.68 (which = y-random-correlation for $y = 0.8$).

[20] It is not the same as y-random-correlation, or z-random-correlation. Nor does it define any sense of votes being independent of each other, but rather defines independence of OL-agreement from voting correctly.

[21] E.g., if $y = 0.99$, then independence is met if $x = 0.999$. This makes BPD competence $= 0.98902$, z-min $= 0.989$, z-max $= 0.991$.

[22] While they are sufficient, they are not necessary, since in several cases there is also a z-threshold ≤ 0.5. In those cases, there is another way of preserving independence, namely, getting at least as bad as $1 - y$.

[23] See Grofman and Feld (1988), p. 571, and citations found there.

[24] This shows how to drop the simplifying assumption that OL's don't vote.

[25] There are serious obstacles, however. The central one is the difficulty of knowing whether the average competence is above the 0.5 threshold. I press this epistemological problem against the Jury Theorem in Estlund (1993).

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