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ORBITAL EQUIVALENCE OF SINGULAR POINTS OF VECTOR FIELDS ON THE PLANE

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This note is a continuation of [1]. The theorems formulated herein yield the moduli of the singular points of vector fields on the plane relative to smooth orbital equivalence (see Sec. 5 in [1]).

Notation. Let V denote the Lie algebra of germs at $(0) \in \mathbf{R}^2$ of vector fields of class C^∞ on the plane, Ω_2 (Ω_1) the R-algebra of germs at $(0) \in \mathbf{R}^2$ ($(0) \in \mathbf{R}$) of functions of class C^∞ on the plane (on the line), $\mathfrak{M}^r$, r an integer, $r \geqslant 2$ (V_r, r = 1, . . . , ∞) the r-th power of the maximal ideal $\mathfrak{M}$ in Ω_2 generated by the germs that vanish at $(0) \in \mathbf{R}^2$ (the r-flat germs of vector fields on the plane, i.e., the $v \in V$, such that $v(\mathfrak{M}^p) \subset \mathfrak{M}^{p+r-1}\ \forall p \in \mathbf{Z},\ p \geqslant 1$).

Let J_k denote the space of k-jets of germs in V ($J_k = V/V_{k-1}$), $k \geqslant 0$ an integer; π_k will denote the natural projection π_k: $V \to J_k$.

Let γ be the germ at $(0) \in \mathbf{R}^2$ of a fibering $\overline{\gamma}$ of class C^∞, $\bar{\gamma}$: $\mathbf{R}^2 \to \mathbf{R}$, $\bar{\gamma}(0) = 0$. Let $\gamma^*(\Omega_1) \subset \Omega_2$ denote the image of Ω_1 under the induced mapping γ^*: $\Omega_1 \to \Omega_2$.

Let $\zeta_1, \ldots, \zeta_m \in V$. Let $\gamma^*(\zeta_1, \ldots, \zeta_m)$ denote the $\gamma^*(\Omega_1)$-module generated by the germs in V of the form $v = \sum_{i=1}^{m} f_i \cdot \zeta_i,\ f_i \in \gamma^*(\Omega_1)$.

THEOREM 1. For every germ $v \in V$ (with the exception of a set of germs in V of codimension ∞ in the space V), there exists an integer $k \geqslant 0$, germs $\zeta_1, \ldots, \zeta_m \in V$ and a germ γ at (0) of a fibering of class C^∞ $\bar{\gamma}$: $\mathbf{R}^2 \to \mathbf{R}$, $\bar{\gamma}(0) = 0$, such that the germ v is C^∞-orbitally equivalent to a germ of the form

$$w = P_k + h_k + \varkappa, \tag{1}$$

where P_k is the germ of a polynomial field of degree at most k, $\pi_k P_k = \pi_k v$, $h_k \in V_{k+1} \cap \gamma^*(\zeta_1, \ldots, \zeta_m)$, $\varkappa \in V_\infty$.

Remark 1. Formula (1) may be regarded as a "normal form" with moduli in the form of functions of a single variable, though some germs w of type (1) belong to a single C^∞-orbital orbit. In some cases, one can construct a C^∞-orbital polynomial normal form with finitely many parameters (moduli) (see Theorem 2).

Definition 1. A germ $v \in V_1$ is called a germ with nontrivial linear part if the eigenvalues of the matrix $\pi_1 v \in J_1$ do not vanish simultaneously.

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THEOREM 2. 1) If the C^∞-orbital orbit of a germ $v \in V_1$ has finite codimension in the space V, then it is the orbit of a germ with nontrivial linear part.

2) If $v \in V_1$ is a germ with nontrivial linear part, then either the C^∞-orbital orbit of v has finite codimension in the space V, or v belongs to a set of codimension ∞ in V.

Theorem 3 below, together with Definitions 2-5, is a refinement of Theorem 1.

Notation. Let $\partial_1 \wedge \partial_2$ denote the outer product of basis vector fields $\partial_1 = \partial/\partial x_1, \partial_2 = \partial/\partial x_2$ and for every $\xi \in V$ define a germ $\alpha_v(\xi) \in \Omega_2$ by the condition $\xi \wedge v = \alpha_v(\xi)\, \partial_1 \wedge \partial_2$ (here $\wedge$ denotes the outer product of vectors).

Definition 2. The derived ideal I(v) of a germ $v \in V$ is the following ideal in the algebra Ω_2:

$$I(v) = \alpha_v(V_1) = \{f \in \Omega_2 : f = \alpha_v(\xi),\ \xi \in V_1\}.$$

Definition 3. A germ $v \in V$ is said to be of finite multiplicity if the factor algebra $\mathfrak{M}/I(v)$ is finite-dimensional (over **R**).

The multiplicity of the singular point $(0) \in \mathbf{R}^2$ of a germ $v \in V_1$ is defined as $\mu(v) = -1 + \dim_{\mathbf{R}} \mathfrak{M}/I(v)$ $(\mu(v) = 1, 2, \ldots, \infty)$.

Definition 4. The r-jet $q = \pi_r v \in J_r$ of a germ $v \in V_1$ of finite multiplicity is said to be stable if

$$\forall f \in \mathfrak{M}^{r+1}\ \exists\ \xi \in V_1 : \alpha_v(\xi) = f \pmod{\mathfrak{M}^{r+2}}. \tag{2}$$

Lemma 1 and Remark 2 below show that Definition 4 is well-founded.

LEMMA 1 (see [2]). A germ $v \in V_1$ is of finite multiplicity if and only if there exists an integer r > 0 such that condition 2 is satisfied.

Remark 2. Let $v \in V_1$ be a germ and r > 0 an integer such that (2) holds. Then (2) is also true for any germ $\tilde{v} \in V_1$ such that $\pi_r \tilde{v} = \pi_r v$.

LEMMA 2. For almost all germs in V (with the exception of a set of germs of codimension ∞ in the space V), there exists a stable jet and the multiplicity of these germs is finite.

Definition 5. Let ord (v) denote the integer or ∞ defined by ∞, $\operatorname{ord}(v) = \max\{r : v \in V_r\}$. Following Frommer (see [3]), we say that the k-jet of a germ v is singular if $\alpha_v(x_1\partial_1 + x_2\partial_2) = 0 \pmod{\mathfrak{M}^{\operatorname{ord}(v)+2}}$, and nonsingular otherwise.

THEOREM 3. Let $q = \pi_r v \in J_r$ be a stable r-jet. Then there exist an integer $k \geqslant r$ and a free $\gamma^*(\Omega_1)$-module $\gamma^*(\zeta_1, \ldots, \zeta_m)$ such that every germ $u \in V, \pi_r u = q$, has (1) $h_k \in V_{k+1} \cap \gamma^*(\zeta_1, \ldots, \zeta_m)$, in normal form (1), where m = ord (v) if the ord (v)-jet of v is nonsingular and m = ord (v) + 1 otherwise.

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