

The Data General MRDOS operating system is more flexible and allows (as a user choice) both actual and virtual loading [3].

It is seen from the above analysis that pseudoloading is needed only when condition (1) is not satisfied and when the time parameters of peripheral devices exceed the DMA channel capabilities.

If condition (1) is obeyed, an overlay module can be loaded into memory at any time in the course of operation of data logging and storing processes, provided the relations (6) and (9) are observed.

Application of the above relations made it possible to use overlay substitutions as an efficient method of memory control in a fast data processing system based on an ETALON minicomputer.

#### LITERATURE CITED

1. S. Mednick and J. Donovan, Operating Systems [Russian translation], Mir, Moscow (1978), pp. 239-241.
2. G. I. Kornienko and O. N. Sheverda, "Control of resources in processing the results of natural experiments," Kibernetika, No. 5, 41 (1978).
3. L. L. Mytus, "Real-time operating systems used in automatic control systems of technological processes," Programmirovaniye, No. 6, 44 (1977).

#### SYSTEMWISE OPTIMIZATION

V. M. Glushkov

UDC 001.57:62-52

Present-day optimization theory and practice are based on the classical statement of the optimization problem, whereby we aim to discover, in a preassigned, fixed, admissible domain  $P$ , the point (or set of points)  $p$  at which a given scalar target function  $f(p)$  takes its extremal value.

There are many economic planning and structural design problems for which such a statement is unsatisfactory in at least two respects. First, the target function  $f(x)$  is a vector and not a scalar function, and moreover, is not reducible in practice to a scalar form by any a priori procedure (such as weighting the different components of the initial vector function). Second, the admissible domain  $P$  may change during the optimization process; and indeed, the essence of the process may often lie precisely in purposeful variation of the admissible domain.

Since the feasible types of variation of the domain  $P$  are usually specified by a system model, it seems natural to speak here of a systemwise approach to optimization problems. With this new approach, the constraints specifying the admissible domain in the parameter space are usually varied as a result of a sequence of solutions, chosen from a discrete set of feasible solutions. This set of feasible solutions is not usually fully defined at the start of the optimization process, and its definition is completed in the course of a dialog with people (planners or designers) who are in possession of only partially formalized devices for generating new solutions.

Let us describe a typical formalized statement of the systemwise optimization problem. Since the underlying idea is more easily grasped when open to graphical illustration, we shall consider the two-criterion case. Also, we shall assume that the relevant solution is uniquely defined by choosing the values of these criteria. In other words, the solution is sought directly in the space  $K$  of optimization criteria (call them  $x_1$  and  $x_2$ , see Fig. 1).

The process of solution starts with choosing in the space  $K$  a point  $A_0$  with coordinates  $a_0$  and  $b_0$ , representing a desirable solution of the problem. We next construct the initial constraints  $F_1^{(0)}(x_1, x_2) \geq 0, \dots, F_n^{(0)}(x_1, x_2) \geq 0$ , specifying the initial admissible domain  $P_0$ . We can check directly whether or not the point  $A_0$  belongs to domain  $P_0$ . If it does, we can in principle use the ordinary (classical) optimization procedure, either with respect to one of the criteria  $x_1, x_2$ , or with respect to some combination of them. But with the systemwise approach, we usually employ an entirely different method: in accordance with the model  $M$  of highest level, controlling the choice of criteria, the point  $A_0$  is withdrawn from the admissible domain  $P_0$ , as indicated in Fig. 1.

---

Translated from Kibernetika, No. 5, pp. 89-90, September-October, 1980. Original article submitted July 12, 1980.

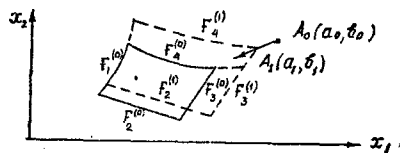


Fig. 1

After this, we isolate the constraints which are not satisfied at point  $A_0$  (in our present case, these are  $F_3^{(0)}$  and  $F_4^{(0)}$ ). Turning to models  $M_3$  and  $M_4$ , generating these constraints, we try out in the dialog mode the solutions which vary the corresponding constraints in the necessary direction (provided that such variation is possible). By a necessary direction we mean here one which reduces the absolute value of the negative discrepancies  $F_i^{(0)}(a_0, b_0)$  [in our present case,  $F_3^{(0)}(a_0, b_0)$  and  $F_4^{(0)}(a_0, b_0)$ ].

It has to be borne in mind that the constraints  $F_i$  are often interconnected, so that variation of one involves variation of certain others. The control of the choice of solutions for variation of the constraints is determined here by the minimization of some penalty function  $g_0(a_0, b_0)$ . As this function we usually choose the maximum absolute value of the negative discrepancies  $\lambda_i \cdot F_i^{(0)}(a_0, b_0)$  (where  $\lambda_i$  are positive weights). If there are no such discrepancies, we put by definition  $g_0(a_0, b_0) = 0$ .

As a result of the control there appears a series of solutions  $R_1, \dots, R_m$ , leading to reduction of the value of the penalty function; after the  $m$ -th solution, we denote this latter by  $g_m(a_0, b_0)$ . On varying the constraints, each of the accepted solutions leads to a corresponding variation of the admissible domain. In Fig. 1 we show two such variations. The first varies the constraints  $F_3^{(0)}$  and  $F_2^{(0)}$ , by replacing them, respectively, by constraints  $F_3^{(1)}$  and  $F_2^{(1)}$ . The second variation affects only the one constraint  $F_4^{(0)}$ , replacing it by constraint  $F_4^{(1)}$ . The domain  $P_2$  obtained after these variations is bounded by the lines  $F_1^{(0)}$ ,  $F_2^{(1)}$ ,  $F_3^{(1)}$ , and  $F_4^{(1)}$ , while the corresponding value of the penalty function is equal to  $g_2(a_0, b_0)$ . Notice that it is not possible to make a rapid choice of final admissible domain because the sequence of domains  $P_0, P_1, \dots$  cannot be ordered with respect to inclusion. Moreover, rapid performance of the work is prevented by the vast labor of generating new constraints, inasmuch as a great deal of superfluous work is needed in the way of varying inessential constraints.

If, as is the case in Fig. 1,  $g_2(a_0, b_0) \neq 0$ , and there are no solutions leading to further reduction of the penalty function, a return is made to the highest model  $M$ , controlling the choice of desirable solution  $A(a, b)$  of the problem. By means of a series of successive decisions  $D_1, D_2, \dots, D_k$  on variation of the initial solution  $A_0(a_0, b_0)$ , the latter is replaced successively by  $A_1(a_1, b_1), \dots, A_k(a_k, b_k)$ , till the last point  $A_k(a_k, b_k)$  is in the admissible domain ( $k = 1$  in Fig. 1). The decisions on variation are chosen from the admissible set of decisions with the aim of minimizing the penalty function. This process closely resembles the classical optimization process, except for the fact that the steps are not arbitrarily chosen but are fixed in conformity with the solutions admitted by model  $M$ .

Finally, after point  $A_k$  has hit the last admissible domain  $P_m$ , we can use a supplementary optimization procedure with respect to some combination of criteria  $x_1$  and  $x_2$  within the admissible domain. The only difference from the classical procedure is that the choice of optimization steps is not arbitrary, but is controlled by the model  $M$  of highest level. If further improvement of the chosen criterion is prevented by certain constraints which are nonetheless amenable to further variations in the required direction, then the optimization process can be continued by including in it successive decisions on such variations.

It is not as unusual as might at first sight appear for the solution of the problem to be uniquely determined by a choice of the values of all the optimization criteria. This happens e.g., in economic-planning problems, where the (vector) criterion is the pure output of the different types of production, while the solution of the problem is the total output (see [1]). In cases where determination of the solution is not unique, the space in which the solution is sought may have other coordinates apart from those corresponding to the optimization criteria. The optimization process described above is then more complicated, inasmuch as the points  $A_i(a_i, b_i)$  are replaced by hyperplanes. The definition of the penalty function is then likewise more complicated: we can then define it, e.g., as the distance from a chosen hyperplane to the next admissible domain, in a space with given compressions (expressions) along the axes corresponding to the optimization criteria.

In the most general case, instead of hyperplanes there may appear point sets of arbitrary type. Statements are possible in which the values of the criteria are not uniquely defined on these sets, while to distinguish more or less desirable solutions in these sets, appropriate weight functions are specified (by means of the model  $M$  of highest level). But there is another important feature of systemwise optimization, apart from

the presence of several criteria and the possibility of varying the admissible domain, which is retained whatever the type of approach, namely, the interaction of the models of the different levels. In the case of economic-planning problems, the solutions are obtained in these models by controllers of different levels, while in the case of structural design problems they are obtained by designers operating on different parts of the overall design.

The author has developed a concrete optimization scheme based on the above principles; it embraces the so-called "Displan" systems (see [1]).

#### LITERATURE CITED

1. V. M. Glushkov, *Macroeconomic Models and (OGAS) Design Principles* [in Russian], Statistika, Moscow (1975).

#### MINIMIZING METHOD FOR FUNCTIONS THAT SATISFY THE LIPSCHITZ CONDITION

A. M. Gupal

UDC 519.47

In this article a numerical method is developed to solve the general problem of mathematical programming:

$$\min f_0(x) \quad (1)$$

with the constraints

$$f_i(x) \leq 0, \quad i = 1, \dots, m, \quad (2)$$

where the functions  $f_\nu(x)$ ,  $\nu = 0, 1, \dots, m$  satisfy a local Lipschitz condition. The general plan of the method agrees with the plan of the linearization method given in [1].

We note that the problem of Eqs. (1) and (2) is equivalent to the problem

$$\min f(x) \quad (3)$$

with the condition

$$h(x) \leq 0, \quad (4)$$

where  $h(x) = \max_{1 \leq i \leq m} f_i(x)$ . Therefore in what follows we will consider the problem of Eqs. (3) and (4).

The method of solving Eqs. (3) and (4) is defined by the relationships

$$x^{k+1} = x^k + \rho_k s^k, \quad (5)$$

where  $s^k$  is the solution of the quadratic programming problem

$$\min (z_f^k, s) + \frac{1}{2} \|s\|^2 \quad (6)$$

with the additional constraint  $(z_h^k, s) + h(x^k) \leq 0$  if the condition  $h(x^k) \geq 0$  is fulfilled.

Here  $x^0$ ,  $z_f^0$ , and  $z_h^0$  are arbitrary initial approximations:

$$z_f^{k+1} = z_f^k + a_k (\Theta_f(x^k, k) - z_f^k),$$

$$z_h^{k+1} = z_h^k + a_k (\Theta_h(x^k, k) - z_h^k),$$

$$\Theta_f(x^k, k) = \frac{1}{2\alpha_k} \sum_{i=1}^n [f(\bar{x}_1^k, \dots, x_i^k + \alpha_k, \dots, \bar{x}_n^k) - f(\bar{x}_1^k, \dots, x_i^k - \alpha_k, \dots, \bar{x}_n^k)] e_i,$$

Translated from *Kibernetika*, No. 5, pp. 91-94, September-October, 1980. Original article submitted February 22, 1979.