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Lognormal Kriging—The General Case[1]

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A theoretical study of the general case of the estimation of regionalized variables with a lognormal distribution is presented. The results of this study are compared to those obtained assuming conservation of lognormality. The numerical significance of the different solutions is illustrated by several simple examples.

KEY WORDS: kriging, lognormal estimation, conservation of lognormality.

INTRODUCTION

Rendu (1979) presented Marechal's (1974) comparison of the mathematical theory for the estimation of regionalized variables with a known normal or lognormal frequency distribution. The major assumption in the presentation for the lognormal case was that of the *conservation of lognormality* which states that if sample values are lognormally distributed then the average value of a number of samples, such as a block average value, is also a lognormally distributed variable.

Although the assumption of conservation of lognormality is statistically invalid (the probability distribution of a linear combination of lognormal variates is not lognormal) it is frequently observed in practice and has been verified for small blocks (Krige, 1951).

The objects of this paper are

(i) to show that even when conservation of lognormality can be assumed, Rendu's presentation of the solution to the problem is only an approximation, and
(ii) to present the mathematical theory for the general case of lognormal kriging (i.e., without the assumption of conservation of lognormality) and to compare results obtained with and without the assumption of conservation of lognormality.

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NOTATIONS

For the sake of continuity Rendu's notations, as reproduced below, are used throughout this paper.

b_i	weight given to the ith sample value x_i
$D^2\{(\cdot)\}$	variance of $(\cdot)$
$E\{(\cdot)\}$	expected value of $(\cdot)$
C	a constant
n	number of samples
W	a block of support W; the size (area or volume) of this block
w_i	a sample of support w_i, the size (area or volume) of this sample
x	a regionalized variable
x_i	value of the sample w_i; average value of x in w_i
β	additive constant
λ	Lagrange multiplier
μ	average value of the orebody: $\mu = E[x]$
μ_k	Kriging estimator of μ_w
μ_W	value of the block W; average value of x in W
$\hat{\mu}_W$	an estimator of μ_W
$\bar{\sigma}(W;W)$	variance of μ_W
$\bar{\sigma}(w_i;W)$	covariance of x_i and μ_W
$\bar{\sigma}(w_i;w_j)$	covariance of x_i and x_j
$\bar{\sigma}_e(W;W)$	variance of $\ln \mu_W$
$\bar{\sigma}_e(w_i;W)$	covariance of $\ln x_i$ and $\ln \mu_W$
$\bar{\sigma}_e(w_i;w_j)$	covariance of $\ln x_i$ and $\ln x_j$
σ_k^2	kriging variance of estimation of μ_W: $\sigma_k^2 = E[(\mu_k - \mu_W)^2]$
σ_{ke}^2	logarithmic kriging variance: $\sigma_{ke}^2 = E[(\ln \mu_k - \ln \mu_W)^2]$ an orebody; the domain within which the regionalized variable is defined

Lognormal Formulas

If the variable x_i, defined on the support w_i, is lognormally distributed with mean μ and variance $\bar{\sigma}(w_i;w_i)$

$$E[x_i] = \mu$$

$$D^2[x_i] = \bar{\sigma}(w_i;w_i)$$

then $\ln x_i$ is normally distributed with mean μ_e and variance $\bar{\sigma}_e(w_i;w_i)$

$$E[\ln x_i] = \mu_e$$

$$D^2[\ln x_i] = \bar{\sigma}_e(w_i;w_i)$$

and the following relationships exist

$$\mu = e^{\mu_e + \frac{1}{2}\bar{\sigma}_e(w_i; w_i)} \tag{1}$$

$$\bar{\sigma}(w_i; w_i) = \mu^2 (e^{\bar{\sigma}_e(w_i; w_i)} - 1 \tag{2}$$

If x_i and x_j, defined on identical supports, w_i and w_j, respectively, are joint lognormally distributed with mean μ and covariance $\bar{\sigma}(w_i; w_j)$ then $\ln x_i$ and $\ln x_j$ are joint normally distributed with covariance $\bar{\sigma}_e(w_i; w_j)$ where

$$\bar{\sigma}(w_i; w_j) = \mu^2 (e^{\bar{\sigma}_e(w_i; w_j)} - 1) \tag{3}$$

COVARIANCE CALCULATIONS

It has been shown (Matheron, 1971) that if the covariogram $\sigma(h)$ of point values is known, the covariance $\bar{\sigma}(w; w')$ between any two blocks or samples w and w', or the variance $\bar{\sigma}(w; w)$ of any block or sample w is equal to the average value of the covariogram in the blocks or samples. The point covariogram $\sigma(h)$ is the function that relates the covariance between sample values with infinitely small support to the vectorial distance h between the supports.

The point covariogram can be obtained from the covariogram of sample values with finite supports. If z and z' are two points in Ω and zz' the distance between these points, $\sigma(zz')$ is the covariance between two point values at distance zz' and $\bar{\sigma}(w; w')$ is calculated as follows

$$\bar{\sigma}(w; w') = \frac{1}{ww'} \int_{z \text{ in } w} \int_{z' \text{ in } w'} \sigma(zz')\, dz'\, dz \tag{4}$$

The same formula applies to the calculation of $\bar{\sigma}(w; w)$. This formula is true, whatever the frequency distribution of the values in Ω.

In practice, when values are lognormally distributed, the logarithms of sample values are used to calculate a logarithmic covariogram, $\sigma_e(h)$, which is related to the covariogram, $\sigma(h)$ of point values by eq. (3)

$$\sigma(h) = \mu^2 (e^{\sigma_e(h)} - 1)$$

If $\sigma_e(zz')$ is the covariance between the logarithms of two point values at distance zz' then

$$\sigma(zz') = \mu^2 (e^{\sigma_e(zz')} - 1)$$

substituting in (4) gives

$$\bar{\sigma}(w; w') = \mu^2 \cdot \frac{1}{ww'} \int_{z \text{ in } w} \int_{z' \text{ in } w'} (e^{\sigma_e(zz')} - 1)\, dz \tag{5}$$

If conservation of lognormality is assumed, then

$$\bar{\sigma}(w; w') = \mu^2 (e^{\bar{\sigma}_e(w;w')} - 1) \tag{6}$$

and thus

$$\sigma_e(w; w') = \ln \left(\frac{1}{ww'} \int_{z \text{ in } w} \int_{z' \text{ in } w'} e^{\sigma_e(zz')} \, dz' \, dz \right) \tag{7}$$

LOGNORMALITY AND SUPPORT

In practice, lognormality is observed for values measured on a given (sample size) support and, unless the assumption of conservation of lognormality is invoked, lognormality cannot be assumed for any different (whether smaller or larger) supports. It cannot, for example, be assumed for sample size supports and, at the same time, be assumed for point supports for the purpose of calculating covariances (in eq. 4 lognormality cannot be assumed for the point supports z, z' and for the nonpoint supports w, w').

In fact, values defined on point supports cannot be lognormally distributed under any circumstances (Matheron, 1962). A grade defined on a point support can take only one of two values: zero if it is inside a grain of waste (nonmineralized grain) or the grade, p of pure ore if it is inside a mineralized grain. If the mean grade over Ω is μ then the regionalized variable takes the value p with probability μ/p and the value zero with probability $1 - (\mu/p)$; its variance is $\mu(p - \mu)$. Such a variable is obviously neither lognormally nor normally distributed. There is no reason, however, why variables defined on supports that are large with respect to the granulometry cannot be lognormally distributed.

For the purpose of this paper it is assumed that the sample supports, w_i, are quasi-point supports; that is, they are

(i) small with respect to the support, W, which is to be estimated
(ii) close enough to point supports for formulas such as eq. (4) to be used, at least as numerical approximations
(iii) large enough with respect to the granulometry for lognormality to be assumed

When these conditions apply the sample-sample covariance, $\bar{\sigma}(w; w')$, defined in eq. (4) becomes

$$\bar{\sigma}(w; w') = \sigma(zz')$$

where zz' is the distance separating the two quasi-point supports w, w'.

Conditions (i)–(iii) are very often fulfilled in practice. At the very least the estimation problem can usually be interpreted in such a way that the conditions are satisfied as, for example, when a three-dimensional problem of estimating a block by drill core intersections is interpreted as estimating a two-dimensional panel by point samples.

The assumption of conservation of lognormality means that not only are the distributions of the individual sample grades and the block grade assumed to be lognormal (i.e., their marginal laws are lognormal) but that the joint distribution of sample grades and block grade is also assumed to be lognormal.

LOGNORMAL ESTIMATION

Consider an orebody, Ω, of average value μ in which quasi-point support sample values are lognormally distributed. The value μ_W of a block of ore, W, is estimated by a log-linear estimator, $\hat{\mu}_W$, which satisfies the following equation

$$\ln \hat{\mu}_W = C + \sum_i b_i \ln x_i \tag{8}$$

where C and the b_is are constants to be calculated. If the mean μ is known, μ_W is chosen such that

$$\ln (\hat{\mu}_W/\mu) = C' + \sum_i b_i \ln (x_i/\mu) \qquad \text{where} \tag{9}$$

$$C' = C - \left(1 - \sum_i b_i\right) \ln \mu \tag{10}$$

The log-linear estimator, $\hat{\mu}_W$, is given by

$$\hat{\mu}_W = \exp\left(C + \sum_i b_i \ln x_i\right) \tag{11}$$

If the $\ln x_i$ are joint normally distributed then the exponent $C + \Sigma_i b_i \ln x_i$ is a normally distributed variable and from the properties of the moment generating function of the normal distribution the expected value of $\hat{\mu}_W$ is

$$E[\hat{\mu}_W] = \exp\left(C + \sum_i b_i \{\ln \mu - \tfrac{1}{2}\, \bar{\sigma}_e(w_i; w_i)\} + \tfrac{1}{2} \sum_i \sum_j b_i b_j \bar{\sigma}_e(w_i; w_j)\right) \tag{12}$$

The error of estimation of μ_W is $\hat{\mu}_W - \mu_W$ and its expectation is

$$\begin{aligned} E[\hat{\mu}_W - \mu_W] &= \exp\left(C + \sum_i b_i \{\ln \mu - \tfrac{1}{2}\, \bar{\sigma}_e(w_i; w_i)\}\right. \\ &\qquad \left. + \tfrac{1}{2} \sum_i \sum_j b_i b_j \bar{\sigma}_e(w_i; w_j)\right) - \exp \ln \mu \qquad (13) \\ &= \exp \ln \mu \left[\exp\left(C + \ln \mu \left(\sum_i b_i - 1\right)\right.\right. \\ &\qquad \left.\left. - \tfrac{1}{2} \sum_i \sum_j b_i b_j \bar{\sigma}_e(w_i; w_j) - \tfrac{1}{2} \sum b_i \bar{\sigma}_e(w_i; w_j)\right) - 1\right] \qquad (14) \end{aligned}$$

The estimator, $\hat{\mu}_W$, is an unbiased estimator of μ_W if and only if

$$C = \left(1 - \sum_i b_i\right) \ln \mu - \tfrac{1}{2} \sum_i \sum_j b_i b_j \bar{\sigma}_e(w_i, w_j) + \tfrac{1}{2} \sum_i b_i \bar{\sigma}_e(w_i; w_j) \quad (15)$$

For this value of C

$$E[\hat{\mu}_W] = \mu \quad (16)$$

Lognormal Kriging Assuming Conservation of Lognormality

If conservation of lognormality is assumed, then μ_W is a lognormal variable with mean μ and variance $\bar{\sigma}(W; W)$; $\ln \mu_W$ is a normal variable with variance $\bar{\sigma}_e(W; W)$, and (from eq. 2), the following relationship exists

$$D^2[\mu_W] = \bar{\sigma}(W; W) = \mu^2 (e^{\bar{\sigma}_e(W; W)} - 1) \quad (17)$$

where (from eq. 7)

$$\bar{\sigma}_e(W; W) = D^2[\ln \mu_W] = \ln \left(\frac{1}{WW} \int_{z \text{ in } W} \int_{z' \text{ in } W} e^{\sigma_e(zz')} \, dz' \, dz \right) \quad (18)$$

The variance of the estimator $\ln \hat{\mu}_W$ is

$$D^2[\ln \hat{\mu}_W] = \sum_i \sum_j b_i b_j \sigma_e(w_i; w_j) \quad (19)$$

and, as $\hat{\mu}_W$ is a lognormal variable

$$D^2[\hat{\mu}_W] = \mu^2 \left(\exp\left(\sum_i \sum_j b_i b_j \sigma_e(w_i; w_j) \right) - 1 \right) \quad (20)$$

If $\ln \hat{\mu}_W$ and $\ln \mu_W$ are joint normally distributed (this is an additional assumption) then the covariance between $\hat{\mu}_W$ and μ_W is

$$\begin{aligned} \text{cov}\,[\hat{\mu}_W; \mu_W] &= \text{cov}\left[\exp\left(C + \sum_i b_i \ln x_i\right) \cdot \mu_W\right] \\ &= \frac{1}{W} \int_{z \text{ in } W} \text{cov}\left[\exp\left(C + \sum_i b_i \ln x_i\right) \cdot z\right] dz \\ &= \mu^2 \frac{1}{W} \int_{z \text{ in } W} \left(\exp \text{cov}\left[\left(C + \sum_i b_i \ln x_i\right) \ln z\right] - 1\right) dz \end{aligned}$$

and thus

$$\text{cov}\,[\hat{\mu}_W; \mu_W] = \mu^2 \frac{1}{W} \int_{z \text{ in } W} \left(\exp\left(\sum_i b_i \sigma_e(w_i; z) \right) - 1 \right) dz \quad (21)$$

and, from (6) and (7)

$$\operatorname{cov}[\ln \hat{\mu}_W; \ln \mu_W] = \ln\left(\frac{1}{W}\int_{z \text{ in } W} \exp \sum_i b_i \sigma_e(w_i; z)\, dz\right) \tag{22}$$

Note that Rendu (1979) calculates this covariance as

$$\operatorname{cov}[\ln \hat{\mu}_W; \ln \mu_W] = \sum_i b_i \bar{\sigma}_e(w_i; W), \qquad \text{where} \tag{23}$$

$$\bar{\sigma}_e(w_i; W) = \ln\left(\frac{1}{W}\int_{z \text{ in } W} e^{\sigma_e(w_i; z)}\, dz\right) \tag{24}$$

This method does not require the assumption of joint normality between $\ln \hat{\mu}_W$ and $\ln \mu_W$; it requires only the assumption of bivariate normality between $\ln x_i$ and z. However, the assumption of joint normality between $\ln \hat{\mu}_W$ and $\ln \mu_W$ must be made for the next step which is the calculation of cov $[\hat{\mu}_W, \mu_W]$

$$\operatorname{cov}[\hat{\mu}_W \mu_W] = \mu^2\left(\exp\left(\sum b_i \bar{\sigma}_e(w_i; W)\right) - 1\right) \tag{25}$$

Rendu's covariance calculation (23 and 24) is thus an approximation to the covariance calculation in (22).

From (17), (20), and (21) the variance of the estimation error is thus

$$D^2[\hat{\mu}_W - \mu_W] = \mu^2\left(\exp \bar{\sigma}_e(W; W) + \exp \sum_i \sum_j b_i \sigma_e(w_i; w_j) - 2\frac{1}{W}\int_{z \text{ in } W} \exp \sum_i b_i \sigma_e(w_i; z)\, dz\right) \tag{26}$$

Using Rendu's approximation (eqs. 23 and 24) the variance of the estimation error is

$$D^2[\hat{\mu}_W - \mu_W] = \mu^2\left(\exp \bar{\sigma}_e(W; W) + \exp \sum_i \sum_j b_i b_j \sigma_e(w_i; w_j) - 2 \exp \sum_i b_i \bar{\sigma}_e(w_i; W)\right) \tag{27}$$

where, in both expressions, $\bar{\sigma}_e(W; W)$ is given by eq. (18) and $\bar{\sigma}_e(w_i; W)$ is given by eq. 24.

The logarithmic error variance is

$$D^2[\ln \hat{\mu}_W - \ln \mu_W] = \bar{\sigma}_e(W; W) + \sum_i \sum_j b_i b_j \sigma_e(w_i; w_j) - 2 \ln\left(\frac{1}{W}\int_{z \text{ in } W} \exp \sum b_i \sigma_e(w_i; z)\, dz\right) \tag{28}$$

Using Rendu's approximation the logarithmic error variance is

$$D^2 [\ln \hat{\mu}_W - \ln \mu_W] = \bar{\sigma}_e(W; W) + \sum_i \sum_j b_i b_j \sigma_e(w_i; w_j) - 2 \sum b_i \sigma_e(w_i; W) \tag{29}$$

Assuming conservation of lognormality, the variances of the estimation errors given in eqs. 26 and 27 will be minimum when the respective logarithmic error variances in eqs. 28 and 29 are minimized. In other words, the kriging estimator, μ_k, is the unbiased log-linear estimator with the smallest logarithmic error variance.

Known Mean

If the mean μ of the orebody is known the estimator given in eq. 9 is used

$$\hat{\mu}_W = \exp \left[\left(1 - \sum_i b_i\right) \ln \mu + \sum_i b_i \ln x_i + \tfrac{1}{2} \sum_i b_i \bar{\sigma}_e(w_i; w_i) - \tfrac{1}{2} \sum_i \sum_j b_i b_j \bar{\sigma}_e(w_i; w_j)\right] \tag{30}$$

Rendu (1979) minimizes the logarithmic error variance given in eq. 29. This results in the kriging weights b_j which are the solutions to the following system of equations

$$\sum_j b_j \bar{\sigma}_e(w_i w_j) = \bar{\sigma}_e(w_i; W) \tag{31}$$

The logarithmic kriging error variance is

$$\sigma_{ke}^2 = \bar{\sigma}_e(W; W) - \sum_i b_i \bar{\sigma}_e(w_i; W) \tag{32}$$

From eqs. 30 and 31

$$\mu_k = \exp \left[\left(1 - \sum_i b_i\right) \ln \mu + \sum_i b_i \ln x_i + \tfrac{1}{2} \sum b_i \bar{\sigma}_e(w_i; w_i) - \tfrac{1}{2} \sum_i b_i \bar{\sigma}_e(w_i; W)\right] \tag{33}$$

The kriging error variance is

$$\sigma_k^2 = D^2 [\mu_k - \mu_W]$$

$$= \mu^2 \left(\exp \bar{\sigma}_e(W; W) - \exp \sum_i b_i \bar{\sigma}_e(w_i; W)\right) \tag{34}$$

$$= \mu^2 e^{\bar{\sigma}_e(W; W)} (1 - e^{-\sigma_{ke}^2}) \tag{35}$$

Minimizing the logarithmic error variance given in eq. 28 results in the kriging weights b_j which are the solutions to the following system of equations

$$\sum_j b_j \sigma_e(w_i; w_j) = \frac{\displaystyle\int_{z \text{ in } W} \sigma_e(w_i; z) \exp \sum_j b_j \sigma_e(w_j; z)\, dz}{\displaystyle\int_{z \text{ in } W} \exp \sum_j b_j \sigma_e(w_j; z)\, dz} \tag{36}$$

Obviously there is no straightforward solution to the system of equations in (36) but they can be solved iteratively.

There is no simple expression for the corresponding kriging error variance, σ_k^2, which is obtained by substituting the values of the kriging weights, b_j (obtained by solving eq. 36) into eq. 26.

Unknown Mean

If the mean μ is unknown, the estimator $\hat{\mu}_W$ (eq. 8) will be unbiased (eq. 15) if and only if

$$\sum_i b_i = 1 \tag{37}$$

Thus

$$C = \tfrac{1}{2} \sum_i b_i \bar{\sigma}_e(w_i; w_i) - \tfrac{1}{2} \sum_i \sum_j b_i b_j \bar{\sigma}_e(w_i; w_j) \tag{38}$$

and the unbiased estimator is

$$\hat{\mu}_W = \exp\left(\sum_i b_i \ln x_i + \tfrac{1}{2} \sum_i b_i \bar{\sigma}_e(w_i; w_i) - \tfrac{1}{2} \sum_i \sum_j b_i b_j \bar{\sigma}_e(w_i; w_j) \right) \tag{39}$$

The kriging estimator μ_k is the unbiased log-linear estimator (i.e., satisfies eq. 37) with the minimum logarithmic error variance. Using Rendu's expression for the logarithmic error variance (eq. 29), the kriging weights, b_j, are obtained by solving the following system of equations

$$\sum_j b_j \bar{\sigma}_e(w_i; w_j) = \bar{\sigma}_e(w_i; W) + \lambda \tag{40}$$

$$\sum_j b_j = 1 \tag{41}$$

where λ is a Lagrange multiplier.

The logarithmic kriging error variance is

$$\sigma_{ke}^2 = \bar{\sigma}_e(W; W) - \sum_i b_i \bar{\sigma}_e(w_i; W) + \lambda \tag{42}$$

From eqs. 39, 40, and 41

$$\mu_k = \exp\left(\sum_i b_i \ln x_i + \tfrac{1}{2}\sum_i b_i\bar{\sigma}_e(w_i; w_i) - \tfrac{1}{2}\sum_i b_i\bar{\sigma}_e(w_i; W) - \tfrac{1}{2}\lambda\right) \quad (43)$$

The kriging error variance is

$$\sigma_k^2 = \mu^2\left(\exp \bar{\sigma}_e(W; W) + \exp\left(\sum_i b_i\bar{\sigma}_e(w_i; W) + \lambda\right) - 2\exp\sum_i b_i\bar{\sigma}_e(w_i; W)\right) \quad (44)$$

$$= \mu^2 e^{\bar{\sigma}_e(W; W)}[1 + e^{\lambda - \sigma_{ke}^2}(e^{\lambda} - 2)] \quad (45)$$

As the mean μ is unknown only the relative variance σ_k^2/μ^2 can be calculated.

Minimizing the logarithmic error variance given in eq. 28 subject to $\Sigma_j b_j = 1$ results in the kriging weights, b_j, obtained by solving the following system of equations

$$\sum_j b_j\bar{\sigma}_e(w_i; w_j) = \frac{\displaystyle\int_{z \text{ in } W} \sigma_e(w_i; z)\exp\sum_j b_j\sigma_e(w_j; z)\,dz}{\displaystyle\int_{z \text{ in } W} \exp\sum_j b_j\sigma_e(w_j; z)\,dz} + \lambda \quad (46)$$

$$\sum_j b_j = 1 \quad (47)$$

which can be solved iteratively.

The resulting kriging variance, σ_k^2, is obtained by substituting the values of b_j which satisfy 46 and 47 into eq. 26.

Lognormal Kriging–The General Case

Without the assumption of conservation of lognormality for the blocks W the relationships given in eqs. 17 and 22 are not valid. As a result the logarithmic error variance cannot be defined and even if it could the minimization of the logarithmic error variance $D^2[\ln \hat{\mu}_W - \ln \mu_W]$ would not ensure the minimization of the error variance $D^2[\mu_W - \mu_W]$ and the latter must be minimized directly.

The error variance is

$$D^2[\hat{\mu}_W - \mu_W] = D^2[\hat{\mu}_W] + D^2[\mu_W] - 2\,\mathrm{cov}\,[\hat{\mu}_W, \mu_W]$$

$\hat{\mu}_W$ is a lognormal variable and thus

$$D^2[\hat{\mu}_W] = \mu^2\left(\exp\sum_i\sum_j b_ib_j\bar{\sigma}_e(w_i; w_j) - 1\right) \quad (48)$$

By definition

$$D^2[\mu_W] = \frac{1}{W}\int_{z \text{ in } W} \sigma(z)\,dz$$

$$= \mu^2 \frac{1}{W}\int_{z \text{ in } W} (e^{\sigma_e(z)} - 1)\,dz, \qquad \text{and} \tag{49}$$

$$\text{cov}\,[\hat{\mu}_W \mu_W] = \mu^2 \frac{1}{W}\int_{z \text{ in } W} \left(\exp \sum_i b_i \sigma_e(w_i; z) - 1\right) dz \tag{50}$$

From eqs. 48, 49, and 50, the error variance is

$$D^2[\hat{\mu}_W - \mu_W] = \mu^2 \left(\exp \sum_i \sum_j b_i b_j \sigma_e(w_i; w_j) - 1\right)$$

$$+ \mu^2 \frac{1}{W}\int_{z \text{ in } W} (\exp \sigma_e(z) - 1)\,dz$$

$$- 2\mu^2 \frac{1}{W}\int_{z \text{ in } W} \left(\exp \sum_i b_i \sigma_e(w_i; z) - 1\right) dz \tag{51}$$

This expression for the error variance can be compared with that given in eq. 16 where conservation of lognormality is assumed.

Known Mean

When the mean, μ, of the orebody is known, the estimator, $\hat{\mu}_W$, is given in eq. 12. As conservation of lognormality is not assumed the kriging weights, b_i, are those that minimize the error variance given in eq. 51.

Differentiating eq. 51 with respect to b_i and equating the resulting expression to zero, gives

$$\sum_j b_i \bar{\sigma}_e(w_i; w_j) \exp \sum_i \sum_j b_i b_j \bar{\sigma}_e(w_i; w_j)$$

$$- \frac{1}{W}\int_{z \text{ in } W} \sigma_e(w_j; z) \exp \sum_i b_i \sigma_e(w_i; z)\,dz = 0 \tag{52}$$

and the weights b_i are the solutions to the following system of equations

$$\sum_i b_i \bar{\sigma}_e(w_i; w_j) = \frac{\dfrac{1}{W}\displaystyle\int_{z \text{ in } W} \sigma_e(w_j; z) \exp \sum_i b_i \sigma_e(w_i; z)\,dz}{\exp \displaystyle\sum_i \sum_j b_i b_j \bar{\sigma}_e(w_i; w_j)} \tag{53}$$

This system of equations can be compared to that given in 18 for the case when conservation of lognormality is assumed.

Obviously there is no straightforward solution to the system of equations in 53, but they can be solved readily by iteration.

There is no simplified form for the resulting kriging variance σ_k^2, which is obtained by substituting the values of the kriging weights, b_i, (obtained by solving eq. 53) into eq. 51. The kriging estimator, μ_k, is obtained by a similar substitution in eq. 17.

Unknown Mean

If the mean μ is unknown, the estimator, $\hat{\mu}_W$ will be unbiased if the condition given in eq. 37 is imposed. The resulting unbiased estimator is given in eq. 39.

The kriging estimator, μ_k, is the log-linear estimator with smallest error variance, $D^2[\hat{\mu}_W - \mu_W]$ (eq. 51), which also satisfies the unbiasedness constraint (eqs. 37 and 38).

Differentiating eq. 51 with respect to b_i, subject to the constraint $\Sigma b_i = 1$, results in the following system of equations

$$\sum_i b_i \bar{\sigma}_e(w_i; w_j) = \frac{\frac{1}{W}\int_{z \text{ in } W} \sigma_e(w_j; z) \exp \sum_i b_i \sigma_e(w_i; z)\, dz}{\exp \sum_i \sum_j b_i b_j \bar{\sigma}_e(w_i; w_j)} + \lambda \tag{54}$$

$$\sum_i b_i = 1 \tag{55}$$

where λ is a Lagrange multiplier. The system of equations given in 54 and 55 can be solved by iteration.

There is no simplified form for the resulting kriging variance, σ_k^2, which is obtained by substituting the values of the kriging weights, b_i (obtained by solving eqs. 54 and 55), into eq. 51. The kriging estimator, μ_k, is obtained by a similar substitution in eq. 39.

Ordinary Kriging

If the lognormality of the data is ignored, the estimator, $\hat{\mu}_W$, is obtained by simple linear kriging.

When the mean, μ, is known the estimate is

$$\hat{\mu}_W = \sum b_i(x_i - \mu) + \mu \tag{56}$$

and the b_i are found from

$$\sum_j b_j \bar{\sigma}(w_i; w_j) = \bar{\sigma}(w_i; W) \qquad \forall_i \tag{57}$$

When the mean, μ, is unknown, the estimator, $\hat{\mu}_W$, is

$$\hat{\mu}_W = \sum b_i x_i \tag{58}$$

and the b_i are found from

$$\sum_j b_j \bar{\sigma}(w_i; w_j) = \bar{\sigma}(w_i; W) + \lambda \qquad \forall_i$$

$$\sum b_i = 1 \tag{59}$$

Comparison of Solutions

Obviously the systems of eqs. (31 and 40) resulting from Rendu's approximation (23 and 24) differ from those (36 and 40) obtained without the approximation. In addition, both solutions differ from those (53 and 54) obtained for the general case when conservation of lognormality is not assumed.

To illustrate the numerical significance of the different solutions, a square panel has been estimated from four corner samples and a central sample, the values of which come from a lognormal distribution. It has been assumed that the covariance, $\sigma_e(h)$, of the logarithms of the sample values is spherical

$$\sigma_e(h) = C_0\delta(h) + C\left[1 - \frac{3}{2}\frac{h}{a} + \frac{h^3}{2a^3}\right], \qquad h \leqslant a$$

$$= 0, \qquad \text{where} \quad h \geqslant a, \quad \text{and where}$$

$$\delta(h) = \begin{cases} 1, & \text{if} \quad h = 0 \\ 0, & \text{if} \quad h \neq 0 \end{cases}$$

The covariance, $\sigma(h)$, of the sample values is given by

$$\sigma(h) = \mu^2(e^{\sigma_e(h)} - 1)$$

Where μ is the mean value of the lognormal distribution from which the sample values are taken.

The square panel has been estimated by

(i) ordinary kriging, that is, solving eq. 57 when the mean μ is known and eq. 59 when it is unknown
(ii) lognormal kriging assuming conservation of lognormality and using Rendu's approximation, that is, solving eq. 31 when the mean μ is known and eq. 40 when it is unknown.
(iii) lognormal kriging assuming conservation of lognormality for the general case, that is, solving eq. 36 when the mean μ is known and eq. 40 when it is unknown.
(iv) lognormal kriging for the general case, that is, solving eq. 53 when the mean μ is known and eq. 54 when it is unknown.

The results for various sill values $(C_0 + C)$, nugget variances (C_0), and panel sizes are summarized in Tables 1-9.

CONCLUSIONS

The results for an unknown mean are given in Tables 1-6. For small values of $C(\leqslant 1)$ the general case of lognormal kriging assuming conservation of lognormality gives results which are not significantly different from those obtained without the assumption of conservation of lognormality. As C increases the kriging variances obtained from both methods remain very similar but the differences in kriging weights become increasingly significant.

Rendu's approximation consistently underestimates the kriging variance even for relatively small panels (e.g., sides equal to 20% of the range).

Ordinary kriging consistently overestimates the kriging variance.

All methods give similar results for very small panels (sides of 5% or less of the range) except when a nugget variance is present; then ordinary kriging results differ significantly from the others.

As C_0 increases the results obtained from Rendu's approximation approach those obtained without the assumption of conservation of lognormality, although the approximation still significantly underestimates the kriging variance. The significance of the differences in the results obtained from ordinary kriging and from the other methods increases as the nugget variance (C_0) increases.

When the mean is known, Rendu's approximation can give negative kriging variances for large sill values $(C_0 + C)$ as shown in Table 8.[3] Apart from this, the same conclusions can be drawn from the case of a known mean with the additional comment that differences are even more significant than in the corresponding case of an unknown mean.

The parameters of the covariance function used to obtain the results in Tables 6 and 9 were taken from a case study of an alluvial tin deposit in Indonesia. The estimated mean value of the deposit is 1.17 kg/m^3 and the variance is 208 $(kg/m^3)^2$.

The covariance/variogram of the data values appeared to be almost random with an estimated nugget effect of approximately 2.0. The covariance/variogram of the logarithms of the data values had a nugget variance (C_0) of approximately 1.0, a sill value $(C_0 + C)$ of approximately 5.0, and a range of 100 meters. Except for very small block sizes, the results in Table 6 indicate that the choice of the valuation method to be used for block estimation is critical. If the mean is assumed known, Tables 8 and 9 indicate that the choice of method is even more critical, Rendu's approximation being unacceptable even for very small blocks.

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[3]This is a consequence of using the approximation given in eq. 23 and relaxing the restriction that the sum of the weights must be unity.

Table 1. Kriging Weights and Variances for the Estimator of a Square Panel by Four Corner Samples and A Central Sample[a]

	Ordinary kriging			Lognormal kriging–conservation of lognormality: Rendu's approximation			Lognormal kriging–conservation of lognormality: General case			Lognormal kriging nonconservation of lognormality		
L/a	b_1	b_2	σ_k^2	b_1	b_2	σ_k^2	b_1	b_2	σ_k^2	b_1	b_2	σ_k^2
1.0	.148	.408	.066	.142	.432	.044	.140	.440	.053	.141	.436	.054
0.95	.146	.316	.064	.141	.436	.042	.139	.444	.051	.140	.440	.053
0.90	.145	.420	.062	.141	.436	.041	.138	.448	.050	.139	.444	.052
0.85	.144	.424	.060	.140	.440	.040	.137	.452	.049	.138	.448	.051
0.80	.143	.428	.057	.140	.440	.039	.136	.456	.047	.137	.452	.049
0.75	.142	.432	.054	.140	.440	.037	.136	.456	.046	.137	.452	.047
0.70	.142	.432	.051	.141	.436	.035	.137	.452	.043	.137	.452	.044
0.65	.142	.432	.048	.142	.432	.032	.138	.448	.040	.138	.448	.041
0.60	.142	.432	.044	.143	.428	.030	.138	.448	.037	.139	.444	.038
0.50	.141	.436	.037	.144	.424	.025	.140	.440	.031	.140	.440	.032
0.40	.142	.432	.029	.145	.420	.021	.141	.436	.024	.141	.436	.026
0.30	.142	.432	.022	.146	.416	.016	.142	.432	.018	.142	.432	.020
0.20	.143	.428	.014	.146	.416	.012	.143	.428	.011	.143	.428	.013
0.10	.144	.424	.007	.145	.420	.006	.144	.424	.005	.144	.424	.007
0.05	.144	.424	.003	.145	.420	.003	.144	.424	.002	.144	.424	.003

[a]The variogram of the logarithms of sample values is spherical with zero nugget variance, a sill value of 0.5, and range a. The mean value of the deposit is assumed to be unknown. N.B. all variances in the table are relative.

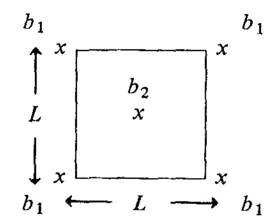


Table 2. Kriging Weights and Variances for the Estimation of a Square Panel by Four Corner Samples and a Central Sample[a]

	Ordinary kriging			Lognormal kriging–conservation of lognormality: Rendu's approximation			Lognormal kriging–conservation of lognormality: General case			Lognormal kriging nonconservation of lognormality		
L/a	b_1	b_2	σ_k^2	b_1	b_2	σ_k^2	b_1	b_2	σ_k^2	b_1	b_2	σ_k^2
1.0	.154	.384	.208	.143	.428	.096	.138	.448	.146	.140	.440	.147
0.95	.153	.388	.202	.143	.428	.093	.137	.452	.144	.139	.444	.145
0.90	.151	.396	.196	.142	.432	.091	.136	.456	.142	.138	.448	.144
0.85	.149	.404	.190	.142	.432	.089	.135	.460	.140	.137	.452	.142
0.80	.148	.408	.183	.143	.428	.086	.135	.460	.137	.137	.452	.139
0.75	.147	.412	.178	.143	.428	.083	.135	.460	.133	.136	.456	.134
0.70	.146	.416	.166	.144	.424	.078	.135	.460	.127	.136	.456	.128
0.65	.145	.420	.156	.145	.420	.073	.136	.456	.120	.137	.452	.121
0.60	.144	.424	.145	.146	.416	.069	.137	.452	.112	.137	.452	.114
0.50	.143	.428	.123	.148	.408	.060	.139	.444	.096	.138	.448	.098
0.40	.142	.432	.099	.149	.404	.052	.140	.440	.079	.139	.444	.081
0.30	.142	.432	.074	.149	.404	.043	.142	.432	.061	.140	.440	.062
0.20	.142	.432	.048	.148	.408	.033	.143	.428	.041	.142	.432	.043
0.10	.143	.428	.023	.146	.416	.019	.144	.424	.020	.143	.428	.022
0.05	.144	.424	.011	.146	.416	.010	.144	.424	.009	.144	.424	.011

[a]The variogram of the *logarithms* of sample values is spherical with zero nugget variance, unit sill, and range a. The mean value of the deposit is unknown. N.B. all variances in the table are relative.

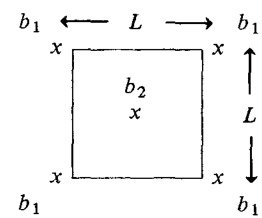


Table 3. Kriging Weights and Variances for the Estimation of a Square Panel by Four Corner Samples and a Central Sample[a]

	Ordinary kriging			Lognormal kriging–conservation of lognormality: Rendu's approximation			Lognormal kriging–conservation of lognormality: General case			Lognormal kriging nonconservation of lognormality		
L/a	b_1	b_2	σ_k^2	b_1	b_2	σ_k^2	b_1	b_2	σ_k^2	b_1	b_2	σ_k^2
1.0	.167	.332	.347	.154	.384	.163	.150	.400	.217	.154	.384	.217
0.95	.166	.336	.342	.154	.384	.162	.150	.400	.217	.154	.384	.217
0.90	.166	.336	.337	.155	.380	.161	.150	.400	.217	.154	.384	.217
.85	.165	.340	.332	.155	.380	.160	.150	.400	.217	.154	.384	.217
.80	.164	.344	.326	.156	.376	.159	.151	.396	.215	.154	.384	.215
.75	.164	.344	.319	.157	.372	.157	.152	.392	.212	.155	.380	.213
.70	.164	.344	.310	.159	.364	.153	.153	.388	.208	.156	.376	.208
.65	.165	.340	.300	.161	.356	.149	.155	.380	.202	.157	.373	.202
.60	.165	.340	.289	.162	.352	.146	.157	.372	.196	.159	.364	.196
.50	.166	.336	.267	.166	.336	.142	.161	.356	.184	.162	.352	.184
.40	.167	.332	.241	.169	.324	.139	.165	.340	.171	.166	.336	.172
.30	.172	.312	.214	.172	.312	.136	.170	.320	.158	.171	.316	.158
.20	.178	.288	.184	.178	.288	.132	.177	.232	.143	.178	.288	.144
.10	.187	.252	.153	.186	.256	.124	.186	.256	.127	.186	.256	.128
.05	.193	.228	.137	.191	.236	.118	.192	.232	.119	.192	.232	.119

[a]The variogram of the logarithm of sample values is spherical with a nugget variance of 0.2, a sill of 1.2, and a range of a. The mean value of the deposit is unknown. N.B. the variances in the table are relative.

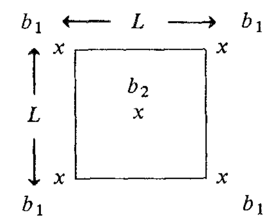


Table 4. Kriging Weights and Variances for the Estimation of a Square Panel by Four Corner Samples and a Central Sample[a]

	Ordinary kriging			Lognormal kriging–conservation of lognormality: Rendu's approximation			Lognormal kriging–conservation of lognormality: General case			Lognormal kriging nonconservation of lognormality		
L/a	b_1	b_2	σ_k^2	b_1	b_2	σ_k^2	b_1	b_2	σ_k^2	b_1	b_2	σ_k^2
1.0	.171	.316	.423	.158	.368	.195	.155	.380	.251	.159	.364	.251
0.95	.171	.316	.418	.158	.368	.194	.155	.380	.251	.159	.364	.251
0.90	.171	.316	.414	.159	.364	.194	.155	.380	.251	.159	.364	.251
0.85	.170	.320	.409	.159	.364	.194	.155	.380	.251	.160	.360	.251
0.80	.169	.324	.403	.160	.360	.193	.156	.376	.251	.160	.360	.251
0.75	.170	.320	.396	.162	.352	.191	.157	.372	.249	.161	.356	.249
0.70	.170	.320	.387	.164	.344	.188	.159	.364	.244	.162	.352	.244
0.65	.171	.316	.377	.165	.340	.185	.161	.356	.239	.164	.344	.239
0.60	.171	.316	.366	.167	.332	.182	.163	.348	.234	.165	.340	.234
0.50	.173	.308	.343	.171	.316	.180	.167	.332	.224	.169	.324	.224
0.40	.175	.300	.317	.174	.304	.179	.171	.316	.213	.173	.308	.213
0.30	.179	.284	.288	.178	.288	.179	.176	.296	.202	.178	.288	.202
0.20	.184	.264	256	.183	.268	.178	.182	.272	.191	.184	.264	.191
0.10	.191	.236	.224	.189	.244	.176	.190	.240	.179	.190	.240	.179
0.05	.195	.220	.207	.194	.224	.172	.194	.224	.173	.194	.224	.173

[a]The variogram of the logarithm of sample values is spherical with a nugget variance of 0.3, a sill of 1.3, and a range a. The mean value of the deposit is unknown. N.B. all variances in the table are relative.

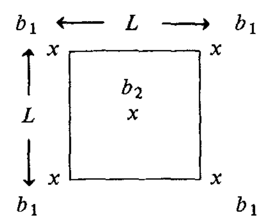


Table 5. Kriging Weights and Variances for the Estimation of a Square Panel by Four Corner Samples and a Central Sample[a]

	Ordinary kriging			Lognormal kriging–conservation of lognormality: Rendu's approximation			Lognormal kriging–conservation of lognormality: General case			Lognormal kriging nonconservation of lognormality		
L/a	b_1	b_2	σ_k^2	b_1	b_2	σ_k^2	b_1	b_2	σ_k^2	b_1	b_2	σ_k^2
1.0	.184	.264	9.890	.155	.380	.511	.124	.504	3.544	.136	.456	3.503
0.95	.182	.272	9.798	.156	.376	.502	.123	.508	3.672	.135	.460	3.634
0.90	.181	.276	9.698	.158	.368	.501	.122	.512	3.799	.133	.468	3.771
0.85	.179	.284	9.585	.159	.364	.506	.122	.512	3.932	.132	.472	3.908
0.80	.177	.292	9.454	.161	.356	.514	.122	.512	4.051	.131	.476	4.036
0.75	.175	.300	9.287	.163	.348	.521	.122	.512	4.151	.130	.480	4.144
0.70	.172	.312	9.108	.165	.340	.527	.124	.504	4.221	.130	.480	4.223
0.65	.170	.320	8.885	.167	.332	.536	.125	.500	4.267	.130	.480	4.279
0.60	.167	.332	8.621	.168	.328	.552	.127	.492	4.295	.129	.484	4.315
0.50	.161	.356	7.943	.170	.320	.609	.130	.480	4.274	.130	.480	4.297
0.40	.155	.380	7.002	.169	.324	.703	.133	.468	4.078	.131	.476	4.103
0.30	.149	.404	5.724	.165	.340	.849	.136	.456	3.626	.133	.468	3.647
0.20	.144	.424	4.052	.160	.360	1.008	.139	.444	2.820	.136	.456	2.841
0.10	.141	.436	2.032	.152	.392	0.947	.142	.432	1.593	.140	.440	1.618
0.05	.142	.432	0.975	.149	.404	0.649	.143	.428	0.783	.142	.432	0.851

[a]The variogram of the logarithms of sample values is spherical with a sill value of 4.0, zero nugget variance, and range a. The mean value of the deposit is unknown. N.B. all variances in the table are relative.

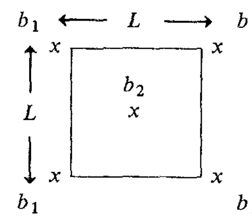


Table 6. Kriging Weights and Variances for the Estimation of a Square Panel by Four Corner Samples and a Central Sample[a]

	Ordinary kriging			Lognormal kriging–conservation of lognormality: Rendu's approximation			Lognormal kriging–conservation of lognormality: General case			Lognormal kriging nonconservation of lognormality		
L/a	b_1	b_2	σ_k^2	b_1	b_2	σ_k^2	b_1	b_2	σ_k^2	b_1	b_2	σ_k^2
1.0	.194	.224	28.826	.163	.348	1.254	.145	.420	4.570	.162	.352	4.353
0.95	.194	.224	28.770	.165	.340	1.286	.145	.420	4.774	.161	.356	4.638
0.90	.193	.228	28.711	.166	.336	1.336	.145	.420	4.994	.161	.356	4.860
0.85	.192	.232	28.645	.168	.328	1.401	.145	.420	5.229	.161	.356	5.097
0.80	.192	.232	28.575	.170	.320	1.479	.146	.416	5.465	.161	.356	5.340
0.75	.191	.236	28.486	.172	.312	1.568	.148	.408	5.695	.161	.356	5.580
0.70	.190	.240	28.374	.174	.304	1.664	.150	.400	5.909	.162	.352	5.806
0.65	.190	.240	28.238	.176	.296	1.786	.152	.392	6.123	.163	.348	6.033
0.60	.189	.244	28.072	.178	.288	1.952	.155	.380	6.351	.164	.344	6.273
0.50	.187	.252	27.617	.180	.280	2.448	.160	.360	6.855	.167	.332	6.786
0.40	.186	.256	26.907	.182	.272	3.256	.166	.336	7.407	.172	.312	7.339
0.30	.186	.256	25.794	.183	.268	4.527	.172	.312	8.031	.178	.288	7.952
0.20	.188	.248	24.085	.184	.264	6.418	.179	.284	8.804	.185	.260	8.711
0.10	.192	.232	21.640	.188	.248	8.974	.188	.248	9.958	.192	.232	9.863
0.05	.196	.216	20.193	.192	.232	10.434	.193	.228	10.790	.196	.216	10.747

[a]The variogram of the logarithms of sample values is spherical with a sill value of 5.0, nugget variance of 1.0, and a range a. The mean value of the deposit is unknown. N.B. all variances in the table are relative.

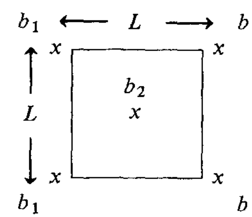


Table 7. Kriging Weights and Variances for the Estimation of a Square Panel by Four Corner Samples and a Central Sample[a]

	Ordinary kriging			Lognormal kriging–conservation of lognormality: Rendu's approximation			Lognormal kriging–conservation of lognormality: General case			Lognormal kriging nonconservation of lognormality		
L/a	b_1	b_2	σ_k^2	b_1	b_2	σ_k^2	b_1	b_2	σ_k^2	b_1	b_2	σ_k^2
1.0	.076	.324	.155	.133	.422	.095	.106	.430	.137	.105	.428	.139
0.95	.080	.338	.156	.137	.426	.093	.109	.439	.138	.108	.438	.139
0.90	.084	.353	.157	.141	.429	.091	.112	.446	.137	.111	.445	.139
0.85	.088	.366	.157	.144	.431	.089	.115	.454	.137	.114	.452	.138
0.80	.093	.379	.156	.148	.431	.086	.118	.458	.135	.117	.457	.136
0.75	.098	.391	.154	.152	.429	.082	.122	.461	.131	.122	.460	.133
0.70	.104	.401	.150	.156	.423	.077	.127	.460	.126	.127	.459	.128
0.65	.110	.410	.144	.160	.417	.071	.133	.456	.119	.133	.456	.121
0.60	.115	.417	.137	.163	.410	.065	.137	.452	.112	.137	.453	.114
0.50	.125	.427	.120	.164	.402	.057	.142	.444	.096	.143	.444	.098
0.40	.132	.433	.098	.161	.398	.049	.145	.436	.079	.145	.435	.081
0.30	.138	.435	.074	.157	.400	.042	.146	.430	.060	.146	.432	.062
0.20	.142	.433	.048	.152	.405	.032	.146	.426	.041	.146	.427	.042
0.10	.144	.428	.023	.148	.413	.019	.145	.424	.020	.145	.424	.022
0.05	.144	.425	.011	.146	.417	.010	.145	.423	.009	.145	.423	.011

[a]The variogram of the logarithms of sample values is spherical with zero nugget variance, a sill value of 1.0, and a range a. The mean value of the deposit is assumed known. N.B. all variances in the table are relative.

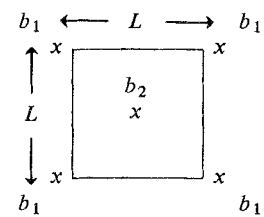


Table 8. Kriging Weights and Variances for the Estimation of a Square Panel by Four Corner Samples and a Central Sample[a]

	Ordinary kriging			Lognormal kriging–conservation of lognormality: Rendu's approximation			Lognormal kriging–conservation of lognormality: General case			Lognormal kriging nonconservation of lognormality		
L/a	b_1	b_2	σ_k^2	b_1	b_2	σ_k^2	b_1	b_2	σ_k^2	b_1	b_2	σ_k^2
1.0	.015	.101	2.228	.236	.430	-0.956	.093	.486	3.361	.091	.473	3.372
0.95	.017	.112	2.438	.242	.418	-1.260	.096	.495	3.514	.094	.481	3.522
0.90	.019	.125	2.671	.245	.404	-1.529	.100	.500	3.665	.097	.489	3.676
0.85	.022	.139	2.928	.247	.389	-1.763	.103	.506	3.819	.101	.495	3.829
0.80	.026	.155	3.208	.248	.373	-1.968	.107	.509	3.961	.105	.499	3.973
0.75	.030	.172	3.509	.248	.355	-2.157	.111	.509	4.088	.109	.500	4.101
0.70	.034	.191	3.824	.249	.337	-2.346	.117	.506	4.188	.115	.498	4.200
0.65	.040	.212	4.145	.247	.319	-2.435	.122	.500	4.257	.121	.493	4.270
0.60	.047	.234	4.461	.242	.307	-2.363	.127	.493	4.297	.126	.489	4.313
0.50	.062	.283	5.004	.226	.296	-1.829	.134	.479	4.278	.132	.473	4.297
0.40	.082	.335	5.255	.206	.303	-0.986	.138	.465	4.068	.139	.465	4.099
0.30	.103	.384	4.949	.187	.324	-0.073	.141	.452	3.605	.139	.447	3.640
0.20	.122	.421	3.861	.170	.353	0.667	.142	.441	2.797	.140	.432	2.940
0.10	.137	.436	2.025	.156	.387	0.893	.144	.431	1.583	.144	.431	1.609
0.05	.142	.432	0.975	.150	.405	0.640	.144	.426	0.780	.144	.426	0.847

[a]The variogram of the logarithms is spherical with zero nugget variance, a sill value of 4.0, and a range a. The mean value of the deposit is assumed known. N.B. all variances are relative.

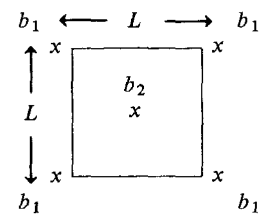
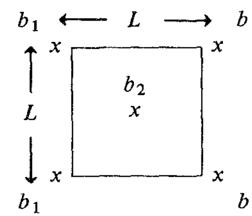


Table 9. Kriging Weights and Variances for the Estimation of a Square Panel by Four Corner Samples and a Central Sample[a]

	Ordinary kriging			Lognormal kriging–conservation of lognormality						Lognormal kriging nonconservation of lognormality		
				Rendu's approximation			General case					
L/a	b_1	b_2	σ_k^2	b_1	b_2	σ_k^2	b_1	b_2	σ_k^2	b_1	b_2	σ_k^2
1.0	.006	.037	2.610	.195	.359	1.182	.086	.378	3.960	.085	.372	3.975
0.95	.007	.041	2.913	.201	.353	1.117	.091	.386	4.200	.089	.380	4.214
0.90	.008	.046	3.263	.205	.346	1.072	.096	.392	4.452	.094	.386	4.468
0.85	.009	.051	3.668	.208	.337	1.047	.101	.396	4.720	.100	.390	4.736
0.80	.010	.057	4.134	.211	.327	1.042	.107	.398	4.995	.106	.393	5.013
0.75	.012	.063	4.670	.212	.317	1.052	.113	.397	5.276	.112	.392	5.295
0.70	.015	.071	5.281	.214	.305	1.072	.121	.393	5.554	.120	.389	5.572
0.65	.018	.079	5.976	.214	.294	1.161	.128	.387	5.827	.127	.383	5.847
0.60	.021	.088	6.761	.212	.284	1.361	.135	.379	6.101	.134	.376	6.123
0.50	.031	.108	8.606	.203	.272	2.089	.146	.360	6.655	.146	.359	6.677
0.40	.045	.131	10.770	.194	.266	3.212	.155	.338	7.201	.154	.335	7.229
0.30	.065	.154	13.027	.185	.263	4.671	.162	.312	7.746	.161	.310	7.778
0.20	.091	.172	14.838	.180	.256	6.391	.170	.282	8.286	.169	.279	8.352
0.10	.121	.174	15.351	.181	.236	8.185	.178	.244	8.846	.177	.243	8.917
0.05	.135	.165	14.876	.185	.218	8.960	.184	.220	9.099	.183	.220	9.172

[a]The variogram of the logarithms is spherical with a nugget variance of 1.0, a sill value of 5.0, and a range a. The mean value of the deposit is assumed known. N.B. all variances are relative.



Iteration Programming

The kriging equations for nonconservation of lognormality (53, 54, 55) and the general case of lognormality (36, 46, 47) have been solved by iteration.

For example the equations given in (36) for the general case of conservation of lognormality with a known mean can be solved by beginning with an initial solution provided by the values of b_j which satisfy Rendu's formulation of the problem, that is, 31 and 24.

These values of b_j are then substituted into the right-hand side of (36) and the set of equations is solved for the unknown b_js on the left-hand side. This procedure is repeated until the weights b_j converge to some acceptable percentage of their value on the previous iteration.

This simple iterative method works satisfactorily for all cases except that of nonconservation of lognormality with a known mean (eq. 53). For large variances ($C_0 + C \geqslant 2$) the number of iterations becomes prohibitively large and for $C_0 + C \geqslant 4$ the procedure will not converge to a solution. The method was therefore adapted to include a directed search at each step of the iteration and this always resulted in a solution after a maximum of 10 iterations. The number of iterations was subsequently reduced to a maximum of 4 by using the method of successive corrections, for example, Noble (1964).

In practice, it has been found that from one to four iterations are sufficient for the weights and Lagrange multipliers to converge to less than 1% of their value on the previous iteration.

On the average, computing time for the solution involving iterative procedures was 40% more than that required by the noniterative procedures (ordinary kriging and Rendu's approximation) which seems a small price to pay for the enhanced estimates.

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