Synergetics

Are Cooperative Phenomena Governed by Universal Principles?

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Synergetics is concerned with the cooperation of individual parts of a system that produces macroscopic spatial, temporal, or functional structures. The following article deals with an aspect of synergetics of particular interest: Are there general principles that govern the self-organized production of macroscopic structures irrespective of the nature of the individual parts?

Unification of seemingly diverse phenomena and ideas under a general idea has been a main goal of science since ever. Be it philosophical approaches to construct a unique picture of the world, or, more specifically, theories belonging to individual sciences. In physics, well-known examples of such views arc Newton's theory of gravitation or Maxwell's theory of electromagnetism. A quite recent development has been the unification of theories of weak and electromagnetic interaction by Glashold, Salam and Weinberg. Further examples of physics out of many others are Einstein's unifying theory of space, time and matter. From other fields, we may quote Mendelejev's table of the chemical elements, or Mendel's laws of inheritance as classical examples. While this search for unifying ideas is still pursued, science itself splits into more and more disciplines each of which speaks its own language and uses its own methods. The original attempt at unification seems to be more and more buried under a giant wave of new, more and more specialized results. In view of this it might seem absurd to search for any concepts capable to bring about new links between different fields of research. But this is exactly what synergetics^{*} does. In this article I wish to describe some of its aims, its success and its limitations.

When we ask ourselves what have the natural sciences, but also other disciplines, such as ecology, economy, sociology etc. in common, we find the following feature: Most of the objects studied by these sciences are themselves composed of many subsystems. All macroscopic bodies in physics are composed of many parts, down to the atoms. In chemical reactions great numbers of molecules participate, biological entities are composed of cells, or a society is composed of many individuals. Roughly speaking, there are two main methods to cope with complex systems. One method can be called the analytic one. It decomposes the systems more and more. A quite typical example for such a procedure is elementaryparticle physics but also molecular biology. The other way of approach is based on the following observation. In many cases the properties of a total system are not a mere superposition of the properties of the individual systems. Through the cooperation of subsystems new qualities of the total system are produced. Often these qualities cannot be even formulated by means of the subsystems alone. While a number of scientists feel that the analytic method leads to more "fundamental" results, prominent scientists, such as Phil Anderson [2], have convincingly stressed the vital role of the second approach, namely the study of cooperative effects. Indeed in many sciences the importance of the second approach is fully appreciated. In many cases both approaches are needed and represent intellectual challenges of equal difficulty.

There is, of course, by no means a need to found a new science just to stress the importance of cooperative effects. Indeed, in the realm of synergetics the existence of cooperative effects is just a prerequisite. The research object of synergetics is more ambitious asking questions like the following ones: Do systems show similar patterns of behavior in the large in spite of the fact that the systems themselves may be entirely different? Are there general principles governing col-

The word synergetics is composed of Greek words and means "science of cooperation". I introduced this term in my lecture in 1970 at Stuttgart University to characterize the research topic I am describing in my present article. See also [1]

lective effects irrespectively of the nature of the subsystems? Are there general theoretical methods to cope with these effects? This goal seems to be surprising, because the systems may be composed of elements as different as atoms, molecules, photons, cells, animals, computers, humans, etc. The ways these elements interact with each other are equally diverse. Nevertheless, over the past years large classes of systems belonging to quite different disciplines have been found which exhibit striking analogies in their macroscopic behavior. These analogies become visible when we adopt a certain level of abstraction. Interestingly enough, they are particularly pronounced when the (different) systems undergo dramatic macroscopic changes. But still other analogies have been discovered recently and I shall describe them below.

I shall try to put general concepts into the foreground and refer the reader interested in more technicalities to my book on synergetics [5] and recent literature [6 10]. One should bear in mind that the border line between general concepts and technicalities is by no means a rigid one but has to be shifted depending on the kind of problem treated. This border line may also depend on the individual scientist. Let me first try to specify possible answers to the goals mentioned above more clearly. First of all there is an example of a unifying concept referring to systems composed of many subsystems provided the subsystems act entirely independently of each other and the total action of the system is a sum of the actions of its parts. In this case one may apply the law of big numbers due to Gauss. This law makes precise statements on the total outcome in terms of the individual subsystems independently of their nature.

In this article we wish to address ourselves to the question whether there are general laws provided there is an *interaction between subsystems.* We shall see that the interaction of subsystems gives rise to structures. We shall not consider those structures as given but rather how those structures evolve in a self-organized way. This attitude is reminiscent of Darwinism in biology where Darwin produced a revolution of thought. Indeed, in this theory the animate world was conceived through its evolution.

At least in the natural sciences the spontaneous occurrence of order has been a puzzle to many scientists because it seems to be in conflict with fundamental laws of physics. According to thermodynamics, in a closed system disorder should become bigger and bigger. The detailed study of explicit examples of disorder-order transitions has revealed, however, that there is no contradiction to those fundamental laws. The best studied example is probably the laser, a novel light source, where the entirely disordered radiation of a lamp is replaced by an entirely coherent radiation. We know that the formation of patterns becomes possible because energy is pumped through these systems. These systems are *open.*

As we shall see below there are numerous disciplines in which self-organization processes take place and can be understood by principles I shall outline now.

Macroscopic Changes, Order Parameters, and Slaving

In many cases a system changes its macroscopic state when external conditions are changed. To quote some examples: In physics the energy or matter flux through a system may be altered. In economy innovations may be introduced. In ecology climate or pollution changes. In sociology, new opinions of people are introduced. At certain critical values of the parameter describing external conditions even a minor change of such a parameter can cause dramatic changes of the total system. In the above-mentioned examples from physics, for instance the disordered light of lamps is quite suddenly replaced by the entirely ordered laser light (compare Fig. 1). Similarly

Fig. 1. Self-organization of the laser. When the laser, a novel light source, is energetically pumped only weakly, it acts like a usual lamp. It emits a random sequence of light wave tracks $[(a)$, the electric field strength E of the light is plotted versus time $t]$. Above a critical pump strength, suddenly a single completely ordered (" coherent") light wave is emitted (b). This ordered wave is a manifestation of the self-organized ordered state of the light-emitting atoms of the laser. The field E serves as order parameter, as explained on p. 123

in economy a change of prices can cause a new product to appear or another one to disappear. We have found that in many cases the mechanism for the dramatic change of the systems behavior is as follows. Under given external conditions the individual parts of the system have found certain equilibrium configurations or stationary motions. These may be static configurations or oscillations of individual parts in physics or chemistry. But these configurations may equally well be certain attitudes of people of a society or certain stationary processes in economy. Such configurations or modes of action are usually stable against small perturbations imposed on the system. Or in other words, the individual parts of the system relax to their former state once the perturbation is removed, or they change their behavior only slightly when the perturbation persists. At the above-mentioned critical points of external parameters this stability property can get lost, however. In such a case the total system tries to find a new global configuration or a new kind of collective motion of its individual parts. The way in which the new macroscopic state is reached seems to be of a rather universal nature. On account of internal fluctuations the system tests different kinds of configurations or kinds of collective motion or processes or behavior patterns. We shall call these different kinds of configurations (in the widest sense indicated) "modes". Competition between such different kinds of modes sets in and eventually one or few kinds of modes win. As can be shown by a precise mathematical formulation the winners of this competition are able to entirely prescribe what the subsystems have to do. Thus by change of external parameters the system arranges itself in a way which can be described at two different levels. On the *macroscopic* level collective modes appear which define the order of the total system. The quantities describing these collective modes are called order parameters. Such order parameters can be material, such as the amplitude of a physical wave, but equally well immaterial, such as ideas or symbols describing certain configurations (modes). On the other hand once these order parameters are established they prescribe the actions of the subsystems or, using a terminus technicus, they *slave* the subsystems on the *microscopic* level. It should be noted that this terminology *does not imply* any *ethical statements.* Let me illustrate these statements by a few examples. When a baby is born it is subjected to the language. It learns the language and it is in this way *slaved* by the language. Similarly a member of a religious or ideological group is slaved by religion or ideology. I stress that these examples are chosen to make it quite clear that slaving only describes a certain relationship between a given order on a macroscopic scale and the behavior of an individual. The example of man and language shows a relationship between order parameters and slaved systems which is quite typical for all kinds of self-organization processes. While the language slaves the individuals it cannot exist without the latter ones. Order parameters and the behavior of individuals condition each other. The usual law of causality does not seem applicable. The distinction between order parameters and slaved systems implies an enormous reduction of the number of variables or degrees of freedom. Indeed, in many practical cases the number of order parameters is by many orders of magnitude smaller than the number of subsystems. This often allows us to describe the behavior of even complex systems in terms of a few variables, which in a number of cases can be rigorously treated using advanced methods of mathematics or theoretical physics.

The proper selection of order parameters appears to be a central issue when we have to deal with complex systems. Indeed in most complex systems the number of variables or individuals is so large that it is entirely impossible for any human being to cope with the enormous amount of information describing the detailed behavior of all the individual parts. Rather we are forced to select features of the system which describe it adequately. In quite a number of selforganizing systems such features can be found through the order parameter concepts and can be calculated by rigorous algorithms provided the corresponding science deals with measurable quantities. However, even if that is not the case we must find means to select relevant features. This is in particular true if the observer cannot handle all information describing the subsystems or if not all information is accessible. In such a case very often statistical methods including those of information theory must be used to make proper guesses about the behavior of ensembles of subsystems. Such situations occur, among others, when we deal with partially structured systems for which the behavior of a fluid may stand as an example. While it is structured on a macroscopic scale showing a honeycomb structure (Fig. 2), on the microscopic scale the general motion of the molecules is a random one being superimposed by small components giving rise to the macroscopic pattern. A warning should be added. The selection of relevant features need not always be unique; it may depend on the context or on the research objective. In this author's opinion the search for relevant features and adequate order parameters is a major task for future research in biology.

Let us briefly mention a further consequence of our above considerations. In earlier times, the structures occurring in nature, especially those of the animate

Fig. 2. Bénard cells in a fluid (photograph taken by Koschmieder). A layer of a fluid is heated from below. At a small temperature difference between the lower and upper boundary of the fluid, it remains macroscopically at rest. Above a critical, well-defined temperature difference a macroscopic motion of the fluid occurs. An example of the resulting pattern is shown. In the language of synergetics, three order parameters, each governing a plane wave, cooperate together to establish this pattern. Also quite different patterns can be formed by fluids depending on various circumstances. Fluctuations decide upon the orientation of patterns etc.

Fig. 3. The potential model. The position coordinate q of the ball symbolizes the size of the order parameter. (a) Ball in a valley with a single minimum, (b) deformation of the "mountains" indicating a change of external control parameters leads to two minima giving rise to symmetry breaking

world, were conceived as once given and static. In the next step this picture was replaced by conceiving structures as a result of the dynamics of evolution. In our times this concept is sometimes overstressed. Scientists are thinking mainly of *sequential* processes, one event following another one being caused by the former. However, in the realm of synergetics we have learned that processes may condition each other while going on in parallel. Such processes are now well known in physics (for instance the coherent emission of light waves by laser atoms) and they seem to play a role in morphogenesis and in pattern recognition. They are of increasing importance in the problem of parallel computing by computer networks and they have been stressed as a major process in the brain by Crick [11] quite recently.

Symmetry Breaking and Conflicts

As mentioned before there are numerous examples where close to "critical points" we can discuss the behavior of complex systems in terms of few variables or quantities, namely the order parameters. Even the most simple case of only one order parameter reveals striking analogies between different systems. One of the most important features is symmetry breaking. To explain it let us consider an example in which a ball may move in a valley (compare Fig. 3). In case (a) of Fig. 3 it can occupy only one equilibrium position. In case (b) the overall situation is still symmetric. When we mirror the right hand side of that figure into the left hand side and vice versa the whole figure does not change at all. Thus, the ball has now two equivalent equilibrium positions at its disposal, namely the left one *or* the right one. The symmetry is *broken* by the actual position the ball takes.

Over the past years it has become clear that symmetry breaking is a widespread feature in the behavior of complex systems including our own brain. This is easily substantiated by looking at Fig. 4. When we take the black spots as foreground we recognize devils, when looking at the white spots as foreground we recognize angels. Thus in this case our brain may exhibit in principle two different states giving us two different answers. In a way, and more generally speaking, pattern recognition can often be viewed as a sequence of symmetry-breaking events, where at each branching point new information is needed to break the symmetry, i.e., to make a unique decision possible. In the case of Fig. 4 this additional information was that the black spots (white spots) are the foreground. In the general, *a priori* "unsymmetric" case, such additional information is contained in the pattern itself. As just mentioned, the occurrence of two or even a multiplicity of new states is quite common

Fig. 4. Angels or devils? Symmetry breaking in perception. A drawing of the famous artist Escher

to self-organizing systems. When external parameters are changed and the old structure becomes unstable, the newly evolving structure has in general different choices among different ordered states, i.e., macroscopic structures. In such situations only fluctuations, specific initial conditions or additional instructions can make the decision which ordered state is eventually chosen.

The occurrence of entirely equivalent possibilities for self-organizing systems has far-reaching consequences. We are usually thinking of unique solutions. Here, however, we recognize that systems may show different kinds of answers to a given situation. Exactly the same phenomenon happens in fields of politics, economy and psychology, though there one does not speak of symmetry breaking but rather of conflict situations. Indeed we are confronted with problems which have two or more equivalent "solutions" one solution excluding the others. Each of these "solutions" has certain advantages but simultaneously disadvantages. In psychology or politics, these conflict situations lead to an undecidedness which induces an instability. Small groups with some preference of one solution may induce a large fraction of society to do exactly the same, breaking in this way the symmetry.

An Epistemologieal Speculation

Since I have published a series of papers dealing with applications of synergetics to the natural sciences I

should rather like to use the opportunity here to apply these thoughts to science itself. At a first glance, science seems to be self-stabilizing in certain wellknown ways. I wish to give just an example. A young scientist may find it difficult to publish an unconventional idea because a good deal of the referees are working along the traditional lines. Scientific disciplines are stabilized by their established methods and scientific language. Without such common means generally agreed upon scientists could not communicate with each other and there would be no science. Being a collective coherent endeavor, science becomes a "rigid" structure. But simultaneously, it gets accessible to viewing it with the eyes of synergetics. Through the ever developing new ideas and new experimental facts, science is an open system. Thus the old system of ideas may become unstable and is replaced by a new building of ideas which may or may not incorporate the former ideas or theories. The mechanism of scientific revolutions has been so beautifully elucidated by Thomas S. Kuhn [12] and need not be repeated here. In addition, the general scheme is well fitting into synergetics. But when we take our above general observations serious, we are led to a question which to the best of my knowledge has not been discussed yet in epistemology : can the evolution of a scientific field pass through a point where symmetry breaking (" bifurcation") occurs? Of course, there may be alternative but equivalent *formulations,* but let aside such rather obvious (and probably unimportant) cases. The question is rather whether there may be situations where scientists choose one line of thought which in a deep sense is different and even exclusive of another line of thought (or scientific approach). At a first glance we might think that "truth" is unique. But when dealing with really complex systems the "unique truth" might not always be so obvious and different schools of thought might develop, one eventually winning (though the loser had his equal virtues). I am fully aware of the fact that such statements are at present purely speculative, but may be worth to be taken into consideration at some instances.

Chaos

So far this article may have given the impression that synergetics deals merely with the occurrence of ordered states out of disordered states, where the order is prescribed by order parameters. Over the past years it has become more and more evident that in a number of important cases even systems governed by few order parameters can behave in a chaotic manner. The reason is that even few order parameters can themselves undergo a chaotic motion. "Chaotic" means that the behavior is quite irregular and a very small change of initial conditions can cause an entirely different pattern of behavior. Again it is interesting to note that such chaotic behavior can be found in quite different disciplines such as physics (fluids, lasers, solid-state physics), chemistry (Belousov-Zhabotinsky reaction), biology (insect populations). The occurrence of chaotic processes is by no means an esoteric issue. For instance, in metereology the question is being discussed, whether long- or even medium-term weather forecast is impossible *in principle.* Chaos can easily be caused by external controls exerted on self-organizing systems, as can be substantiated by experimental facts from physics and chemistry and by explicit models of processes. Such effects must certainly show up in other domains, for instance in management or in certain kinds of administrative regulations imposed on otherwise self-organizing and well-functioning institutions. To my great amazement theories of chaotic behavior in those fields as well as in economy seem to be lacking up to now.

A List of Examples of Synergetic Processes

In its beginning synergetics concentrated its attention on those cases in which the macroscopic pattern of behavior of a system changes dramatically. In physics we found new kinds of transitions called nonequilibrium phase transitions which exhibited pronounced analogies to phase transitions of systems in thermal equilibrium, such as freezing or boiling of water or the occurrence of ferromagnetism. Soon it turned out that such kind of dramatic changes occur in many other systems and I shall list a few of them. Some of these phenomena have been known since long (e.g., the Bénard instability), while others have been found recently. But all of them have in common that they are different manifestations of processes governed by universal principles, as outlined above. Lack of space does not allow me to go into details, but I hope the reader will get a feeling how (seemingly), diverse the phenomena and systems are which can be treated in a unified way. Readers interested in more details can find them in the literature to which [3-10] may serve as a guide.

The examples are as follows:

Physics [pattern formation of fluids, such as the convection and Taylor instabilities, certain cloud formations, some geological formations, certain plasma instabilities (see Fig. 5), lasers, rasers, parametric oscillators, Gunn oscillators and tunnel diodes in solidstate physics], astrophysics (patterns of star surfaces),

Fig. 5. Convection cells in a magneto-hydrodynamic (MHD) plasma. Plotted are lines of equal vertical velocity (after Haken and Klenk, to be published)

mechanical engineering (deformation and post-buckling of shells), chemistry (macroscopic temporal or spatio-temporal patterns, chemical networks). Biology is a vast field with many future application of these concepts. Some examples are morphogenesis (Fig. 6), some aspects of prebiotic evolution and population dynamics, neural networks, drug-induced hallucinations.

Further fields are:

Electrical engineering (oscillators of different kinds, coupled oscillators, networks), economy (competition between goods, monetary flows etc.), ecology (population dynamics, competition and cooperation of species), politics (formation of public opinion), epistemology (new theories, mechanisms of scientific revolutions), history (development of society caused by new developments, e.g., industrial).

Other disciplines with problems similar to those treated by synergetics comprise informatics (computer nets), psychology (conflict situations) and pattern recognition [cooperation of primitives (features)].

From the viewpoint of methodology, there is an equally rich variety of links between synergetics and other disciplines. Among those are general systems theory, dynamic systems theory, bifurcation theory, catastrophe theory, cybernetics (control theory), the theory of stochastic processes, irreversible thermodynamics, dissipative structures, phase transition theory and statistical mechanics. Again, it is beyond the scope of this article to discuss these links and I must refer the interested reader to my book [3] and my

Fig. 6. A model of morphogenesis has been developed by Gierer and Meinhardt [13]. These figures show the growth of the concentration of an "activator" substance, A , plotted over a two-dimensional ensemble of cells at various stages (after Haken and Olbrich [14] who solved the Gierer-Meinhardt equations by methods indicated in this article)

articles [7, 8]. A quite general statement can be made, however. While each of these disciplines shares important aspects with synergetics, each of them misses other aspects of at least equal importance.

Some Other Analogies

So far I have dealt with systems showing dramatic changes of their behavior when certain external parameters are changed. More recently it became apparent that further far-reaching analogies exist which will help to widen the concept of synergetics. In the realm of physics we now know that nonlinear wave propagation described by the soliton concept has a wide range of applications. Another convergence of ideas has recently been achieved linking the Eigen-Schuster hypercycle [15] with the theory of games of v. Neumann and Morgenstern. Here again two seemingly different fields come now together [16]. While the Eigen-Schuster theory of the hypercycle deals with the evolution of biomolecules and their selection due to competition, the theory of games deals with the behavior of individuals playing games against each other. Here one should understand the word game in a wide sense not only meaning, say card games, but also the behavior of individuals in economy, politics etc. When one translates the theory of certain classes of games (sumconserving games) into differential equations these equations have exactly the same structure as those of the hypercycle [16]. Incidentally, this relation underlines the idea of evolutionary games, presented so clearly by Eigen and Winkler [17].

Future Steps and Limits

Though it is always difficult to make predictions in science and though we are again and again surprised by unexpected developments, a few next steps in the field of synergetics are visible. It appears that a number of complex systems are describable in terms of hierarchies of order parameters, where a single order parameter may govern total processes which can be described, for instance, as subroutines. Biology is a vast field of possible applications but further steps must be done here. For instance we observe a continuous interplay between function and structure which condition each other. Structures seem to help to freeze in dynamic processes and thus to enable a system to stabilize itself, to learn etc. On the other hand, structures are the backbone of new kinds of functions. The methods of synergetics and its results shed also new light on the problem of reductionism. I think it has become quite clear that at each hierarchical level we need new concepts and we have to deal with new qualities.

Other new ideas to be developed are the interplay between different order parameters which implies switching between states of complex systems. In some cases, this interplay between order parameters can

best be described by the formalism of logics. Because order parameters have their properties irrespective of the nature of the slaved subsystems, quite different substrate systems (physical, chemical, biological etc.) can perform the same logical processes. Another possibility will be a combination of order parameter concepts with aspects of the theory of games, where the "players" are the order parameters.

What is the range of applicability and what are the limitations of the concepts (and the corresponding mathematical tools) I have sketched above? In physics, chemistry and other sciences, where we deal with measurable quantities, we have for quite a number of cases concepts and algorithms (order parameters and slaving) which allow us to predict the behavior of the systems quantitatively. In the "soft" sciences, I think these concepts are still valid though the algorithms will be lacking.

When dealing with theories of large ranges of applicability one may ask about the value of such a theory reminding oneself of a basic law of logics : The greater the range of applicability of a statement the smaller is its logical content. Thus we may raise the question whether we don't lose too much when trying to find general principles for self-organization.

The possible answers to this question depend in a profound way on our own attitude towards science and, more generally, towards "reality". Are we interested in deep-lying unifying principles or in the equally fascinating diversity? Let me give two examples. In physics, as we know, all the atoms of matter consist of electrons and nuclei which in turn consist of protons and neutrons. It is clear that a statement of such a kind gives us an enormous insight into the structure of the world *because* of the generality of this statement. On the other hand, the animate world provides one with an enormous richness of phenomena and at least some biologists see the study of this variety as a main objective of biology. Undoubtedly, synergetics belongs to the first class.

We now know of well-defined common features of certain classes of phenomena. The exploration of other classes will be an exciting future field of research. In this search, the notion of classes will play an important role, when we try to define equivalent behavior of different systems. There are deep-lying mathematical theorems by Goedel telling us that there is no *universal* approach which should permit us to decide by a finite number of steps, whether say, two processes are equivalent in a certain sense. But problems of equivalence can be solved when we restrict ourselves to certain classes.

But even within each class, there is a price to be paid for synergetics. Namely, the similarity of behavior patterns of quite different systems implies that, at least in general, we cannot draw unique conclusions for elementary mechanisms from observed macroscopic phenomena. Thus in this sense synergetics will not and never can replace individual sciences. On the other hand, it gives us a means at hand to cope with complex systems and to learn from one field for another one. To say the least, in complex problems it can give us hints of how to find out the essential features and to make guesses.

The growing interest in synergetics is reflected by the fact that book series and periodicals are now dealing with it. In particular, the Springer Series on Synergetics and the new journal "Nonlinear Phenomena" (North-Holland), stressing nonlinearity as the common feature, should be mentioned.

From my above article it hopefully transpires that we presently observe a remarkable convergence of ideas when dealing with self-organizing systems in fields ranging from mathematics to business management. On the other hand, we must be careful to keep our mind open to look for new concepts which may draw a still quite unexpected, more general, and more beautiful picture of these processes. I hope that synergetics is at least a first step towards that goal.

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Received November 12, 1979