SYSTEM OPTIMIZATION IN MULTITEST LINEAR PROGRAMMING PROBLEMS

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STATEMENT OF THE PROBLEM

A majority of planning and economic problems are formulated as problems of mathematical programming with a vectorial and not scalar goal function [1, 2]. The principal difficulties with vector optimization problems are usually associated with ambiguous choice of a unique solution; they are surmounted by introducing additional information from the decision maker (DM) [3], calling for the construction of man–machine procedures.

Such procedures are based on a variety of techniques of consecutive refinement of evaluation of the relative importance of individual criteria with the objective of restricting the region of acceptable decisions to one decision that would possess the values of the test functions that are desirable for DM [4]. It is assumed that at each step of the man–machine procedure the region of acceptable decisions remains invariable – so that it is necessary for DM to modify his preferences on the set of test functions.

If the region of acceptable solutions of vector optimization problems may vary, the search for a unique solution that would have the test values desirable for DM can be effected by using the notions of system optimization [1].

We will consider a system optimization procedure with particular reference to a multitest linear programming problem based on man–machine procedures presented in [4].

Let a set of linear goal functions

$$f=\{f_i(x),\ i\in I=\{1,\dots,M\}\},\ \text{where}$$
$$f_i(x)=c_1^i\cdot x_1+\dots+c_j^i\cdot x_j+\dots+c_n^i\cdot x_n, \tag{1}$$

be defined, as well as the set of acceptable decisions determined by the region

$$D_0=\left\{x: g_i(x)=\sum_{j=1}^{n} a_{ij}^0\cdot x_j\leqslant b_i^0,\quad i\in Q=\{1,\dots,m\},\ \ x_j\geqslant 0,\ j=1,\dots,n\right\}. \tag{2}$$

We assume that all functions of the set f can be maximized in the region D_0.

The parameters a_{ij}^0, b_i^0, $i \in Q$, $j = 1, \dots, n$, which characterize region D_0, can vary by quantities Δa_{ij}, Δb_i, to which, during the course of solution, additional restrictions are imposed that describe the limited region P_0 and which are defined on the basis of the technological capacities of the planning or economic problem under consideration.

We suppose that the decision maker (the planner or designer), in accordance with the philosophy of Displan [2], defines the solution that is desirable for him, either via the plan x^* with the values of test functions $f_i^* = f_i(x^*)$, $i \in I$, or solely via the desirable values of test functions $f^* = \{f_i^*,\ i \in I\}$ without specifying the corresponding plan x^*. It is natural to assume here that x^* does not belong to the initial region of acceptable decisions D_0.

According to the methodology of system optimization [1], we must construct a new region D_1 in accordance with the originally defined domain of variation of parameters P_0 such that either the point x^* be immediately a solution of the vector optimization problem (designated as x^k), i.e., $x^* = x^k$, or a solution x^k exists on the region D_1 for which $f_i(x^k) \geq f_i^*$, $i \in I$, where at least one inequality is strict.

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METHODS OF CONSTRUCTION OF A NEW REGION OF ACCEPTABLE SOLUTIONS OF THE INITIAL PROBLEM WITH DM SPECIFYING THE DESIRABLE PLAN

According to [3, 5], definition of the desirable plan x^* determines the direction of the search for the compromise solution of the multitest optimization problem, which is defined by the vector $\rho^* = \{\rho_i^*, i \in I\}$, where

$$\rho_i^* = \Big(\prod_{\substack{j \neq i \\ j \in I}} w_j^*\Big) \Big/ \Big(\sum_{q \in I} \prod_{j \in I} w_j^*\Big). \tag{3}$$

In formula (3), w_j^* denote the components of the point w_i^*, $i \in I$ defined by the chosen transformations of tests $w_i(x) = (f_j^0 - f_i(x))/(f_i^0 - f_{i(\min)})$ for $f_i(x) = f_i(x^*)$, $i \in I$, where f_i^0 and $f_{i(\min)}$ designate the best and worst values of tests on the region D_0. Let x_0^k be an effective solution of the problem of multitest optimization (1)-(2) for a given vector $\rho = \rho^*$, found as a result of solution of the problem

$$\min_{x \in D_0} \max_{i \in I} \rho_i \cdot w_i(x). \tag{4}$$

Since the desirable plan x^* does not belong to the region D_0, we assume that

$$f_i^* = f_i(x^*) \geqslant f_i(x_0^k),\ i \in I, \tag{5}$$

and at least one of the inequalities is strict. In that case, we will say that x^* is "better" than x_0^k, and write $x^* > x_0^k$. In the opposite case, the value of tests for the plan x_0^k is better than for x^* ($x_0^k > x^*$) according to [4], and there is no need to construct the region D_1.

We isolate the numbers of restrictions (2) that are violated by substitution $x = x^*$ and denote the set of these restrictions by Q_0.

We denote $g_i'(x) = (a_{i1}^0, \ldots, a_{in}^0)$, $i \in Q$, and $f_i'(x) = (c_{i1}, \ldots, c_{in})$, $i \in I$, respectively, the gradients of restrictions (2) and test functions of the set f at a certain point x.

<u>THEOREM 1</u>. When $x^* > x_0^k$, the acceptable solutions of system (2) lying on hyperplanes with numbers from sets Q_0 are effective solutions with respect to the set of tests f.

<u>Proof.</u> Suppose the contrary: that the acceptable plan x_0' lying on the hyperplanes with numbers from the set $Q_0(x_0') \subset Q_0$ is ineffective, where $Q_0(x_0')$ are numbers of restrictions that become equality at $x = x_0'$.

Since $x^* > x_0^k$, i.e., $f_i(x^*) \geq f_i(x_0^k)$, $i \in I$ and at least one inequality is strict and the functions $f_i(x)$ are linear, then $(f_i'(x_0^k), x_0^k - x^*) = f_i(x_0^k) - f_i(x^*) \leq 0$. This means that the vector $S = (x_0^k - x^*) = (x_{01}^k - x_1^*, \ldots, x_{0n}^k - x_n^*)$ defines the direction in which the values of the tests of the set f can be improved. In the linear case, this direction is retained at each acceptable point of the region D_0. Therefore, in the direction S_1 that emerges from point x_0' and is parallel to and has the same direction as the vector S, there must exist an effective solution $\tilde{x}$ such that $\tilde{x} > x_0'$. Then, the direction S_1 will have the form $S_1 = (x_0' - \tilde{x}) = (x_{01}' - \tilde{x}_1, \ldots, x_{0n}' - \tilde{x}_n)$ and $S_1 = \beta S$, where β is a nonnegative number. Hence, the scalar products $(g_i'(x), S_1) = \beta(g_i'(x), S)$, $i \in Q_0(x_0')$.

Taking into account the representations of the vectors $g_i'(x)$, S, S_1, the scalar products in the right- and left-hand sides (rhs and $l$hs) of the latter equalities are defined as follows:

$$(g_i'(x), S_1) = \sum_{j=1}^{n} a_{ij}^0 \cdot x_{ij}' - \sum_{j=1}^{n} a_{ij}^0 \cdot \tilde{x}_j,\ i \in Q_0(x_0');$$

$$(g_i'(x), S) = \sum_{j=1}^{n} a_{ij}^0 \cdot x_{0j}^k - \sum_{j=1}^{n} a_{ij}^0 \cdot x_j^*,\ i \in Q_0(x_0').$$

Since

$$\sum_{j=1}^{n} a_{ij}^0 \cdot x_{0j}' = b_i^0,$$

$$\sum_{j=1}^{n} a_{ij}^0 \cdot \tilde{x}_j \leqslant b_i^0,\ i \in Q_0(x_0'),$$

then

$$(g_i'(x), S_1) \geqslant 0,\ i \in Q_0(x_0'),$$

and since

$$\sum_{j=1}^{n} a_{ij}^{0} \cdot x_{0j}^{k} \leqslant b_i^0,$$

$$\sum_{j=1}^{n} a_{ij}^{0} \cdot x_j^{*} > b_i^0, \quad i \in Q_0(x_0'),$$

then

$$(g_i'(x),\ S) \geqslant 0, \quad i \in Q_0(x_0').$$

Then,

$$(g_i'(x),\ S_1) \neq \beta (g_i'(x),\ S), \quad i \in Q_0(x_0') \quad \text{for} \quad \beta \geqslant 0,$$

which contradicts the identical direction of vectors S and S_1. This in turn, disagrees with the retention of the direction of improved values of tests at each admissible point D_0, which proves the theorem.

Thus, in order for the plan x* to become an acceptable solution, we must modify the region of problem solutions D_0 by altering the restrictions of the set Q_0, since the latter lead to improved values of tests of the set f. We impose on the variations of parameters Δa_{pj} and Δb_p of the restrictions of the set Q_0 the following conditions:

$$\sum_{j=1}^{n} x_j^{*} \cdot \Delta a_{pj} - \Delta b_p \leqslant b_p^0 - \sum_{j=1}^{n} a_{pj}^{0} \cdot x_j^{*}, \quad p \in Q_0; \tag{6}$$

$$\begin{aligned} &\Delta b_p > -b_p^0, \quad \text{if} \quad b_p^0 > 0, \ p \in Q_0; \\ &\Delta b_p < |b_p^0|, \quad \text{if} \quad b_p^0 < 0, \ p \in Q_0; \\ &\Delta a_{pj} > -a_{pj}^0, \quad \text{if} \quad a_{pj}^0 > 0, \ j = 1, \ldots, n, \ p \in Q_0; \\ &\Delta a_{pj} < |a_{pj}^0|, \quad \text{if} \quad a_{pj}^0 < 0, \ j = 1, \ldots, n, \ p \in Q_0. \end{aligned} \tag{7}$$

Let us denote the variation domain of the parameters of restrictions of the set Q_0 described by (6) and (7) by P. It can be readily seen that the variation domain of the parameters P is unlimited and can have an infinite set of solutions. The choice of restrictions (6) and (7) to this domain is associated with the fact that restrictions (6) enable the point x* to become acceptable in the new region D_1 (x* in that region will satisfy the restrictions of Q_0, while restrictions Q/Q_0 are satisfied by it by condition), and restrictions (7) are necessary for the traces of hyperplanes on the axes x_j, j = 1, . . . , n in the space R^n to remain on the same axes (so that the physical sense of the restrictions is not violated). For finding the parameters Δa_{pj}, Δb_p, j = 1, . . . , n, $p \in Q_0$, we construct the intersection of the regions P_0 and P.

If $P \cap P_0 \neq \varnothing$, then the variation domain of the parameters of the model will be limited, and this makes it possible to solve the problem of constructing the new region D_1 in which the point x* will already be acceptable. If, however, $P \cap P_0 = \varnothing$, in that case one has either to modify restrictions P_0 (i.e., technology), or select a new desirable point x*.

In order to find the parameters Δa_{pj}, Δb_p, j = 1, . . . , n, $p \in Q_0$, we formulate an additional optimization problem (as shown in [1]) in which it is allowed to select as tests the costs involved in modification of model parameters: $c(\Delta\bar{a},\ \Delta\bar{b})$, where $\Delta\bar{a} = \{\Delta a_{pj}, p \in Q_0, j = 1, \ldots, n\}$, $\Delta\bar{b} = \{\Delta b_p,\ p \in Q_0\}$. Then the problem of choice of parameters $\Delta\bar{a}$ and $\Delta\bar{b}$ is reduced to optimization problem

$$\min_{\Delta\bar{a}, \Delta\bar{b}} c(\Delta\bar{a},\ \Delta\bar{b}),\ \Delta\bar{a},\ \Delta\bar{b} \in P \cap P_0. \tag{8}$$

If it is impossible to construct the cost function, the problem of finding the parameters of the new region can be formulated as a multitest problem; in such a problem, each parameter appears as an individual test which, depending on the physical sense of the parameter, can be maximized or minimized. We denote by

$$f^{(p)} = \{\Delta a_{lj},\ \Delta b_p,\ l \in Q_0^{(a)},\ j \in J_{(l)},\ p \in Q_0^{(b)}\}$$

the set of tests formed by parameters, the optimization of which involves construction of a new region ($Q_0^{(a)}$ being the set of numbers of restrictions in which these parameters occur; and $Q_0^{(b)}$, $J_{(l)}$ being the sets of numbers of parameters of rhs and $l$hs for the l-th restriction, respectively). Then, the problem of choice of parameters $\Delta\bar{a}$ and $\Delta\bar{b}$ is reduced to that of multitest optimization with respect to the set of tests

$$f^{(p)} = \{\Delta a_{lj},\ \Delta b_p,\ l \in Q_0^{(a)},\ j \in J_{(l)},\ p \in Q_0^{(b)}\} \tag{9}$$

subject to restrictions $\Delta\bar{a},\ \Delta\bar{b} \in P \cap P_0$.

We denote by

$$a'_{pj} = a^0_{pj} + \Delta a_{pj},\ b'_p = b^0_p + \Delta b_p,\ p \in Q_0,\ j = 1, \ldots, n, \tag{10}$$

the new values of parameters for restrictions of the set Q_0, where Δa_{pj}, Δb_p, $p \in Q_0$, $j = 1, \ldots, n$ are found by solution of problems (8) or (9). The new region of acceptable solutions D_1 will appear as

$$D_1 = \left\{x: \sum_{j=1}^{n} a'_{pj} \cdot x_j \leqslant b'_p,\ p \in Q_0,\ \sum_{j=1}^{n} a^0_{pj} \cdot x_j \leqslant b^0_p,\ p \in Q/Q_0,\ x_j \geqslant 0,\ j = 1, \ldots, n\right\}. \tag{11}$$

Statement 1. If among restrictions of the region D_0 are such that $a^0_{ij} > 0$, $j = 1, \ldots, n$, $b^0_i > 0$, the region D_1 is limited and closed.

In effect, let us construct the parallelepiped $\Pi = \{x : 0 \leq x_j \leq B,\ j = 1, \ldots, n\}$, where B is a positive constant. Then, choosing the value

$$B = \max_{i \in Q_>} \left\{ \max_{j=1,\ldots,n} \left\{\frac{b^0_i}{a^0_{ij}}\right\},\ \max_{j=1,\ldots,n} \left\{\frac{b^0_i + \Delta b_i}{a^0_{ij} + \Delta a_{ij}}\right\}\right\},$$

if $Q_> \cap Q_0 \neq \varnothing$, $Q_>$ is the set of indices of restrictions for which $a^0_{ij} > 0$, $j = 1, \ldots, n$, $b^0_i > 0$, either

$$B = \max_{i \in Q_>} \left\{ \max_{j=1,\ldots,n} \left\{\frac{b^0_i}{a^0_{ij}}\right\}\right\},$$

if $Q_> \cap Q_0 = \varnothing$, or

$$B = \max_{i \in Q_>} \left\{ \max_{j=1,\ldots,n} \left\{\frac{b^0_i + \Delta b_i}{a^0_{ij} + \Delta a_{ij}}\right\}\right\},$$

if $Q_> \cap Q_0 = Q_0$, we obtain that the region $D_1 \subset \Pi$. The statement is thus proven.

If the set $Q_>$ is empty, and the region D_0 is closed and limited, then, for constructing the region D_1 that would also be closed and limited, we would proceed as follows: We construct the inequality $\sum_{j=1}^{n} x_j - b_c < 0$, which is a consequence of the system (2) [6]. The value b_c is found as $\max_{x \in D_0} \sum_{j=1}^{n} x_j$. In that case, the new region of acceptable solutions is defined in the following manner:

$$D_1 = \left\{x: \sum_{j=1}^{n} a'_{cj} x_j \leqslant b'_c,\ \sum_{j=1}^{n} a'_{pj} x_j \leqslant b'_p,\ p \in Q_0,\ \sum_{j=1}^{n} a^0_{pj} \cdot x_j \leqslant b^0_p,\ p \in Q/Q_0,\ x_j \geqslant 0,\ j = 1, \ldots, n\right\}, \tag{12}$$

where $a'_{cj} = 1 + \Delta a_{cj}$, $b'_c = b_c + \Delta b_c$, $j = 1, \ldots, n$.

The variation values of parameters Δa_{pj}, Δb_p, Δa_{cj}, Δb_c, $p \in Q_0$, $j = 1, \ldots, n$ are defined from the region P_c, which is constructed by adding to the region P the inequalities

$$\begin{gathered} \sum_{j=1}^{n} \Delta a_{cj} x^*_j - \Delta b_c \leqslant b_c - \sum_{j=1}^{n} x^*_j,\ j = 1, \ldots, n, \\ \Delta a_{cj} > -1, \quad j = 1, \ldots, n, \\ \Delta b_c > -b_c. \end{gathered} \tag{13}$$

In this case, the parameters of the region D_1, described by (12), are found at the solution of problem of the form (8) or (9), considering that they belong to the region that is described by the intersection of P_c and P_0. It is readily seen that the region D_1 constructed in this fashion will be closed and limited by virtue of Statement 1 (since it contains one restriction which has all positive coefficients).

Since the new region of acceptable solutions D_1 is closed and limited, there exist optimal solutions $x_{i(1)}$, $i \in I$ on this region for each of the tests of (1) with values $f^0_{i(1)}$, $i \in I$ and also effective solutions of problems (1), (12).

Statement 2. In the region D_1, there exists a solution $x_{(1)}^{(k)}$ for which

$$f_i(x_{(1)}^k) \geqslant f_i(x^*), \qquad i \in I.$$

An instance of such a solution is, in particular

$$x_{(1)}^k = \arg\min_{x \in D_1} \max_{i \in I} \rho_{i(1)}^* \cdot w_{i(1)}(f_i(x)), \tag{14}$$

where $w_{i(1)}(f_i(x))$, $i \in I$, are constructed in accordance with the region D_1 and $\rho_{i(1)}^*$, $i \in I$, is defined by formula (3) for this region.

Proof. Suppose the contrary: The point $x_{(1)}^k$ will not be better than x^*, i.e., $f_i(x^*) \geq f_i(x_{(1)}^k)$, $i \in I$, and at least one inequality is strict. Then, since $x^* \in D_1$, then $x^* > x_{(1)}^k$, which contradicts the effectiveness of $x_{(1)}^k$. This proves the statement.

The approach presented above thus allows one to construct a new region D_1 in which the desirable plan will be acceptable, which ensures the existence of solution of the problem of multitest optimization on the new region with values of test functions that are not worse than the required ones. The ambiguity of construction of such a region is eliminated by solution of additional optimization problems with choosing the necessary variations of parameters of the initial region.

METHOD OF CONSTRUCTION OF A NEW REGION OF ACCEPTABLE SOLUTIONS OF THE INITIAL PROBLEM WITH DM SPECIFYING DESIRABLE VALUES OF TEST FUNCTIONS

Suppose that DM has specified the desirable values of the test functions defined by the set $f^* = \{f_i^*,\ i \in I\}$. We will assume that $f_i^* \in [f_{i(\min)}, f_i^0]$, $i \in I$, since in the opposite case it would be necessary initially to modify the region of acceptable solutions D_0 so that the maximum value of such a test on the new region would be greater than f_i^*, $i \in I$. Suppose also, for the desired values of tests, that the inequalities (5) are fulfilled, i.e., the point $w^* = \{w_i^* = (f_i^0 - f_i^*)/(f_i^0 - f_{i(\min)}),\ i \in I\}$ in the space of transformed test values does not belong to the region W_{D_0} of the values of modified tests for acceptable solutions D_0. Therefore, as in the first case, there is a need to modify the model parameters so that they satisfy the given values of goal functions.

To this end, we will first establish compatibility of the system of inequalities

$$\sum_{j=1}^{n} c_j^i \cdot x_j \geqslant f_i^*, \qquad i \in I. \tag{15}$$

If the system is compatible, it is possible to find its acceptable solution x^*. Since $x^* \notin D_0$, it is possible to repeat with respect to x^* the foregoing procedure of modification of the region D_0. If such a region can be constructed, then in the new region D_1 there will always exist a solution $x_{(1)}^k$ of the initial multitest problem for which $f_i(x_{(i)}^k) \geq f_i^*$, $i \in I$. When selecting x^* that satisfies (15), we are not concerned with obtaining any solution, but only with one that would be best in terms of solution of the multitest problem. To this end, we proceed as follows. We define arbitrary variation limits of each variable, i.e., $x_j \in [0, h_j]$, where h_j is a sufficiently large arbitrary number. First we apply to these ranges the reduction procedure that is analogous to procedure of elimination of solutions of discrete optimization problems described in [7, 8]. Let at the k-th step of application of this procedure the following variation limits for the variables be obtained:

$$h_{j(H)}^{(k)} \leqslant x_j \leqslant h_{j(B)}^{(k)}, \qquad j = 1, \ldots, n. \tag{16}$$

The variation limits of variables at the (k + 1)-th step are defined, then, as follows:

$$h_{j(H)}^{(k+1)} = \max\left\{\max_{i \in J_j^+}\left\{\frac{1}{c_j^i}\left(f_i^* - \sum_{\substack{l \in J_i^+ \\ l \neq j}} c_l^i \cdot h_{l(B)}^{(k)} - \sum_{\substack{l \in J_i^- \\ l \neq j}} c_l^i \cdot h_{l(H)}^{(k)}\right)\right\}, \quad \text{if} \quad c_j^i > 0,\ h_{j(H)}^{(k)}\right\},$$

$$h_{j(B)}^{(k+1)} = \min\left\{\min_{i \in J_j^-}\left\{\frac{1}{c_j^i}\left(f_i^* - \sum_{\substack{l \in J_i^+ \\ l \neq j}} c_l^i \cdot h_{l(B)}^{(k)} - \sum_{\substack{l \in J_i^- \\ l \neq j}} c_l^i \cdot h_{l(H)}^{(k)}\right)\right\}, \quad \text{if} \quad c_j^i < 0,\ h_{j(B)}^{(k)}\right\}, \tag{17}$$

$$J_j^+ = \{i / c_j^i > 0\}, \quad J_j^- = \{i / c_j^i < 0\},$$

where J_i^+ and J_i^- designate the sets of numbers of variables for which $c_j^i > 0$ and $c_j^i < 0$, respectively. At the zero step $h_{j(H)}^{(0)} = 0$, $h_{j(B)}^{(0)} = h_j$. The procedure stops when $\varepsilon = \max_j \{h_{j(H)}^{(k+1)} - h_{j(H)}^{(k)}, h_{j(B)}^{(k)} - h_{j(B)}^{(k+1)}\}$ is sufficiently small. We designate by l the number of the step of the application of the procedure and by $\Pi^{(l)} = \prod_{j=1}^{n} [h_{j(H)}^{l}, h_{j(B)}^{l}]$ the parallelepiped constructed by variation ranges of variables at the l-th step. Here, if $h_{j(B)}^{(l)} = h_j$, then the region of variables described by inequalities (15) is unlimited. If the region (15) is closed and limited, then, according to [8], the procedure will eliminate no single acceptable solution of inequalities system (15). In particular, if $\Pi^{(l)} = \emptyset$, i.e., $\exists i \in I$,

$$\max_{x \in \Pi^{(l)}} \sum_{j=1}^{n} c_j^i \cdot x_j < f_i^*, \tag{18}$$

then system (15) is incompatible.

We denote:

$$x^{*(k)} = \arg \min_{x \in \Pi^{(l)}} \max_{i \in I} \rho_i^{*(\Pi)} \cdot \hat{\omega}_i^{(\Pi)}(x), \tag{19}$$

if conditions (18) are not met, where $\hat{w}_j^{(\Pi)}(x)$, $i \in I$, are the earlier introduced transformations calculated for the parallelepiped $\Pi^{(l)}$ and $\rho_i^{*(\Pi)}$, $i \in I$, are weights determined by (3) for the point f^* in the space of functions $\hat{w}_i^{(\Pi)}(x)$, $i \in I$.

Statement 3. If

$$f_i(x^{*(k)}) \leqslant f_i^*, \qquad i \in I, \tag{20}$$

and at least one of the inequalities is strict, system (15) is incompatible.

Proof. Let us suppose the opposite: Inequalities (20) are met and system (15) is compatible. Let the plan $\tilde{x}$ satisfy inequalities (15). We denote: $\tilde{k}_0 = \max_{i \in I} \rho_i^{(\Pi)} \cdot \hat{\omega}_i^{(\Pi)}(\tilde{x})$; $k_0^{*(k)} = \max_{i \in I} \rho_i^{*(\Pi)} = \hat{\omega}_i^{(\Pi)}(x^{*(k)})$. If $\tilde{k}_0 \leq k_0^*$, this contradicts that $x^{*(k)}$ is a unique solution of problem (19). Let $\tilde{k}_0 \geq k_0^{*(k)}$; then $\tilde{k}_0 > k_0^*$, since $k_0^{*(k)} > k_0^*$ [here $k_0^* = \rho_i^{*(\Pi)} \cdot \hat{w}_i^{(\Pi)}(f_i^*)$, $i \in I$]. Considering that $\tilde{k}_0 > k_0^*$, we obtain:

$$\rho_i^{*(\Pi)} \cdot \hat{\omega}_i^{(\Pi)}(\tilde{x}) > \rho_i^{*(\Pi)} \cdot \hat{\omega}_i^{(\Pi)}(f_i^*), \qquad i \in I.$$

Hence, $f_i(\tilde{x}) \in f_i^*$, $i \in I$, which contradicts the assumption that $\tilde{x}$ satisfies inequalities (20); this proves the statement.

If it is ascertained, therefore, that system of inequalities (15) is incompatible, then, as in the previous case, there is a need for system optimization in controlling the model of goal functions $f = \{f_i(x) = c^i \cdot x\}$, $i \in I$. For the point with respect to which this system optimization problem should be solved, it is convenient to take the point $x^{*(k)}$, since it is the best point on the parallelepiped $\Pi^{(l)}$ for the set of criteria f and for the preference $\rho^{*(k)}$, specified by DM and defined by the point $f^* = \{f_i^*, i \in I\}$. We denote by I^0 the set of numbers of inequalities (15) which are not fulfilled at point $x^{*(k)}$. The variation region of the coefficients of the goal functions of the set I^0 and of the values of goal functions desirable for DM will be defined, on analogy with the region (6), (7), in the following manner

$$\sum_{j=1}^{n} \Delta c_j^i \cdot x_j^{*(l)} - \Delta f_i^* \geqslant f_i^* - \sum_{j=1}^{n} c_j^i \cdot x_j^{*(l)}, \quad i \in I^0; \tag{21}$$

$$\begin{aligned} &\Delta c_j^i > -c_j^i, \ \text{if} \quad c_j^i > 0, \qquad j \in J_i^+, \ i \in I^0; \\ &\Delta c_j^i < |c_j^i|, \ \text{if} \quad c_j^i < 0, \qquad j \in J_i^-, \ i \in I^0; \\ &\Delta f_i^* > -f_i^*, \qquad i \in I^0. \end{aligned} \tag{22}$$

We denote by P^c the variation range of coefficients c_j^i, f_i^*, $j = 1, \ldots, n$, $i \in I^0$ described by (21) and (22) and by P_0^c their variation range as defined proceeding from physical considerations. Then, if $P^c \cap P_0^c \neq \emptyset$, one can, in order to find the values Δc_j^i and Δf_i^*, state problems analogous to (8) or (9). In that case, we are not concerned that the region described by inequalities (15) be closed and limited – here we are only concerned with ensuring that the inequalities be compatible. At $P^c \cap P_0^c = \emptyset$, it is necessary to modify the region P_0^c if $h_{j(B)}^{(l)} \neq h_j$ at least for one j, $j = 1, \ldots, n$, or find a new point $\tilde{x}^{*(k)} \in \Pi^{(l)}$ that would be worse than $x^{*(k)}$ in terms of the given set of tests f and preference $\rho^{*(\Pi)}$.

If the inequalities of system (15) is compatible, or if we have modified the goal functions model in such a manner that point $x^{*(k)}$ satisfies (15), it is possible to solve the problem of system optimization in modifying the region D_0 with respect to the point $x^{*(k)}$ as described above. If when constructing the new region with respect to point $x^{*(k)}$ it turns out that $P \cap P_0 = \varnothing$, in that case one must also either find a new point $\tilde{x}^{*(k)} \in \Pi^{(l)}$ or modify the region P_0.

In conclusion, the approach described above allows one to construct a formalized scheme of modification of the region of acceptable problem solutions and organize the man–machine procedure of solution searching in multitest linear programming problems without modification of the original preference defined by DM on the set of tests.

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INVERSE FIBONACCI TRANSFORMATION

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The finding of procedures inverse to a given one is an essential aspect of contemporary applied mathematics. One case in point is the use of inverse Fourier transformations. In this paper, we investigate the inverse Fibonacci transformation, which consists in finding, by a given number, a minimal base and number k such that the given number is the k-th term in the Fibonacci series generated by the base. The procedure proposed here could be used in various data processing systems that utilize Fibonacci numbers.

Let a_0 and a_1 be two arbitrary integers such that $0 \le a_0 < a_1$. The series $\{\Phi_i(a_0, a_1)\}_{i \ge 0}$, where $\Phi_0(a_0, a_1) = a_0$, $\Phi_1(a_0, a_1) = a_1$ and for $i \ge 0$, $\Phi_{i+2}(a_0, a_1) = \Phi_{i+1}(a_0, a_1) + \Phi_i(a_0, a_1)$ will be called the Fibonacci series with base $\langle a_0, a_1 \rangle$. We denote by Φ_i the $(i + 1)$-th element of a Fibonacci series with the base $\langle 0, 1 \rangle$ (the i-th element of an ordinary series with the base $\langle 1, 1 \rangle$).

We will say that the base $\langle a_0, a_1 \rangle$ represents the number m if there exists an integer $k \ge 0$ such that $m = \Phi_k(a_0, a_1)$. For an arbitrary natural number m, there exists a finite set B(m) of bases representing the number m. We will define on B(m) the relation of order $<_m$, setting for $\langle a_0, a_1 \rangle$, $\langle b_0, b_1 \rangle \in B(m)$

$$\langle a_0, a_1 \rangle <_m \langle b_0, b_1 \rangle \Leftrightarrow a_1 < b_1.$$

Note that if $\langle a_0, a_1 \rangle$, $\langle b_0, b_1 \rangle$ are two different bases from B(m), then $a_1 \ne b_1$.

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