Journal of Regulatory Economics; 5:233-249 (1993)

Option Value of Emission Allowances

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Abstract
We study the market for emission allowances stipulated in the 1990 Clean Air Act Amendment. We assume that the number of allowances is fixed and that demand is affected by a stochastic parameter that follows a Wiener process ('Brownian motion'). The optimal investment policy for scrubbers is characterized. Investments in scrubbers are reduced if there is greater uncertainty about future market conditions. This is because purchases of emission allowances provide flexibility to adapt to demand conditions in a way that installing scrubbers does not. The price of emission allowances may therefore exceed the marginal cost of scrubbers by an amount called the option value. We derive an explicit formula for the option value and present computational results to illustrate its likely magnitude.

1. Introduction

Under the provisions of the 1990 Clean Air Act Amendment, electric utilities in the United States can emit specified pollutants (mainly sulfur dioxide) only if they obtain sufficient 'emission allowances' to cover the amount emitted. Some allowances are provided to each utility as an endowment and others can be purchased in auctions. In addition, allowances can be exchanged; consequently, a market will be established in 1993, including a market for futures contracts conducted by the Chicago Board of Trade. Various aspects of the market have been investigated by Hahn (1984), Hahn and Hester (1989), and Tietenberg (1985) using a static framework. Our purpose is to develop a model that can help utilities interpret the dynamic behavior of prices in this market. We show that these prices include an option value representing the value of flexibility as compared to the alternative of investing irreversibly in immobile durable equipment that permanently reduces the quantity of emissions.

Each utility faces a basic choice between buying (or selling) allowances, and alternative technologies to reduce emissions. Several alternatives are described by Niemeyer (1990):

- **Alternative Fuels.** Replacement of high-sulfur coal with low-sulfur coal is one of several possibilities for fuel substitution. The fuel market is expected to adjust quickly so that price premia for low-emission fuels reflect the imputed savings in the price of emission allowances, as well as differences in extraction and transport costs.

- **Alternative Technologies.** The cost of emissions derived from the price of emission allowances will provide greater impetus to exploitation of existing low-emission generation technologies, including hydro, nuclear, gas, and combined-cycle sources, as well as experimental renewable technologies relying on power from wind, solar, geothermal, and biomass sources. These technologies may be exploited efficiently through either central dispatch or bulk power exchanges. The development of low-emission technologies is expected to be a small factor over the next decade.
- **Scrubbing Emissions.** The primary alternative for the many coal-fired plants in the United States is the installation of scrubbers on smokestacks to extract noxious fumes as solid residues. A scrubber is an expensive, durable (e.g., 30 years), irreversible investment, but it substantially reduces the need for emission allowances.

Over the next decade, the number of scrubbers installed by utilities will be the main determinant of the market supply of emission allowances. For, after the market for low-sulfur fuels has adjusted, the tradable supply will increase in direct relation to the number of scrubbers installed, since each scrubber releases an equivalent number of emission allowances for sale in the market.

The demand for emission allowances derives from three factors. Emissions are input factors in the production of power so customers' demand for electric power (at rates affected by adding a charge for emission allowances or for recovery of the costs of scrubbers) yields a derived demand for emissions. This demand is then modified by the amount of substitution of low-sulfur fuels based on the prevailing price structure. Here, we consider only the net demand for emissions after the allowance for fuel substitution has already been accounted for. Finally, scrubbers render harmless the emissions from the facilities where they are installed. Consequently, the residual demand for unscrubbed emissions is the demand for emission allowances, since each ton of unscrubbed emissions requires an emission allowance.

To appreciate the implications of this formulation, we describe a possible scenario. For simplicity, suppose that the capital and operating cost of a scrubber over its lifetime is essentially fixed in terms of the expected present value of the unit cost of emission reduction, say $400 per ton. In this case, one might surmise that emission allowances could not trade for long at any higher price. That is, at a price of $500 utilities would find it advantageous to install scrubbers until the price is driven down to the $400 cost of scrubbing. Consequently, as a first approximation one might suppose that emission allowances will trade at prices no more than $400 per ton. This conclusion depends, however, on the implicit assumption that scrubbers are perfect substitutes for emission allowances, which would be true only in the absence of uncertainty about market conditions in the future. In fact, utilities face considerable uncertainty and consequently they perceive scrubbers as inferior substitutes for emission allowances. In contrast to emission allowances, investments in scrubbers are irrevocable commitments lasting decades: purchases of allowances can be adapted to changing conditions whereas a scrubber might be under-utilized if demand falls — or, in retrospect the cost of a scrubber might be excessive after a fall in the prices of low-sulfur fuels or alternative generation technologies. For these reasons, the price of emission allowances will include a premium that recognizes the value of flexibility. Later we use plausible parameter estimates to indicate that this premium, the option value, could be as large as \$100, indicating that the \$500 market price could be sustained without prompting utilities to increase the number of scrubbers installed.

The organization of the remaining sections of the paper is as follows. In section 2, we provide a brief review of the literature which underlies our formulation. In section 3, we present the formulation. In section 4, we derive our main theoretical result. For a simplified model in which expectations derive from a stochastic process that is a Brownian motion, the optimal capital stock and the imputed option value are characterized completely. In section 5 we conclude with summary of our findings and their implications.

2. Option Value

The option value of a substitute for investment is the main topic of this paper. An extensive literature on option value has been developed over the past several decades. In the 1960s, the theory of investment focused on a debate about whether demands for new capital equipment are well defined as other than the increments required to attain optimal capital stocks based on current expectations about future production.[1] The resolution of this debate in favor of the capital-stock viewpoint was affected by a seminal article by Arrow (1968) that identified irreversibility of investment (or costliness of disinvestment) as a principal factor when expectations change stochastically. That is, capital equipment is durable and often it is specialized or otherwise immobile. Consequently, an investment in new equipment runs the risk that a downturn in product demand will leave it idle. Thus, determination of optimal capital stocks must hedge cautiously against risks of unfavorable changes in utilization. The volatility of empirically observed investment rates can therefore be explained in part by variations in optimal capital stocks induced by stochastic variations in expectations. In particular, changes in the perceived risk, as measured by the variance, can be as important as changes in the mean (Bernanke 1983); or, firms may defer investment while they learn more about the stochastic process affecting utilization (Demers 1991). This theme has been repeated in subsequent contributions by Baldwin (1982), Freixas and Laffont (1984), and Henry (1974) that study further the basic implications of irreversibility. In addition to studies of aggregate investment and business cycles, there have been several recent applications of these ideas at the firm or industry level by Dixit (1989; 1991a; 1991d), Grossman and Laroque (1990), Majd and Pindyck (1987), McDonald and Siegal (1985; 1986), and Pindyck (1980; 1988; 1991b). Pindyck (1991a) surveys much of this literature.

These studies identify three basic phenomena. First, a basic implication of irreversibility is called 'hysteresis' by Dixit (1991c): after (say) a favorable shift in conditions, firms may be slow to invest; and then if this shift eventually reverses, they may be left with excess capacity. The second effect is volatility of investment magnitudes and prices for new capital equipment as expectations move stochastically. For instance, in a stochastically stationary environment, firms order new equipment only when expectations reach new highs. The third effect is that substitutes for investment have an 'option value' that would be absent were it not for the irreversibility of investment and uncertainty about utilization. For instance, if labor can substitute for equipment, then it may be advantageous to meet a surge in product demand with labor-intensive production rather than purchasing new equipment.

The possibility of using short-term labor provides flexibility that is valuable because it avoids the risks of irreversible commitment associated with specialized durable equipment; cf., Jones and Ostroy (1984).

The terminology comes from the theory used in finance to predict the prices of financial instruments, including put and call options. Indeed, there is a substantial equivalence, as can be seen by viewing durable equipment as a long-lived option to produce output whenever product demand warrants. Similarly, an exhaustible resource, such as a mine or oil deposit, provides options to start or stop extraction. In turn, this theory stems from the theory of stochastic control that (in the present context) studies either the optimal time and circumstances in which to commit to a particular investment, or the optimal capital stock depending on expectations about the future; cf., Karatzas and Shreve (1988) and Lund and Øksendal (1991).

We turn now to the exposition of the formal model and the derivation of the key results. Initially, the formulation is designed to measure the option value of emission allowances based on an optimal strategy of investments in scrubbers.

3. Model

We model time as continuous with an infinite horizon ($0 < t < \infty$). Future revenue and cost streams are discounted at the instantaneous interest rate r to obtain present values.

To concentrate on the main considerations, we assume that at any time a scrubber can be installed instantly at a constant marginal cost c.[2] This cost is expressed in terms of dollars per ton of annual scrubbing capacity and it represents the expected present value at the time of installation of the stream of all current and future costs. Thus, for a scrubber with capacity to remove K tons per year forever, an amortized (capital, maintenance, and operating) level cost of C dollars per year translates as a marginal cost of $c = C/rK$ dollars per ton. Because we study the industry as a whole, and the addition of any one scrubber is small relative to the industry's stock of scrubbers, we interpret the total industry stock as a continuous variable s. The size of this stock is measured in tons per year of scrubbing capacity.

We assume that the aggregate demand for emissions is a known continuous function $q = D(x, p)$ of a state variable x and the price p, independently of the date t. Since $D(x ,p)$ is a derived demand, uncertainty could enter through output demand, factor supply, the technology, or regulation. The single state variable x summarizes all factors affecting the demand for emissions (induced by the demand for power) as well as other developments, such as low-sulfur fuel costs and technological progress in the design of low-emission generation sources. Thus, the state is a parameter that shifts the demand function in response to changing economic conditions. A simple example, for instance, is the linear demand function $D(x, p) = x - p$ for which the effect of altering the state variable is to raise or lower the demanded quantity at every price, or equivalently, to raise or lower the price at which each quantity is demanded. On the other hand, the scrubbing capacity s indicates the portion of the annual demand for emissions that is satisfied by scrubbing smokestack effluents.

The rate q of emissions is measured in terms of tons per year, of which a portion s is accounted for by utilization of scrubbing capacity; thus, the net rate of unscrubbed emissions is $q - s$. For simplicity, we represent the effect of the 1990 Clean Air Act as a fixed annual quota Q of unscrubbed emissions, which implies the constraint $q - s = Q$. Consequently, given the state x and the scrubbing capacity s, the price p of emission allowances adjusts so that the market clears to satisfy $D(x, p) = Q + s$, or possibly $p = 0$ and $D(x, 0) < Q + s$ if the quota and the scrubbing capacity saturate demand for emissions. [Note that the effect of the 1990 Clean Air Act is essentially to reduce the quota from $Q = \infty$ to the rates specified in the Act.] The price is measured in terms of (dollars per year) per (ton per year), or equivalently dollars per ton of unscrubbed emissions.

We use $d(x, p) = D(x, p) - Q$ to present the net demand in excess of the quota. In particular, if the state x were fixed forever, so that a ton of scrubbing capacity is a perfect substitute for a ton of unscrubbed emissions, then $d(x, p)$ is the induced demand function for scrubbing capacity, and we would expect the stock of scrubbers to be determined by the amortized cost rc of adding a ton of annual scrubbing capacity. That is, $s = d(x, rc)$ if x were fixed.

The effect of the market equilibrium for emission allowances is to maximize the expected present value of total surplus given the state and the scrubbing capacity. In each contingency, this surplus (the area under the demand curve) is a measure of the annual rate of social benefit obtained from emissions. Thus, we use the derived function

$$U(x,q) \equiv \int_0^\infty \min\,[q, D(x,p)]\,dp$$

to specify the annual rate of social benefit, measured in terms of dollars per year, from q tons per year of emissions when the state is x. The marginal rate of social benefit $V(x,q) \equiv \partial U/\partial q$ from an additional ton per year of emissions is the inverse of the demand function, viz., $p = V(x, D(x, p))$. To take account of the assigned quota Q of unscrubbed emissions, we define $u(x, s) \equiv U(x, Q + s)$ and its marginal rate $v(x, s) \equiv \partial u/\partial s$, which is the inverse of the net demand function d. Note that $v(x, s) - rc$ is the net social benefit of an additional unit of scrubbing capacity at every future date, were the state x fixed.

These ingredients imply that if the state x were fixed forever then the scrubbing capacity would increase immediately to the level $s = d(x, rc)$ and then emission allowances would sell at the price $p = rc$. In practice, however, the demand for emissions varies in response to variations in the demand for power, as well as other developments in fuel costs and generation technologies. Here we model this variation as a stochastic process affecting the path of the state variable x over time. In this case, scrubbers are imperfect substitutes for emission allowances because scrubbers are irreversible durable investments, whereas emission allowances can be utilized as needed in response to changes in the state variable. This implies that emission allowances will sell at a price $p > rc$ that includes an option value representing their superior flexibility. Our aim is to obtain an estimate of this option value.

Initially, we derive the simple formula for the option value that occurs in the special case that the stochastic process of the state variable is a Wiener process or 'Brownian motion.' In this case the time path $(x_t)_{t>0}$ of the state variable is the realization of a random walk $\{X_t\}$ for which, over each time interval $[t, t + \tau]$ of duration τ, the state increment $X_{t+\tau} - X_t$ has a Normal distribution with mean $\mu\tau$ and variance $\sigma^2\tau$, independently of prior history. The mean rate of change μ is called the drift rate, and σ^2 is called the instantaneous variance.

With this specification, at any time the portion of past history relevant for the future is fully summarized by the current state and the scrubbing capacity that has been installed so far. Consequently, we use $W(x, s)$ to represent the current measure of future potential welfare: it is the expected present value of the stream of future benefits net of future scrubber costs when the current state is x and the installed scrubbing capacity is s. As mentioned obliquely above, in the absence of any market imperfections, the net effect of equilibria in the markets for scrubbers and emission allowances is to solve the problem of maximizing this measure of welfare at every time based on the current state and scrubbing capacity. Thus, the calculation of $W(x, s)$ is conditioned on an optimal policy of investments in scrubbers.

In the Appendix, we characterize the form of an optimal policy in the case that decisions about scrubber additions are made at discrete times separated by short intervals. Using a dynamic programming formulation, we show that an optimal policy is described by a nondecreasing function $S(x)$ such that whenever $s < S(x)$ enough scrubbers are installed to bring the scrubbing capacity up to the optimum level $S(x)$. If $s \geq S(x)$, then no new scrubbers are installed, and since disinvestment is excluded, the scrubbing capacity remains unchanged. For the continuous-time version addressed here, we restrict attention to policies of this form.

In general, the optimal policy may have flat segments. Indeed it usually has a flat segment for a range of states in which scrubbers would not be cost-effective even if the state were fixed forever. That is, $S(x) = 0$ if $v(x, 0) < rc$. For states where the policy is increasing, however, it is useful to define the inverse function $B(s) = S^{-1}(s)$ and then to extend this definition to the entire range of scrubbing capacities by specifying that $B(s)$ is the least upper bound on those values of the state x for which $S(x) > s$. Thus, if $x < B(s)$, then no scrubbers are installed. For instance, if $v(x, 0) < rc$, then $x < B(0)$ and no scrubbers are installed.

4. Optimal Policy

We present here a heuristic derivation of the optimal policy based on the presumption that the optimal policy and the welfare measure W are (piecewise) differentiable functions of the scrubber capacity; the Appendix adds more detail.

First, we take advantage of some standard results. Define

$$w(x, s) \equiv E\left\{\int_0^\infty u(X_t, s)\, e^{-rt}\, dt \mid X_0 = x\right\},$$

which is what the welfare measure $W(x, s)$ would be if no additional scrubbers were ever installed. Also, let

$$\hat{W}(s) \equiv W(B(s), s) \text{ and } \hat{w}(s) \equiv w(B(s), s)$$

be the values of W and w on the boundary $x = B(s)$ between the region where scrubbers are not added $[x < B(s)]$ and the region where they are $[x > B(s)]$. A useful property of Brownian motion is that if $T(x, B(s))$ is the random time it takes for the state process to travel from $X_0 = x$ initially to $X_T = B(s)$ at time T then the expected discount factor is[3]

$$E\left\{e^{-rT(x,B(s))} \mid X_0 = x\right\} = e^{-[B(s) - x]/\alpha},$$

where the parameter is

$$\alpha \equiv \frac{[\sigma^2/r]}{\sqrt{[\mu/r]^2 + 2[\sigma^2/r]} - [\mu/r]} = \frac{1}{2}\,[[\mu/r] + \sqrt{[\mu/r]^2 + 2[\sigma^2/r]}]\,.$$

This property implies that in the region where $x < B(s)$ the welfare measure satisfies the equation

$$W(x, s) = w(x, s) + [\hat{W}(s) - \hat{w}(s)]\, e^{-[B(s) - x]/\alpha}\,. \tag{1}$$

This equation is a standard recurrence formula: it says that one accumulates benefits at the rate $u(X_t, s)$, as in the definition of $w(x, s)$, until the first time $t = T(x, B(s))$ at which the state $X_t = B(s)$ encounters the boundary, after which one gets the continuation value $\hat{W}(s)$ from an optimal investment policy, rather than continuing to get the stream of values of $u(B(s), s)$ as supposed by the definition of w.

The second observation is that one is indifferent about installing additional scrubbers along the boundary where $x = B(s)$. Consequently,

$$\frac{\partial W}{\partial s}(B(s), s) = c\,, \tag{2}$$

which specifies that on the boundary the marginal benefit and cost of an additional unit of scrubbing capacity are equal.

Combining these facts yields an explicit formula for the welfare measure W, obtained as follows. Write the recursive formula (1) for $W(x, s)$ as

$$W(x, s) = w(x, s) + A(s)e^{x/\alpha}\,, \text{ where } A(s) = [\hat{W}(s) - \hat{w}(s)]\, e^{-B(s)/\alpha}\,.$$

Note that an optimal policy must ensure $A(\infty) = 0$. That is, as $s \to \infty$ it must be that $W(x, s) - w(x, s) \to 0$, because it becomes almost sure that no additional scrubbers will be installed. Assuming that A and therefore W are differentiable, (1) implies that

$$\frac{\partial W}{\partial s}(x, s) = \frac{\partial w}{\partial s}(x, s) + A'(s)e^{x/\alpha}\,,$$

so (2) implies that on the boundary where $x = B(s)$,

$$\frac{\partial w}{\partial s}(B(s), s) + A'(s)\, e^{B(s)/\alpha} = c\,.$$

This differential equation for A has a unique solution satisfying the requirement that $A(\infty) = 0$, from which we obtain the explicit formula

$$W(x, s) = w(x, s) + \int_s^\infty \left[\frac{\partial w}{\partial s}(B(\xi), \xi) - c\right] e^{-[B(\xi) - x]/\alpha}\, d\xi \tag{3a}$$

for the measure of optimal welfare. This formula holds in the region where $x < B(s)$ so that no scrubbers are installed. On the other side of the boundary,

$$W(x, s) = W(x, S(x)) - c[S(x) - s]\,, \tag{3b}$$

since scrubbers are added to bring the capacity up to the level $S(x)$.

The formula (3a) has an intuitive explanation. It says that the optimal welfare measure is the surplus $w(x, s)$ obtained if no further additions were made to capacity, plus the net value $w_s - c$ obtained from each subsequent increment of capacity, multiplied by the expectation of the discount factor at the time at which that increment is installed.

Lastly we take account of the fact that the optimal policy must maximize the welfare measure. From (3a) we see that if the optimal function B is increasing then each value $B(\xi)$ must maximize the integrand for the argument ξ, independently of the position (x, s) from which the welfare measure is calculated. Therefore the optimal policy, expressed in terms of B, is

$$B(s) = \arg\max_{B} \left[\frac{\partial w}{\partial s}(B, s) - c \right] e^{-B/\alpha}, \tag{4}$$

provided this yields a nondecreasing function.[4] For instance, in the monotone case the first- and second-order necessary conditions for optimality reduce to the conditions that

$$\frac{\partial}{\partial s}\left[w(x, s) - \alpha \frac{\partial w}{\partial x}(x, s)\right] = c \text{ , and} \tag{5a}$$

$$\frac{\partial^2}{\partial x \partial s}\left[w(x, s) - \alpha \frac{\partial w}{\partial x}(x, s)\right] \leq 0 , \tag{5b}$$

where $x = B(s)$. Note that condition (5a) generalizes the criterion that would be employed if the state x were fixed forever. In that case, $w(x, s) = u(x, s)/r$ and $\alpha = 0$ so condition (5a) would be the analog of (2), namely $\partial u(x, s)/\partial s = rc$, and the price of emission allowances would be $p = v(x, s) = rc$ if the scrubbing capacity were chosen optimally.

Condition (4) has the advantage of indicating directly the regularity properties required to justify the foregoing derivation. To ensure that $B(s)$ is a differentiable increasing function for $s > 0$ requires that the maximand has a maximizer with this property (Milgrom and Shannon 1991). It suffices, for instance, that $v(x, s) - \alpha v_x(x, s)$ is a decreasing function of the capacity s. The Appendix provides further analysis; in particular, we exploit the general feature of stochastic control problems involving Brownian motion that the welfare measure must have continuous derivatives, even across the boundary.

Examples

To conclude our analysis, we present some examples that illustrate applications of the characterizations (4) and (5) of the optimal policy.

Linear Demand

Suppose the demand function (net of the quota) is linear, say $d(x, p) = [ax - p]/b$, corresponding to $u(x, s) = axs - \frac{1}{2} bs^2$ and $w(x, s) = [a/r][x + \mu/r]s - \frac{1}{2}[b/r]s^2$.[5] Then (5a) indicates that the policy and welfare function are

$$B(s) = \frac{rc + bs}{a} + \theta \quad \text{or} \quad S(x) = \max\left\{\frac{0, [ax - rc - \theta]}{b}\right\}$$

$$W(x, s) = w(x, s) + \left[\frac{a\alpha^2}{br}\right] \exp\{-[B(s) - x]/\alpha\}, \quad \text{where}$$

$$\theta \equiv \frac{\alpha - \mu}{r} \geq 0 .$$

This implies that whenever $x \geq B(s)$ the price of emission allowances is $p = v(x, S(x)) = rc + \theta$. The option value θ represents the price premium that emission allowances command because they avoid the irreversible commitments required by scrubbers. Written out in full it is

$$\theta = \frac{1}{2}\left\{\sqrt{[\mu/r]^2 + 2[\sigma^2/r]} - [\mu/r]\right\},$$

$$\approx \sqrt{\sigma^2/2r} - \frac{1}{2}[\mu/r] \quad \text{if} \quad |\mu| \text{ is small,}$$

$$\approx \frac{1}{2}\sigma^2/|\mu| \quad \text{if } \sigma^2 \text{ is small.}$$

To illustrate quantitative magnitudes, suppose the interest rate is $r = 10\%$ per year, the drift is $\mu = 0$, and the standard deviation of the change in the maximum demand price for emission allowances over a year is $\sigma = \$40$ per ton. The predicted option value is then $\theta = \$89$ per ton. Thus, if the marginal cost of a scrubber is \$400 per ton of scrubbing capacity, then the predicted price of an emission allowance is \$489 per ton. Moreover, if the initial maximum demand price is $v(x, 0) = \$600$ per ton then this option value accounts for a reduction of 12% in the scrubbing capacity installed initially (starting from $s = 0$), as compared to the case that x is fixed forever. These effects are proportional to the magnitude of the standard deviation s: doubling the standard deviation doubles the option value and the percentage reduction in scrubbing capacity.

Note that the price of emission allowances is not always equal to the marginal cost rc plus the option value θ. When $x < B(s)$, the price is less than this because then there is a temporary excess of scrubbing capacity. Over time the price follows a stochastic path that varies in a way that appears somewhat cyclical; moreover, the average price is less than $rc + \theta$. Each time the price would exceed $rc + \theta$, additional scrubbers are installed to keep the price at $rc + \theta$, while at other times the price varies stochastically in the interval below $rc + \theta$. Thus, $rc + \theta$ is properly interpreted as a cap on the price of emission allowances that is binding only at times when the state variable affecting demand is sufficiently large that utilities find it advantageous to install additional scrubbers.

Geometric Brownian Motion

If the inverse demand function is $v(x, s) = e^x P(s)$ then

$$w(x, s) = A^{-1} e^x \int_0^s P(\xi)\, d\xi \quad \text{provided} \quad A \equiv r - \mu - \frac{1}{2}\sigma^2 > 0 .$$

Consequently, the optimality condition (5a) implies that

$$e^x P(S(x))[1 - \alpha] = Ac ,$$

and the market price of emission allowances is $v(x, S(x)) = Ac/[1 - \alpha]$ whenever $s \leq S(x)$, so the option value is

$$\theta = \frac{Ac}{1-\alpha} - rc = \frac{1}{2}rc[\sqrt{[\mu/r]^2 + 2[\sigma^2/r]} - [\mu/r]] .$$

Extensions

Our analysis can encompass a tax on emissions by redefining $u(x, s) = \max_Q U(x, s + Q) - \pi Q$, where π is the tax rate on unscrubbed emissions. If the tax rate is fixed and the quota Q is variable, then the emission rate varies randomly as the state x varies, replicating the effect of a state-contingent quota. On the other hand, in a market for emission allowances with a fixed quota, the price of emission allowances varies as the state varies, replicating the effect of a state-contingent tax rate set to cap emissions. Pure price and pure quantity modes of regulating externalities are two extremes in a spectrum of possibilities that have been analyzed extensively. In general, the choice between them depends on the form of the function that specifies the damage from emissions. In principle, the valuation function $W(x, s)$ derived above could be used in conjunction with a damage function to determine an optimal quota or an optimal tax rate. The provisions of the 1990 Clean Air Act Amendment indicate that Congress has opted for quantity controls that cap unscrubbed emissions. This is consistent with the view that the damage function is strongly convex, independently of the state x.

Opportunities to trade in the emission-allowance market in effect provide utilities with a menu of intertemporal state-contingent contracts. These enable improvements in efficiency by allowing utilities to adjust their investments in scrubbers in response to changing demand conditions. One measure of this gain in efficiency is the difference between $W(x, s)$ and $w(x, s)$ at scrubber capacities s optimized at the initial state x.

5. Concluding Remarks

The reliance on a market mechanism in the 1990 Clean Air Act Amendment marks a milestone in environmental regulation. The success of this scheme depends critically on utilities' incentives to engage in efficient trading of emission allowances. In this paper, we study one aspect: the factors affecting emission-allowance prices and, in parallel, firms' investments in scrubbers. Our formulation omits important practical considerations, such as regulatory uncertainty, banking of allowances, the futures market, lead times for scrubber installations, interactions with bulk power markets, and auctions of extra allowances—all of which require further analyses.

We examine the case that the demand for emissions is affected by an exogenous stochastic process. In this case, the market price of emission allowances can exceed the marginal cost of a scrubber by an amount called the option value. The option value measures the expected present value of the greater flexibility that emission allowances provide, compared to the irreversible commitments required by investments in scrubbers. Our calculation suggests that the option value can be substantial. For the linear-demand case, the option value is roughly proportional to the standard deviation of demand uncertainty. We also show that investments in scrubbers are reduced if there is greater uncertainty about future market conditions. Markets for emission allowances are likely to be used in the future to accomplish other environmental goals. In fact, to counter the effects of global climate change, such markets have been proposed to limit carbon dioxide emissions. In this case, an important feature is that present evidence leaves considerable uncertainty about the mean rates of change in atmospheric gases, global temperature, and consequent effects on sea levels, crops, etc.; that is, the drift rate μ is uncertain. To address this feature, the formulation can be amended to include learning about the drift from accumulating evidence. The analysis is more complicated and closed-form solutions have not been obtained, but our numerical calculations indicate that the main qualitative results are unaffected (Chao and Wilson 1991).

In sum, we illustrate the use of option-value theory to gain basic insights into the dynamics of markets for emission allowances. We surmise that the theory of option values could be used in the future to study other aspects of market design, such as variable quotas or taxes depending on the state of demand, or allowances with variable terms and conditions.

Appendix

The formulation with continuous time used in section 3 can be interpreted as the limit of the following setup formulated with discrete time. In this version, decisions about additions to scrubbing capacity are made periodically at discrete times $t = 0, \delta, 2\delta, \ldots$, separated by periods of duration $\delta > 0$. Over a period $[t, t + \delta)$ in which the initial state is X_t and the scrubbing capacity is S_t, the accrued benefit is $u_t(X_t, s_t)\ \delta$. In addition, a cost $c_t\varepsilon_t$ is incurred (immediately) if scrubbing capacity is increased by an amount $\varepsilon_t \geq 0$. The next period then begins with the state $X_{t+\delta} = X_t + \xi_t$ and the scrubbing capacity $S_{t+\delta} = S_t + \varepsilon_t$. Let $W_t(x, s)$ be the welfare measure representing the maximal expected present value of current and future net benefits when the initial state is $X_t = x$ and the scrubbing capacity is $S_t = s$. From the principle of optimality used in dynamic programming, therefore, we obtain the relation

$$W_t(x, s) = \sup_{\varepsilon \geq 0}\ u_t(x, s)\ \delta - c_t\varepsilon + e^{-r_t\delta} E_t\{W_{t+\delta}(x + \xi_t, s + \varepsilon) \mid x\},$$

where r_t is the interest rate. The sufficiency of the state for the past history of the stochastic process is represented here by the Markovian property that the conditional distribution of the state increment ξ_t depends only on the date t and the current state x. We assume further that the attainable welfare is finite and the supremum is attained as a maximum.[6]

We first establish that the optimal investment policy has the simple form used in section 3.

Lemma: There exists an optimal capacity function $S_t^*(x)$ such that the optimal investment policy is to choose

$$\varepsilon_t = \max\{0, S_t^*(x) - s\}$$

at date t and position (x, s).

Proof: Given any such functions S_t^* specifying investment policies at each date $t = 0, \delta, 2\delta, \ldots$,, let $W_t^*(x, s)$ be the resulting welfare. Because the cost is linear in the scrubbing capacity, we can improve these policies by solving for each (x, s, t) the problem

$$\hat{W}_t(x, s) = u_t(x, s)\delta + c_t s + \max_{\hat{s} \geq s} [- c_t \hat{s} + e^{-r_t \delta} E_t\{W^*_{t+\delta}(x + \xi_t, \hat{s}) \mid x\}] .$$

The optimal solution to this problem has the same form: evidently

$$\hat{s}(x, s, t) = \max \{s, \hat{s}(x, 0, t)\}$$

and therefore the capacity increment $\hat{\varepsilon}(x, s, t) \equiv \hat{s}(x, s, t) - s$ is

$$\hat{\varepsilon}(x, s, t) = \max \{0, \hat{s}(x, 0, t) - s\} .$$

Thus, $\hat{S}_t(x) \equiv \hat{s}(x, 0, t)$ defines a function that specifies a policy of the same form as specified by S^*_t. According to the principle of policy improvement in dynamic programming, repetition of this procedure yields a sequence of policies converging to an optimal policy. Thus, there exists an optimal policy of the required form. Q.E.D.

The foregoing formulation allows nonstationarity of benefits, costs, interest rates, and the distribution of state increments. Usually, such problems can be solved only by numerical methods. A finite horizon is assumed and the problem is solved recursively by the standard techniques of dynamic programming, such as value iteration or policy iteration.

To obtain the analytical results in section 3 we adopt three simplifying assumptions. The first is that all data are independent of the date; we therefore omit the time subscript in all that follows. The second and third are that time is continuous and the distribution of state increments is independent of the current state. This allows few choices for the distribution of state increments and we have chosen to cast our results in terms of ordinary Brownian motion. Geometric Brownian motion is included by using $y = \exp[x]$ as the state. Recall that Brownian motion is defined by the property that the state increment $X_{t+\tau} - X_t$ over any interval $[t, t + \tau]$ has a Normal distribution with mean $\mu\tau$ and variance $\sigma^2\tau$ independently of the date and prior history. For the discrete-time formulation above, this implies that

$$W(x, s) = \max_{\varepsilon \geq 0} u(x, s)\delta - c\varepsilon + e^{-r\delta} E\{W(x + \mu\delta + (\sigma\sqrt{\delta})z, s + \varepsilon)\} ,$$

where the expectation is taken with respect to the standard Normal variate z having mean 0 and variance 1. The continuous-time version is obtained heuristically by taking on the right side a Taylor's series expansion of W as a function of the state and then extracting the limit.[7] Subtracting $W(x, s)$ from both sides and dividing by δ, this yields

$$0 = \lim_{\delta \to 0} \frac{-W(x, s)}{\delta} + u(x, s) - \frac{c\varepsilon}{\delta} + e^{-r\delta}\left[\frac{1}{\delta}\right]$$

$$\times E\{W(x, s') + W_x(x, s') [\mu\delta + (\sigma\sqrt{\delta}) z] + \frac{1}{2} W_{xx}(x, s') [\mu\delta + (\sigma\sqrt{\delta})z]^2 + o(\delta)\} ,$$

$$0 = \lim_{\delta \to 0} u(x, s) - \frac{c\varepsilon}{\delta} - [W(x, s) - e^{-r\delta} W(x, s')] + e^{-r\delta}$$

$$\times \left\{W_x(x, s')\mu + \frac{1}{2} W_{xx}(x, s') [\mu^2\delta + \sigma^2] + O(\delta)\right\} ,$$

using the optimal investment strategy ε and subsequent capacity s'. In the region where no investment is undertaken, $\varepsilon = 0$ and $s' = s$, which yields the characterization

$$rW(x, s) = u(x, s) + \mu W_x(x, s) + \frac{1}{2}\sigma^2 W_{xx}(x, s)\,, \tag{A1}$$

in terms of a differential equation for the welfare function W. On the other hand, in the region where investment is undertaken, we have seen previously that the investment policy consists of moving to the optimal capacity $S(x)$, so $W(x, s) = W(x, S(x)) - c[S(x) - s]$ in that region. For instance, in the linear demand case, say $v(x, s) = x - s$ and standardizing units so that $r = 1$ and $\sigma = 1$, the relevant solutions of (A1) have the form

$$W(x, s) = u(x, s) + \mu s + L \exp\left\{\frac{K - x}{\alpha}\right\},$$

where the coefficient functions K and L might depend on the capacity s. It remains to specify the condition that determines the boundary of the no-investment region, which thereby enables solution of the above differential equation and derivation of the optimal capacity $S(x)$. In the theory of stochastic control, this condition is called variously the 'principle of smooth fit,' 'the high-contact condition,' or in the present case 'the super-contact condition.'[8] Essentially it requires that the terms of the differential equation above must be continuous across the boundary. Here, the crucial requirement is that the second derivative W_{xx} must be continuous across the boundary.

This condition is applied as follows. Recall that on the boundary $\partial W(x, S(x))/\partial s = c$ and therefore also

$$\frac{d}{dx}\frac{\partial W}{\partial s}(x, S(x)) = 0\,.$$

Moreover, on the investment side of the boundary, $W(x, s) = W(x, S(x)) - c[S(x) - s]$. Consequently,

$$\frac{\partial^2 W}{\partial x^2}(x, s) = \frac{\partial^2 W}{\partial x^2}(x, S(x)) + \frac{\partial^2 W}{\partial x \partial s}(x, S(x))S'(x)\,,$$

wherever $S(x)$ is differentiable. The principle of smooth fit says, in effect, that

$$\frac{\partial^2 W}{\partial x^2}(x, s) \rightarrow \frac{\partial^2 W}{\partial x^2}(x, S(x))$$

as $s \rightarrow S(x)$. Consequently, we obtain the condition

$$\frac{\partial^2 W}{\partial x \partial s}(x, S(x)) = 0\,,$$

on the boundary wherever $S(x)$ is differentiable and not locally constant. Applying this condition to the recursive formula (1) yields

$$\frac{\partial^2 w}{\partial x \partial s} - \frac{1}{\alpha^2} \frac{\hat{W} - \hat{w}}{S'(x)} + \frac{1}{\alpha} \frac{d}{ds} [\hat{W} - \hat{w}] = 0 , \tag{A2}$$

evaluated at positions (x, s) on the boundary where $B'(s) = 1/S'(x)$.

Next, we use the result (A2) to show that it implies the optimal policy derived by different methods in section 3. Again we use the recursion relation (1) that holds on the no-investment side of the boundary and consider the effect of a small increment ε in the scrubbing capacity. Let $y = B(s + \varepsilon) - x$ be the state increment required to reach the boundary; then:

$$\begin{aligned}\frac{\partial W}{\partial s}(x, s) &= \lim_{\varepsilon \to 0} \frac{W(x, s+\varepsilon) - W(x, s)}{\varepsilon} \\ &= \frac{w(x, s+\varepsilon) - w(x+y, s+\varepsilon)}{\varepsilon} + [\hat{w}(s+\varepsilon) - \hat{W}(s+\varepsilon)] \frac{1 - e^{-y/\alpha}}{\varepsilon} + \frac{\hat{W}(s+\varepsilon) - \hat{W}(s)}{\varepsilon} , \\ &= -\frac{\partial w}{\partial x} \frac{dy}{d\varepsilon} + [\hat{w}(s) - \hat{W}(s)]\, \alpha \frac{dy}{d\varepsilon} + \frac{d\hat{W}(s)}{ds} .\end{aligned}$$

If the position (x, s) is on the boundary then the state increment before another scrubber is needed is approximately

$$y \approx \frac{\varepsilon}{S'(x)} \quad \text{so} \quad \frac{dy}{d\varepsilon} = \frac{1}{S'(x)} .$$

Consequently, on the no-investment side of the boundary,

$$\frac{\partial W}{\partial s}(x, s) = -\frac{\dfrac{\partial w}{\partial x} + [\hat{W}(s) - \hat{w}(s)]\, \alpha}{S'(x)} + \frac{d\hat{W}(s)}{ds} .$$

Invoking the property $\partial W(x, s(x))/\partial s = c$ on the boundary therefore yields a second condition on the boundary:

$$\frac{\partial w}{\partial x} + \frac{\hat{W} - \hat{w}}{\alpha} = S'(x)\left[\frac{d\hat{W}}{ds} - c\right] . \tag{A3}$$

One can now check that conditions (A2) and (A3) are both satisfied only if the previously stated necessary condition is satisfied:

$$\frac{\partial}{\partial s}\left[w(x, s) - \alpha \frac{\partial w}{\partial x}(x, s) \right] = c . \tag{5a}$$

This shows that satisfaction of (5a) on the boundary is a necessary condition wherever the optimal capacity function S is differentiable and increasing.

For instance, applying condition (5a) to the linear demand case mentioned above yields the coefficient functions $K(s) = B(s) = s + c + \alpha - \mu$ and $L = \alpha^2$, where $x = B(s)$ defines the optimal boundary of the investment region.

Notes

We are grateful to Darrell Duffie, Avinash Dixit, Michael Harrison, and Victor Niemeyer for suggestions. The second author acknowledges research support from National Science Foundation grant SES9207850.

1. See the commentary on the debate between Haavelmo (1960) and Jorgenson (1967) in Moene and Rødseth (1991, 187).

2. If the marginal cost depends on the number of scrubbers installed, then the price of scrubber would be affected by the price of emission allowances. Further, to the extent that the installation of scrubbers requires certain lead time, the installed capacity of scrubbers provides owners with an insurance against contingencies such as that the regulatory agency should curtail the size of emission quota.

3. This formula is derived by Harrison (1985, section 3.2) with $\alpha = 1/\alpha_*(r)$ in his notation.

4. More generally, the optimal function B may be constant over one or more interval domains where (4) would yield a decreasing segment. Over such an interval, the optimal value of B and the endpoints of the interval maximize the average of the maximand in (4) subject to continuity of the function B at the endpoints. This method is described by Mussa and Rosen (1978). Also, (4) can yield multiple solutions over an interval, which indicate an intermediate value of s at which B jumps discontinuously and S is constant over an associated interval of values of x. This case requires a more complicated analysis, which we have not done, to maximize welfare based on the expectation of the stochastic process over the interval.

5. The last formula uses a result in Harrison (1985, section 3.3). The linear-demand case is closely related to the quadratic-loss monotone-follower problem studied by Karatzas (1981), since if $a = b = 1$ then $2u(x, s) = x^2 - [x - s]^2$, of which the first term is independent of the control s.

6. It suffices to assume that the cost and benefit functions are continuous and bounded below and above respectively as s varies, and that the welfare $w_{t(x,}\ s)$ with no further investment is bounded. Also, $c_t >> 0$ and $r_t >> 0$.

7. This heuristic derivation is made rigorous by the theory of the Itô calculus. In terms of the Itô calculus, the stochastic process governing the state variable x is expressed as $dX_t = \mu dt + \sigma dZ_t$ where Z_t is a standard Wiener process. The Itô calculus also applies to the control process dS_t of capacity increments because S_t is nondecreasing.

8. In the technical literature this is called the principle of smooth fit, as in Benes, Shepp, and Witsenhausen (1980), Karatzas (1981), and Karatzas and Shreve (1988, section 5.8); or the super-contact condition as in Lund and Øksendal (1991).

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