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STOCHASTIC DOMINANCE IN MULTICRITERION ANALYSIS UNDER RISK

ABSTRACT. Traditionally, in the literature on the modelling of decision aids one notes the propensity to treat expected utility models and outranking relation models as rivals. It may be possible, however, to benefit from the use of both approaches in a risky decision context. Stochastic dominance conditions can be used to establish, for each criterion, the preferences of a decision maker and to characterise them by a concave or convex utility function.

Two levels of complexity in preference elicitation, designated as clear and unclear, are distinguished. Only in the case of unclear preferences is it potentially interesting to attempt to estimate the value function of the decision maker, thus obtaining his (her) preferences with a reduced number of questions. The number of questions that must be asked of the decision maker depends upon the level of the concordance threshold that he(she) requires in the construction of the outranking relations using the ELECTRE method.

KEY WORDS: Multiattribute decision aid, stochastic dominance, outranking relations, ELECTRE methods, multicriterion analysis under risk.

1. INTRODUCTION

It is easy to imagine many decision contexts in which the performance of the alternatives to be compared is not known with certainty. Nevertheless, little research in multicriterion analysis has considered decisions in a context with uncertain outcomes.

The past work on multiattribute utility (MAUT) theory (Keeney and Raiffa, 1976) suggests the use of a single synthetic criterion approach (Roy, 1985), but the application of this approach is greatly compromised by the necessity of obtaining complete information on the decision maker's preferences. The analytical hierarchical process (AHP), for which Saaty and Vargas (1987) have proposed a version that introduces uncertainty, may also be included in this family of approaches. Recent work in multiobjective mathematical programming (MOMP) under uncertainty by Goicoechea *et al.* (1979), Leclercq (1982), Teghem *et al.* (1986) and Urli (1989) can

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be classified as local interactive judgement approaches with trial and error. However, the power of the MOMP techniques used in these studies is only completely justified when solving problems where the implicit decision set is infinite or very large.

In the family of synthetic outranking approaches can be found the studies by, in chronological order, Jacquet-Lagrèze (1977), Dendrou *et al.* (1980), Martel *et al.* (1982, 1986), Siskos (1983), Mareschal (1986) and D'Avignon and Vincke (1988). Most of these models (except for those of Siskos and Mareschal) use a probabilistic binary relation which varies significantly among the various models. Mareschal (1986) gives very little consideration to risk and does so, on the last page of his article, based on the notion of an expected utility function calculated by using probability distributions over the differences between two performances. Siskos (1983) suggests a stochastic ordinal regression method (stochastic UTA), which could be classified in the family of single synthetic criterion approaches, or even in the family of interactive judgement approaches (except that the interaction is based more on the construction of preferences than on the alternative set (Vincke, 1989)).

Jacquet-Lagrèze (1977) constructs a fuzzy preference relation by solving a linear program expressing probabilistic relations and calculating three trivial preference relations. Dendrou *et al.* (1980) build a non-stochastic matrix of the probabilities that one alternative will dominate another. They then determine concordance and discordance indices in order to construct outranking relations, as in ELECTRE III. Martel *et al.* (1982, 1986), along the same lines, establish a confidence index by using probabilities that one alternative is as favourable as another. They determine doubt indices by using mean deviations that are unfavourable to outranking, criteria weights, and the variation in distributional estimations. As in ELECTRE III, they introduce thresholds (such as indifference, strict preference, veto, etc.) to construct quantified outranking relations, which are qualified as fuzzy.

D'Avignon and Vincke (1988) determine the degrees of probabilistic outranking relations which incorporate performance probability distributions and preference indices relative to these performances, although they are not very explicit on the procedure permitting the calculation of these indices. From these degrees of outranking they derive two types of probability distributions; for each alternative they obtain a distribution expressing its 'strength' and another expressing its 'weakness'. Finally, they propose procedures taken from the ELECTRE and PROMETHEE methods for using these probability distributions.

Even if there are some other works in that direction (Colson and De Bruyn, 1989; Siskos and Assimakopoulos, 1989 etc.), this past research has far from exhausted the avenues which could permit the development of a multicriterion decision aid model in a risky context. This article proposes a multiattribute decision aid model, based on stochastic dominance results which is used to build, as in the ELECTRE methods, outranking relations which consider the possibility of incomparability.

The paper is structured as follows. The problem is formulated in Section 2. Section 3 presents the results emanating from stochastic dominance conditions for each attribute and in Section 4 these results are used to build the outranking relations. The method thus developed is used to solve several examples taken from the literature in Section 5.

2. FORMULATION OF THE PROBLEM

We consider a multiattribute problem which can be represented by the $\mathcal{A}.A.E.$ model (Alternatives, Attributes, Evaluators). The three important elements of this model are as follows:

1. a set $\mathcal{A} = \{a_1, a_2, \ldots, a_m\}$ representing the set of all feasible alternatives;
2. a set $A = \{X_1, X_2, \ldots, X_n\}$, of attributes, an attribute X_i defined in the interval $[x_i^0, x_i^1]$ where x_i^0 is the worst value obtained with the attribute X_i and x_i^1 is the best value;
3. a set $E = \{f_1, f_2, \ldots, f_n\}$ of evaluators, an evaluator $f_i(x_{ij})$ being a probability function associating to each feasible alternative a_j a non-empty set of x_{ij} (a random variable) called the evaluation of a_j relative to the attribute X_i.

These attributes are defined such that a larger value is preferred to a smaller value ('more is better') and that the probability functions are known. We also assume that the attribute set A obeys the additive independence condition (Keeney and Raiffa, 1976; Colson, 1989). This stochastic multiattribute problem is approached by using stochastic dominance conditions to compare the alternatives, two by two, on each attribute considered individually. These comparisons are interpreted in terms of partial preferences. Next, the synthetic outranking approach (Roy, 1968; Roy and Bouyssou, 1993) is used by constructing outranking relations based on a concordance index and a discordance index. With this approach, a majority attribute condition (concordance test) replaces the unamimity condition of classic dominance. Finally, these outranking relations are used to solve the problem, either by choosing the best alternative or by ranking the set of alternatives.

3. PARTIAL PREFERENCES BETWEEN TWO ALTERNATIVES UNDER RISK

As indicated above, the context examined is one in which the performance of each alternative with respect to each attribute is expressed by a probability distribution. Often, it is unnecessary to make completely explicit all the decision-maker's partial preferences (*i.e.,* at one attribute level) in order to decide that: 'alternative a_j is at least as good as a_j^*' with respect to the attribute X_i. In fact, it can be clearly and simply concluded that this proposition is true, as a result of stochastic dominance conditions FSD, SSD and TSD (definitions are given in the Appendix) for a class of concave utility functions with decreasing absolute risk aversion. We refer to these as DARA utility functions. The class of such utility functions will be denoted by U_4.

Formally

$$U_4 = \{U_i(x_i)/U_i'(x_i) > 0, \quad U_i''(x_i) \leqslant 0,\\ U_i'''(x_i) \geqslant 0, \quad r'(x_i) = (-U''(x_i)/U'(x_i))' \leqslant 0,\\ \forall x_i \in R\}$$

where $r'(x_i)$ is a measure of absolute risk aversion (Pratt, 1964).

If a decision-maker's (partial) preference for each attribute X_i can be related by the utility function $U_i \in U_4$, then his preference for the $F_{ij}(x_i)$ distribution associated with alternative a_j for each X_i will be:

$$(1) \qquad g_i(x_i) = \int_{x_i^0}^{x_i^1} U_i(x_i)\,\mathrm{d}F_{ij}(x_i)$$

THEOREM 1 (Hadar and Russel, 1969; Whitmore, 1970). *If* F_{ij} FSD F_{ij}^* *or* F_{ij} SSD F_{ij}^* *or* F_{ij} TSD F_{ij}^* *and* $F_{ij} \geqslant F_{ij}^*$ *then* $g_i(F_{ij}) \geqslant g_i(F_{ij}^*)$ *for all* $U_i \in U_4$, *where* F_{ij} *and* F_{ij}^* *represent cumulative distribution functions associated with* a_j *and* a_j^*, *respectively.*

Theorem 1 allows us to conclude clearly that the proposition a_j is at least as good as a_j^* with respect to the attribute X_i, as a result of the validity of one of the stochastic dominance conditions. Moreover, Bawa (1975) has proposed some simple rules to verify the existence of stochastic dominance for certain families of probability distributions.

We state that this is often not necessary to make the decision-maker's preferences explicit since, according to Levy and Sarnat (1984), first-degree stochastic dominance (FSD) is observed in about 60% of comparisons between two probability distributions. If one adds the second-order (SSD) or third-order (TSD) stochastic dominance cases to the FSD cases (see Appendix A for the definitions), one obtains a sufficiently high percentage of situations in which one can conclude that 'a_j is at least as good as a_j^*', without the necessity for making the decision-maker's preferences completely explicit.

However, the DARA hypothesis must be accepted. According to Arrow (1971), who observed certain economic phenomena, utility functions usually exhibit decreasing and sometimes increasing absolute risk aversion. However, serious doubts about the increasing absolute risk aversion hypothesis have been raised by Stiglitz (1970). An aversion to risk in the overall behaviour of the decision maker with respect to the attribute X_i in a risky context can be sufficient to decide that alternative a_j is at least as good as a_j^*.

4. THE SYNTHETICAL OUTRANKING RELATION APPROACH

The use of stochastic dominance, rather than attempting to systematically make explicit the values of the criterion $g_i(F_{ij})$, is not only simpler but also more informative about the decision maker's behaviour under risk. This information may be important in a constructionist multiattribute approach where incomparability is allowed.

In our approach, two situations are identified; SD identifies stochastic dominance situations consistent with the conditions imposed by Theorems 1, and $\overline{\text{SD}}$ designates those which are not consistent with these dominance conditions. Since the dominance relation is asymmetric, when on one attribute $(a_j, a_j^*) \in$ SD, then, on this same attribute $(a_j^*, a_j) \notin$ SD. In the $\overline{\text{SD}}$ case (incomparability case), one of the FSD, SSD, TSD stochastic dominances cannot be fulfilled and it will be necessary to make explicit the decision maker's value system by deriving his $U_i(x_i)$ function. In fact, two complexity levels are distinguished in the expression for the decision maker's pairwise alternative preferences with respect to each attribute X_i:

1. clear – if one of the dominances is fulfilled, *i.e.,* the SD situation;
2. unclear – if it is the $\overline{\text{SD}}$ situation.

The following question arises: is it always necessary to clarify all the cases in which the decision maker's preferences are unclear in order to make use of a multiattribute aid decision for the statement of the choice or ranking problem? It will soon become clear that this depends on the level of concordance threshold required by the decision maker in the construction of outranking relation according to ELECTRE. The lower that this level is, the more useful it could be to delineate the unclear situations. It is tempting to make explicit a larger number of $U_i(x_i)$ functions, but this would result in a richer graph of outranking relations. Our objective is to reduce this number as far as possible without increasing the risk of erroneous conclusions that 'a_j is at least as good as a_j^*'.

Given the level of concordance threshold desired by the decision maker, the value of the concordance index can be decomposed into two parts:

1. *Explicable concordance.* This results from the cases in which the expression of the decision maker's preferences is trivial or clear:

$$C_E(a_j, a_j^*) = \sum_{i=1}^{n} W_i d_i^E(a_j, a_j^*)$$

(2) $\quad \text{where } d_i^E(a_j, a_j^*) = \begin{cases} 1 & \text{if } F_{ij} \text{ SD } F_{ij}^* \\ 0 & \text{otherwise.} \end{cases}$

and W_i = relative importance accorded the ith attribute, with $W_i \geqslant 0$ and $\sum_{i=1}^{n} W_i = 1$.

2. *Non-explicable concordance.* This corresponds to the potential value of the cases in which the expression of the decision maker's preferences is unclear.

$$C_N(a_j, a_j^*) = \sum_{i=1}^{n} W_i d_i^N(a_j, a_j^*)$$

(3) $\quad \text{where } d_i^N(a_j, a_j^*) = \begin{cases} 1 & \text{if } F_{ij} \overline{\text{SD}} F_{ij}^* \text{ and } F_{ij}^* \overline{\text{SD}}\ F_{ij} \\ 0 & \text{otherwise.} \end{cases}$

This second part of the concordance is only a potential value, as it is not certain that for each of these attributes F_{ij} will be at least as good as F_{ij}^*. We can formulate a condition for which attempts to make explicit the decision maker's value functions $U_i(x_i)$ corresponding to these attributes may be beneficial.

(4) $\quad$ If the condition $0 \leqslant p - C_E(a_j, a_j^*) \leqslant C_N(a_j, a_j^*)$,

where p = the concordance threshold, is fulfilled, then the explanation of the unclear cases can lead to a value of the concordance index such that the concordance test (see condition (6)) is satisfied for the proposition that a_j globally outranks $a_j^*(a_j \text{ S } a_j^*)$.

The discordance index for each attribute X_i *may be* defined as the ratio of the difference between the means of the range of the scale:

(5) $\quad D_i(a_j, a_j^*) = \begin{cases} \frac{(F_{ij}^* - (F_{ij})}{(x_i^1 - x_i^0)} & \text{if } F_{ij}^* \text{ FSD}_i\ F_{ij} \\ 0 & \text{if } F_{ij}^* \text{ not FSD}_i\ F_{ij}. \end{cases}$

The difference between the mean values of two distributions gives a good indication of the difference in performance of the two compared alternatives. If this difference is large enough (in relation to the range of the scale), and FSD is fulfilled on attribute X_i, then the chances are great that a_j is dominated by a_j^*. In that event, we assume a minimum level v_i, called a veto threshold, of the discordance index giving to attribute X_i the power of withdrawing all credibility if this attribute is not in concordance with the proposition that a_j is globally outranks a_j^*.

The discordance test follows from the notion of a veto threshold v_i for each attribute X_i. The sets of concordance and of discordance for the set of potential alternatives $\mathcal{A}$ are formulated in a classical manner:

$$(6) \quad \begin{array}{ll} \forall(a_j, a_j^*) \in \mathcal{A} \times \mathcal{A}, & [(a_j, a_j') \in C_p \leftrightarrow C(a_j, a_j^*) \geqslant p] \\ \forall(a_j, a_j^*) \in \mathcal{A} \times \mathcal{A}, & [(a_j, a_j^*) \in D_v \leftrightarrow \exists_i / D_i(a_j, a_j^*) \geqslant v_i]. \end{array}$$

The set of outrankings results from the intersection between the concordance set and the set of complementary to the discordance set:

$$(7) \quad S(p, v_i) = C_p \cap \overline{D}_v.$$

Next, depending on whether one is confronted with a choice or a ranking statement, either the core of the graph of outrankings is determined or the outranking relations are exploited as in ELECTRE II, for example.

5. APPLICATIONS

Our method will be illustrated by three quoted examples. In the trivial case of D'Avignon and Vincke (1989), the objective is to model the decision maker's preferences over four alternatives, a, b, c, d, which are expressed in the form of probability distributions for each of the three criteria examined (see Table I). It is assumed that each of the three criteria has the same importance.

To apply the approach, it is first necessary to establish the types of pairwise stochastic dominance relations for each pair of alternatives using each criterion. Table II shows that all the stochastic dominance conditions in existence are FSD, and as such the expression of preferences is clear in all cases. The concordance and discordance sets are the following:

$$Q_{2/3} = \{(a,b), (a,c), (a,d), (b,c), (b,d), (c,d)\}$$
$$D_{1/3} = \{(c,d), (c,a), (c,b), (d,b), (b,a)\}, \text{ with all the } v_i \text{ equally fixed at } 1/3.$$

TABLE I

Table of distributional estimations.

Alternatives	$i = 1$			$i = 2$			$i = 3$		
	[0,	1,	2]	[0,	1,	2]	[0,	1,	2]
a	0	0	1	0	0.5	0.5	0.3	0.4	0.4
b	0	0.5	0.5	0.3	0.3	0.4	0.5	0.5	0
c	1	0	0	0.3	0.7	0	0.5	0	0.5
d	0.3	0.7	0	0.5	0.5	0	0.5	0.5	0

TABLE II

Observed dominances for the D'Avignon and Vincke example.

	$i = 1$				$i = 2$				$i = 3$			
	a	b	c	d	a	b	c	d	a	b	c	d
a	*	FSD	FSD	FSD	*	FSD	FSD	FSD	*	FSD	FSD	FSD
b	–	*	FSD	FSD	–	*	FSD	FSD	–	*	–	FSD
c	–	–	*	–	–	–	*	FSD	–	FSD	*	FSD
d	–	–	FSD	*	–	–	–	*	–	FSD	–	*

Finally, the outranking set $S(p, v_i) = C_p \cap \overline{D}_v$ is:

$$S(2/3, 1/3) = \{(a,b), (a,c), (a,d), (b,c), (b,d)\}$$

from which the following order results:

$$\{a\} \rightarrow \{b\} \rightarrow \{c, d\}.$$

This is precisely the order found by D'Avignon and Vincke for a concordance threshold of 2/3 and a discordance threshold of 1/3.

The second example first appeared in Siskos (1983) and concerns the preorder of six candidates for the position of sales manager to be valued on the terms of the following attributes: personality, intellectual level, and experience (see Table III).

Table IV shows that, in this example, the relation between all alternative pairs is explained by either FSD or SSD stochastic dominance type on the three chosen criteria.

TABLE III

Distributional estimations of the six candidates.

Candidate	Personality			Intellectual Level			Experience		
	+	++	+++	+	++	+++	+	++	+++
A	0.3	0.4	0.3	0.1	0.6	0.2	0.3	0.6	0.1
B	0.1	0.1	0.8	0.3	0.5	0.2	0.7	0.2	0.1
C	0.5	0.2	0.3	0	0.2	0.8	0	0.7	0.3
D	0.1	0.3	0.6	0.4	0.4	0.2	0	0.1	0.9
E	0.4	0.4	0.2	0.3	0.5	0.2	0.4	0.4	0.2
F	0.2	0.5	0.3	0.4	0.5	0.1	0.5	0.4	0.1

If it is assumed that the decision maker's utility function for each attribute belongs to the DARA class, the resulting complexity level is clear. The weights of the criteria are the following: $W_1 = 0.3, W_2 = 0.4$, and $W_3 = 0.3$. The values of the explained concordance index appear in Table V.

For a concordance threshold of $p = 0.6$ and a veto threshold $v_i = 0.3$ for each i, the resulting preorder is identical to the one obtained by Siskos, who used the ELECTRE III method with a discrimination threshold of $s = 0.12$. The outranking set is:

$$S(0.6, 0.3) = \{(a,b)(a,e)(a,f)(b,e)(g,f)(c,a)(c,e)\\ (c,f)(d,f)(e,f)\}.$$

The following ranking results:

$$\{c,d\} \rightarrow \{a\} \rightarrow \{b\} \rightarrow \{e\} \rightarrow \{f\}.$$

The third example by Martel, D'Avignon and Couillard (1986), concerns the selection of development projects in university medical centres in Québec. This analysis was limited to fourteen Category A projects (see Table VI). This real-life example is a bit more complicated than the preceding ones. Four kinds of experts were used: clinical physicians (0), hospital administrators (1), University specialists (2) and others (3). A five-level scale was used for each criterion. The relative importances of the fourteen evaluation criteria are the following: $W_{A1} = W_{A2} = W_{C2} = 10/41$ and $W_{A3} = W_{A4} = W_{B1} = W_{B2} = W_{B4} = W_{C1} = W_{C3} = W_{C4} = W_{C5} = W_{C6} = 1/41$.

TABLE IV
Observed dominances for the Siskos example.

	Personality						Intellectual level						Experience					
	A	*B*	*C*	*D*	*E*	*F*	*A*	*B*	*C*	*D*	*E*	*F*	*A*	*B*	*C*	*D*	*E*	*F*
A	–		FSD		FSD		–	FSD		FSD	FSD	FSD	–	FSD			FSD	FSD
B	FSD	–	FSD	FSD	FSD	FSD		–		FSD	FSD	FSD		–				
C			–				FSD	FSD	–	FSD	FSD	FSD	FSD	FSD	–		FSD	FSD
D	FSD		FSD	–	FSD	FSD				–		FSD	FSD	FSD	FSD	–	FSD	FSD
E			SSD		–			FSD		FSD	–	FSD			FSD		–	FSD
F	FSD		FSD		FSD	–						–			FSD			–

TABLE V

Explained concordance $C_E(a_j, a'_j) = C_{\text{FSD+SSD}}$.

	a	b	c	d	e	f
a	×	0.7	0.3	0.4	1	0.7
b	0.3	×	0.3	0.7	0.7	0.7
c	0.7	0.7	×	0.4	0.7	0.7
d	0.6	0.3	0.6	×	0.6	1
e	0	0.7	0.3	0.4	×	0.7
f	0.3	0.3	0.3	0	0.3	×

The explicable concordance and the non-explicable concordance were calculated for the set of alternatives pairs (a_j, a_j^*). The results are shown in Tables VII and VIII. For example, for a concordance threshold of $p = 0.93$ and a discordance threshold of $v_i = 0.3$ for all criteria i, the following pairwise alternative outranking relations result by considering only the explicable concordance:

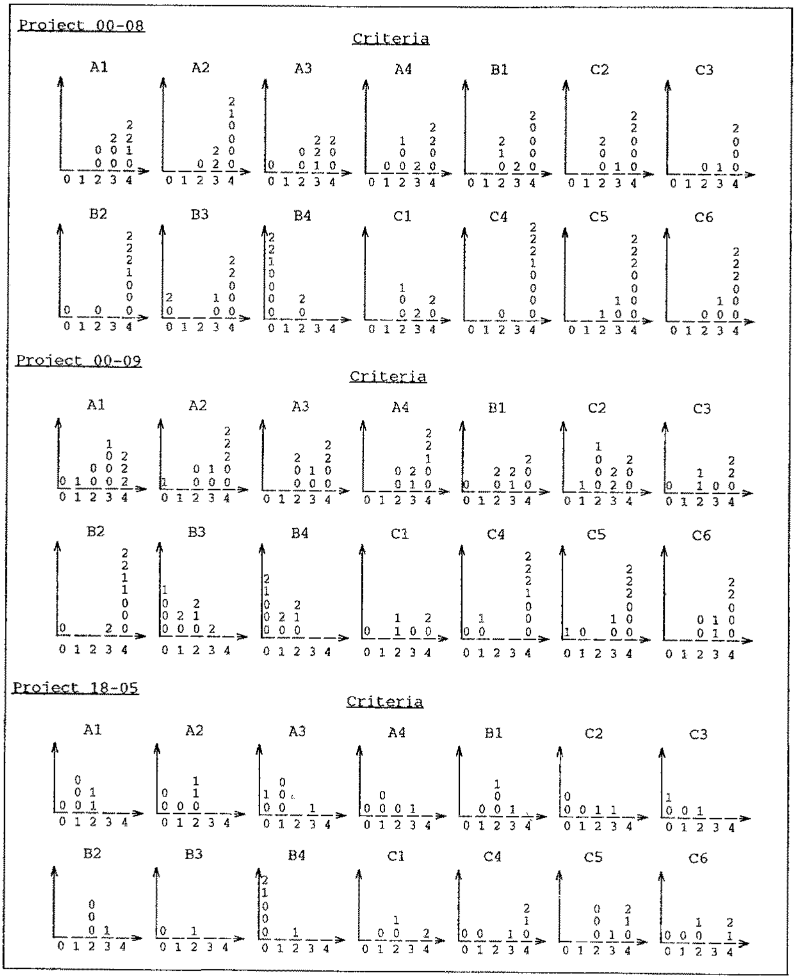
$$(00\text{–}08, 00\text{–}09); (00\text{–}08, 25\text{–}00); (00\text{–}10, 17\text{–}14);$$
$$(00\text{–}10, 19\text{–}01); (15\text{–}39, 25\text{–}00); (25\text{–}21, 17\text{–}13);$$
$$(25\text{–}21, 18\text{–}05); (25\text{–}21, 25\text{–}00); (26\text{–}02, 19\text{–}01);$$
$$(27\text{–}11, 17\text{–}14); (34\text{–}04, 17\text{–}14) \text{ and}$$
$$(34\text{–}04, 18\text{–}05).$$

If the non-explicable concordance is considered, various other outranking relations may also potentially result in cases where $p = 0.93$.

Since $C_E(a_j, a_j^*) = 0.88$ and $C_N(a_j, a_j^*) = 0.05$ for the pairs respecting the non-discordance test (00–09, 26–02) and (17–14, 17–13), by making explicit the decision maker's utility functions $U_i(x_i)$ for the criteria $B1, B2$, and $C1$, it is in fact possible to discern whether or not there exists an outranking relation between these two alternative pairs. The same possibility exists for the following pairs, for which the non-discordance test is also fulfilled:

$$(34\text{–}04, 26\text{–}02) \text{ since } C_E\ (34\text{–}04, 26\text{–}02) = 0.90$$
$$\text{and } C_N\ (34\text{–}04, 26\text{–}02) = 0.05;$$

TABLE VI
Distributional evaluations for three (3) of the fourteen category A projects.



(25–21, 19–01) since C_E (25–21, 19–01) = 0.90
and C_N (25–21, 19–01) = 0.07;
(00–11, 26–02) since C_E (00–11, 26–02) = 0.88
and C_N (00–11, 26–02) = 0.07;
(25–21, 17–14) since C_E (25–21, 17–14) = 0.71
and C_N (25–21, 17–14) = 0.24;
(25–21, 26–02) since C_E (25–21, 26–02) = 0.71

TABLE VII
Explicable concordance $C_E(a_j, a_j^*)$.

	00–08	00–09	00–10	00–11	15–39	17–13	17–14	18–05	19–01	25–00	25–21	26–02	27–11	34–04
00–08	×	⩾0.93	0.37	0.85	0.85	⩾0.93	0.85	⩾0.93	⩾0.93	⩾0.93	0.38	⩾0.93	0.59	0.12
00–09	0.12	×	0.1	0.12	0.59	0.68	0.44	⩾0.93	0.88	0.71	0.02	0.88	0.10	0.05
00–10	0.88	0.85	×	0.63	⩾0.93	⩾0.93	⩾0.93	⩾0.93	⩾0.93	⩾0.93	0.59	⩾0.93	0.56	0.34
00–11	0.07	0.63	0.12	×	0.57	⩾0.93	0.56	⩾0.93	⩾0.93	0.71	0.15	0.88	0.34	0.12
15–39	0.07	0.39	0.02	0.36	×	0.54	0.54	⩾0.93	0.81	⩾0.93	⩾0.93	0.39	0.02	0.10
17–13	0.02	0.05	0.00	0.05	0.02	×	0.08	0.59	0.05	0.08	0.00	0.05	0.00	0.02
17–14	0 00	0.00	0.00	0.00	0.02	0.88	×	0.08	0.59	0.51	0.02	0.34	0.02	0.02
18–05	0.02	0.05	0.00	0.05	0.05	0.34	0.00	×	0.02	0.08	0.02	0.08	0.00	0.00
19–01	0.00	0.08	0.05	0.05	0.08	0.63	0.08	0.88	×	0.15	0.02	0.19	0.00	0.00
25–00	0.00	0.02	0.00	0.02	0.00	0.05	0.24	0.37	0.27	×	0.02	0.08	0.00	0.00
25–21	0.59	0.86	0.10	0.8	0.9	⩾0.93	0.71	⩾0.93	0.90	⩾0.93	×	0.71	0.39	0.61
26–02	0.02	0.05	0.02	0.02	0.02	0.86	0.34	0.87	⩾0.93	0.02	0.24	×	0.07	0.02
27–11	0.34	⩾0.93	0.10	0.39	0.68	0.73	⩾0.93	⩾0.93	⩾0.93	0.86	0.54	⩾0.93	×	0.39
34–04	0.1	0.86	0.10	0.48	0.8	⩾0.93	⩾0.93	⩾0.93	0.86	⩾0.93	0.37	0.90	0.34	×

TABLE VIII
Non-explicable concordance $C_N(a_j, a_j^*)$.

	00–08	00–09	00–10	00–11	15–39	17–13	17–14	18–05	19–01	25–00	25–21	26–02	27–11	34–04
00–08	×	–	–	0.02	0.02	0.02	0.05	0.02	0.05	–	0.07	0.05	0.05	0.78
00–09	–	×	0.02	0.26	0.05	0.24	0.49	0.07	0.05	–	–	0.05	0.05	0.07
00–10	0.02	0.02	×	0.26	0.05	0.02	–	0.05	–	0.02	0.26	0.02	0.29	0.29
00–11	0.02	–	–	×	–	0.02	0.26	0.07	0.07	–	–	0.07	0.07	0.29
15–39	–	–	–	0.05	×	0.02	0.24	0.05	0.05	–	0.02	0.32	0.05	0.02
17–13	–	–	–	–	0.02	×	–	0.02	0.02	0.02	0.05	0.05	0.05	–
17–14	0.05	0.02	0.02	0.02	0.07	0.05	×	0.07	0.05	0.24	0.02	0.05	0.05	0.05
18–05	–	–	–	–	–	–	0.02	×	–	–	–	–	0.02	0.02
19–01	–	–	–	–	0.02	0.29	0.49	0.05	×	–	–	–	–	–
25–00	–	0.29	0.02	0.26	0.05	0.59	0.51	0.31	0.56	×	0.02	0.29	0.02	0.02
25–21	–	–	0.02	–	–	–	0.24	–	0.07	–	×	0.26	0.02	–
26–02	–	–	–	–	–	0.02	0.24	–	–	–	0.02	×	–	–
27–11	–	–	–	0.24	0.02	–	–	0.02	0.02	–	0.26	0.05	×	0.24
34–04	–	–	0.02	0.05	–	0.05	–	–	0.05	–	0.05	0.05	–	×

$$\text{and } C_N \text{ (25–21, 26–02)} = 0.26.$$

If one assumes that an outranking relation is established for these five alternative pairs, the average classification of the fourteen alternatives in this category would be the following:

$$\begin{aligned}\{00\text{–}08\} &\rightarrow \{00\text{–}10, 34\text{–}04, 27\text{–}11, 00\text{–}11\}\\ &\rightarrow \{25\text{–}21, 15\text{–}39, 00\text{–}09\} \rightarrow \{17\text{–}14\}\\ &\rightarrow \{26\text{–}02, 18\text{–}05, 25\text{–}00\} \rightarrow \{17\text{–}13\}\\ &\rightarrow \{19\text{–}01\}.\end{aligned}$$

Seven out of eight projects that were accepted in this category (in the real case from which the example is taken) are at the top of the list; project 27–11 was not chosen but project 26–02 was.

The existence of an outranking relation between the alternatives 00–09 and 26–02 is obviously crucial according to detailed analysis of the outrankings graphs. In effect, if this outranking relation is established, in addition to the outranking relationships existing only on the basis of explained concordances, one obtains as the 'average' preorder

$$\begin{aligned}\{00\text{–}09\} &\rightarrow \{00\text{–}10, 34\text{–}04, 27\text{–}11, 25\text{–}21, 15\text{–}39\}\\ &\rightarrow \{00\text{–}11\} \rightarrow \{00\text{–}09\} \rightarrow \{26\text{–}02\}\\ &\rightarrow \{17\text{–}14, 18\text{–}05, 25\text{–}00, 17\text{–}13\} \rightarrow \{19\text{–}01\}.\end{aligned}$$

The first eight alternatives in this preorder are the same as those in the preceding 'average' preorder, where it was assumed that the five outranking relations were established. For this reason, little benefit would result from any efforts to make explicit the decision maker's utility functions with respect to all attributes, except those made in order to conclude on the relation between the alternatives 00–09 and 26–02.

6. CONCLUSION

In this article we have presented an approach based on the assumption that the compared alternative performances are expressed by a known probability distribution set. It has been proved that in this context it is possible to model the decision maker's preferences by constructing some outranking relations using the stochastic dominance conditions.

Two complexity levels were distinguished within the expression of these preferences, which were denoted clear and unclear. For the first level, the decision maker's preferences can be deduced by using knowledge of his overall behaviour. Only for the second (unclear) level is it actually necessary to make explicit the utility functions $U_i(x)$ of the decision maker. In the first two examples analysed, it was unnecessary to interrogate the decision maker to discern his/her utility function since all the comparisons encountered were at the clear level. In the third example, which included 14 criteria, it would be in fact necessary to question the decision maker(s) to be able to construct his (their) utility functions $U_i(x_i)$ for certain criteria, but this would depend upon the level of concordance threshold deemed necessary for the construction of the outranking relations. In any case, it is generally possible, when using a multicriterion decision aid approach in a risky context, to reduce (sometimes significantly) the number of questions that must be asked of the decision maker without increasing the risk of erroneous advice.

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APPENDIX

We will consider the stochastic dominance set as the following: FSD (First-degree Stochastic Dominance), SSD (Second-degree Stochastic Dominance), TSD (Third-degree Stochastic Dominance). These stochastic dominances are defined for continuous random variables and for discrete random variables in the following way:

DEFINITION 1

$$F_{ij} \text{ FSD } F_{ij}^* \text{ if and only if } F_{ij} \neq F_{ij}^*$$
$$H_1(x_i) = F_i(x_{ij}) - F_i(x_{ij}^*) \leqslant 0 \text{ for all } x_i \in [x_i^0, x_i^1]$$

DEFINITION 2

$$F_{ij} \text{ SSD } F_{ij}^* \text{ if and only if } F_{ij} \neq F_{ij}^* \text{ and}$$
$$H_2(x_i) = \int_{x_i^0}^{x_i} H_i(y)\,\mathrm{d}y \leqslant \text{ for all } x_i \in [x_i^0, x_i^1]$$

DEFINITION 3

$$F_{ij} \text{ TSD } F_{ij}^* \text{ if and only if } F_{ij} \neq F_{ij}^* \text{ and}$$
$$H_3(x_i) = \int_{x_i^*}^{x_i} H_2(y)\,\mathrm{d}y \leqslant \text{ for all } x_i \in [x_i^0, x_i^1].$$

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