THOMAS MORMANN

NATURAL PREDICATES AND TOPOLOGICAL STRUCTURES OF CONCEPTUAL SPACES

ABSTRACT. In the framework of set theory we cannot distinguish between natural and non-natural predicates. To avoid this shortcoming one can use mathematical structures as conceptual spaces such that natural predicates are characterized as structurally 'nice' subsets. In this paper topological and related structures are used for this purpose. We shall discuss several examples taken from conceptual spaces of quantum mechanics ('orthoframes'), and the geometric logic of refutative and affirmable assertions. In particular we deal with the problem of structurally distinguishing between natural colour predicates and Goodmanian predicates like 'grue' and 'bleen'. Moreover the problem of characterizing natural predicates is reformulated in such a way that its connection with the classical problem of geometric conventionalism becomes manifest. This can be used to shed some new light on Goodman's remarks on the relative entrenchment of predicates as a criterion of projectibility.

1. INTRODUCTION

As is well known there are many more sets than properties, but set theory does not give us a tool to distinguish properties, i.e., natural predicates, from non-natural ones (cf. Quine 1969, p. 118). This inadequacy of set theory is demonstrated rather spectacularly in the paradoxes of Hempel and Goodman: from a set-theoretical point of view the predicate 'non-raven' is as good as the predicate 'raven'. Having conceded this, Hempel's paradox arises because now a non-black non-raven (e.g., a green frog) corroborates the law 'All ravens are black'. In a similar vein we may construct non-projectible Goodmanian predicates as 'grue' and 'bleen' that, from a set-theoretical point of view, are as good as their (interdefinable) cousins 'blue' and 'green' (cf. Goodman 1983, Chap. III).

To avoid these and similar shortcomings, instead of set theory we need *specific* frameworks that are better adapted for the logical reconstruction of fragments of (natural and scientific) language.

Gärdenfors (1990) uses the framework of *structured conceptual spaces* to cope with predicates like 'grue' and 'bleen'. He claims that generally 'natural' or 'projectible' predicates like 'blue' and 'green' can be represented by *convex* subsets of conceptual spaces, whereas non-natural ones like 'grue' and 'bleen' cannot. I would like to show that this is

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incorrect. In many cases Gärdenfors's concept of convexity is flawed and does not yield a helpful criterion to distinguish between natural and non-natural colour predicates.

In the case of conceptual spaces for colour predicates, (more basic) topological structures like connectedness and closedness serve better. We can get what we want, i.e., the distinction of natural from non-natural colour predicates, at a lower price. Whereas convexity presupposes that the conceptual space is endowed with a linear or at least a metric structure, the concepts of closure and connectedness only make use of a topological structure that is weaker. Thus, even granted that the convexity criterion works, following a principle of structural economy the use of topological criteria should be preferred. The appeal of topological criteria is made even greater by pointing out that there is an important class of conceptual spaces for which the problem of distinguishing between natural and non-natural predicates lies in using topological structures, to wit, the frames of quantum and similar logics.

In the framework of conceptual spaces the problem of distinguishing between natural and non-natural predicates can be reformulated in such a manner that its connection with the classical problem of geometric conventionalism becomes manifest. Set in this context, Goodman's riddle loses at least some of its sceptical appeal and can be partially defused. More precisely, the connection with geometric conventionalism may be used to shed some new light on Goodman's remarks on the relative entrenchment of predicates as a criterion of projectibility (cf. Goodman 1983, Chap. IV).

Before we go into the details, some general remarks on the role and function of conceptual spaces are needed.

(1) On the one hand the structure of a conceptual space can be used to *enrich* the language fragment considered. With the aid of a topological structure we can speak about continuity, e.g., a *continuous* change of colour. If we only had the language of sets at our disposal this would be impossible. Concepts like continuous change crucially depend on the topological structure of the conceptual space: they cannot be expressed if we consider the conceptual space solely as a set.

(2) On the other hand the structure of a conceptual space can be used to *restrict* the profusion of admissible predicates: a predicate represented by a topologically 'nice' subset of C, e.g., a closed connected

subset, appears to be more natural than a predicate that is represented by a topologically 'wild' subset. More generally, the structure of a conceptual space C can be used to define symmetries and invariances in such a way that only the predicates that are represented by symmetric or invariant subsets of C are considered as natural predicates that allow for an empirical interpretation.[1]

(3) Large parts of the structure of conceptual spaces may not correspond to any elements of reality whatsoever (cf. van Fraassen 1987, 1989). For example, taking the real line as a conceptual space of the colour spectrum, the distinction between rational and irrational points is empirically pointless and without significance. The sets of rational and irrational numbers do not correspond to any reasonable partition of reality. Conceptual spaces can only be partially empirically interpreted.

(4) The linguistic surface behaviour may be explained by quite different conceptual spaces. Generally the available linguistic and behaviouristic data do not completely suffice to determine the underlying conceptual space. As we shall see in what follows this applies for the language fragment of colour sentences that can be explained by various conceptual spaces. Furthermore, for different conceptual spaces different predicates may turn out as natural.

Partiality and plurality of conceptual spaces lead to a principle of structural economy that advises us to use as few structure as possible. For example, consider a language fragment concerning temperature. It deals with sentences like 'X is hotter than Y', 'Z is rather cold', etc. We may explain its logical structure by the assumption that it is based on a conceptual space, the basic ingredient of which is a numerical scale based on the real numbers R. Then the principle of structural economy advises us to consider R as an ordered set rather than as an ordered additive group or a Lie group. Otherwise we would have to look for an empirical interpretation of the additive or even the differentiable structure of R in terms of temperature. As is well known this is a futile task, since temperature is not an extensional quantity.

2. CONCEPTUAL SPACES OF COLOUR LANGUAGE

In this section, I would like to consider various conceptual spaces for colour predicates and discuss how their structure may be used to distinguish between natural and non-natural colour predicates. In particular it will be shown that for many spaces, convexity – as defined in Gärdenfors (1990) – is an ill-defined and unstable concept.

Let us start with the colour circle, the simplest conceptual space discussed in Gärdenfors (1990, p. 85):

(2.1)

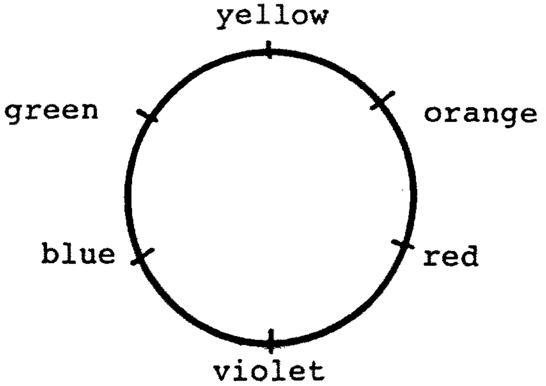

Fig. 1. The colour circle C.

According to Gärdenfors (1990, p. 88), "a convex region is characterized by the criterion that for every pair s_1 and s_2 of points in the region all points between s_1 and s_2 are also in the region". If we took this literally the only convex region of the circle that contains at least two different points would be the circle itself! At the first sight one might be tempted to repair this defect by redefining a convex region as a set that, containing any two points s_1 and s_2, always contains the *shortest* line connecting s_1 and s_2. But this is hardly an acceptable way out.

For example, according to the new 'improved' definition a natural change of colour, which comprises all colours from, say, yellow via orange, red up to blue does not turn out as natural since for many pairs (s_1, s_2) it does not contain the *shortest* line connecting s_1 and s_2.

Moreover, the 'improved' definition is at odds with the concept of complementary colours, i.e., colours like Y (yellow) and V (violet) or

G (green) and R (red), that are directly opposed to each other. According to the 'improved' definition, non-trivial convex sets that contain pairs like G and R or Y and V do not exist. There is, however, no reason why these pairs should be considered as exceptional.

We can avoid these shortcomings if, instead of convexity, we use the more basic topological concept of pathconnectedness. It can be used to conceptualize the concept of a continuous process as follows.

DEFINITION 2.2: Let C be a conceptual space with a topological structure and I the unit interval [0, 1]. Let s_0 and s_1 be elements of C.

(1) A path from s_0 to s_1 is a continuous map $f: I \to C$ with $f(0) = s_0$ and $f(1) = s_1$.

(2) A subset S of C is pathconnected iff for any two elements s_0 and s_1 of S there is a path f in S that connects s_0 and s_1, i.e.,

$$f: I \to C, \quad f(0) = s_0, \quad f(1) = s_1, \quad f(I) \subseteq S$$

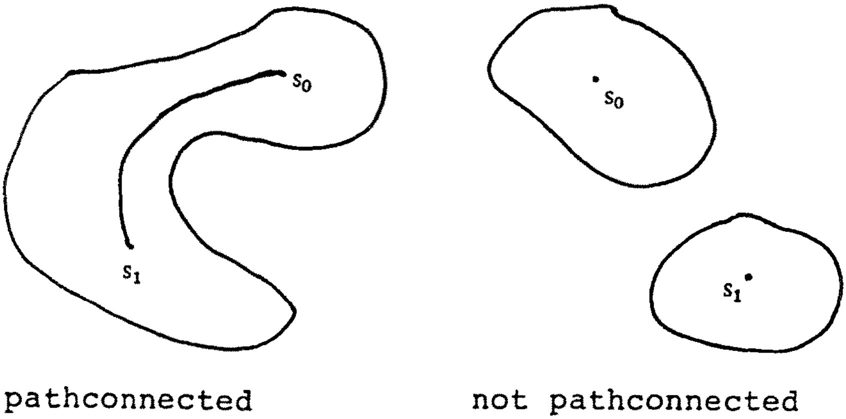

Fig. 2.

Contrary to convexity, connectedness can be defined for any topological space whatever, and not just for metrical or linear spaces. Furthermore, we don't become involved in any quarrel about shorter or the shortest lines. Defining natural predicates for colour spaces by connectedness, we have no problems with complementary points. Thus, in the case of

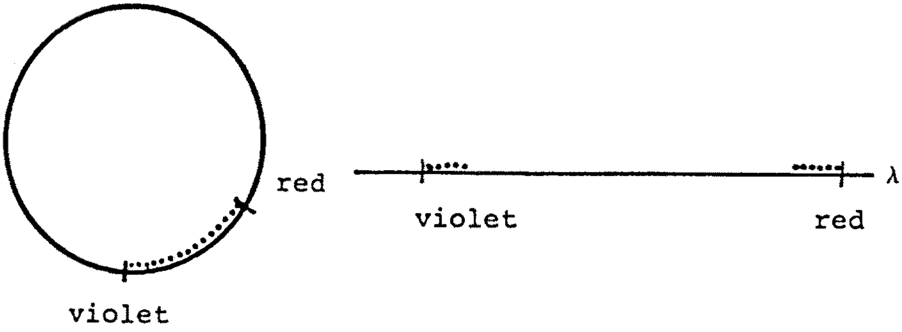

Fig. 3.

conceptual spaces of colour languages, connectedness can cope with the distinction of natural and non-natural predicates at least as satisfactorily as convexity.

Before we move on to the discussion of more complex conceptual spaces of colour languages let us note that even on this elementary level different conceptual spaces yield different natural predicates. If we had used the real line instead of the colour circle as a conceptual space, as is done in van Fraassen (1980, p. 201), the colour 'reddish violet' would not count as a natural predicate since 'red' and 'violet' are located at different ends of the spectrum and do not form a convex set. Using the circle, however, reddish violet comes out as a natural predicate (regardless of whether we use convexity or connectedness).

The line and the circle, however, are by no means the only conceptual spaces used in language fragments concerning colours. Rather, they are just the simplest ones, often replaced by more complex ones.

If we introduce saturation of colours as a further dimension, this results in embedding the colour circle C into the colour disk D, D has C as its circumference, its middle point, Z, represents the 'non-colour' grey.

In D the arc connecting B (blue) and G (green) no longer counts as convex since it does not contain the chord connecting B and G. Thus the new definition of convexity has the drawback of being unstable.

One might object to this argument against convexity that the embedding of C in D should be conceived in a different manner: the

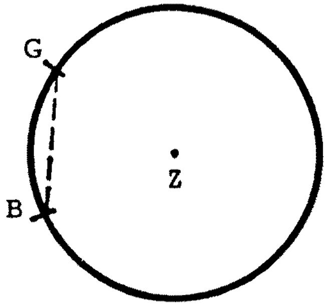

Fig. 4. The colour disk D.

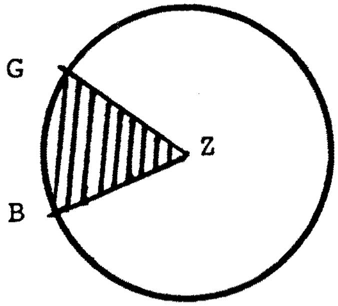

Fig. 5.

counterpart of the arc BG in C is not the arc BG but rather the sector BZG.

It is obvious that BZG is convex, hence, the move from C to D would not have violated convexity. However, this procedure hardly seems acceptable: the substitution of C by D is motivated by the desire to introduce a further conceptual dimension into the discourse about colour. For example, we want to speak of a 'saturated blue' and a 'not-so-saturated blue' as natural predicates. If we accepted only sectors of the colour disk as natural subsets, this would be impossible. Obviously, sectors of D and arcs of C are in one-one correspondence, thus nothing would be gained by moving from C to D.

A different argument against the alleged instability of convexity might object to the fact that we have chosen the standard Euclidean metric on the colour space. If we had taken a spherical metric, considering the disk D as a half sphere with 'Grey' as the North Pole, we could maintain the original (flawed) criterion of convexity, i.e., a set that is convex in C would remain convex if considered as a subset of the larger space D. But in this case as well, the criterion of connectedness scores better. Since the underlying topology is the same, regardless of whether we endow D with the Euclidean metric or with the spherical metric, in both cases the same sets come out as connected. Thus, following the principle of structural economy we can remain neutral with respect to the problem which metric is the correct one.

In summary, we may say that for conceptual spaces of colour theories the criterion of connectedness scores at least as well as the criterion of convexity: it achieves the same at a lower price since it does not depend on a disputable convex structure.[2]

3. CLOSURE STRUCTURES AND NATURAL PREDICATES

In this section we now consider some examples of conceptual spaces for which the class of nice subsets is characterized by the topological concept of closure structures, to wit, the frames of quantum and similar logics. Then we show that closure structures can also be used in the case of colour theories to distinguish non-natural (Goodmanian) predicates, like 'grue' and 'bleen', from natural ones.

DEFINITION 3.1: Let X be a set, and PX its power set (set of all subsets of X). A *closure structure* on X is defined by an operator J: $PX \to PX$ with the following properties (Y, Y′ subsets of X):

(J1)	$Y \subseteq J(Y)$	(reflexivity)
(J2)	$J(J(Y)) = J(Y)$	(transitivity)
(J3)	If $(Y \subseteq Y')$ then $(J(Y) \subseteq J(Y'))$	(monotony)

A set $Y \subseteq X$ is called *closed* (with respect to J) iff it is *invariant* with respect to J, i.e., $J(Y) = Y$. The pair (X, J) is called a closure space.[3] A set $Y \subseteq X$ is called *open* (with respect to J) iff it is the set-theoretical complement of a closed set.

Let (X, J) and (X′, J′) be two closure spaces. A map $f: X \to X'$ is called a *closure map* iff the following holds:

$$f(J(Y)) = J'(f(Y))$$

A closure structure is slightly weaker than a topological structure. As is well known the existence of a topological structure is equivalent to the existence of a closure operator J_t that satisfies the following additional condition:

$$\text{(t-closure)} \qquad J_t(Y \cup Y') = J_t(Y) \cup J_t(Y')$$

Now we would like to discuss some well-known examples of conceptual spaces endowed with a closure structure that is used to single out their nice properties. First we deal with quantum logics. Then we consider the distinction of *refutative* and *affirmative* properties, and finally we treat Goodmanian colour predicates.

Quantum Logics. The conceptual spaces of quantum logics are called *orthoframes*. They are defined as follows (cf., for example, dalla Chiara 1986):

DEFINITION 3.2: (a) An orthoframe is a relational structure $F = (U, R)$, where U is a non-empty set (called the set of worlds to be interpreted as the set of physically possible situations), and R (called the accessibility relation) is a binary reflexive and symmetrical relation on U, i.e., $R \subseteq U \times U$ and for all i, $j \in U$, the following conditions hold:

(1) $(i, i) \in R$
(2) $(i, j) \in R \Leftrightarrow (j, i) \in R$

(b) For any set of worlds $X \subseteq U$ the orthocomplement X^* of X is defined as follows:

$$X^* := \{i \mid \text{for all } j(j \in X \Rightarrow (i, j) \notin R)\}$$

The following lemma is well known:

LEMMA 3.3: Let (U, R) be an orthoframe. Then the operator $J: PU \rightarrow PU$ defined as $J(X) := X^{**}$ is a closure operator on U.

As is common in possible world semantics we identify a proposition with the set X of worlds where it holds. That means we consider it as a property a world has (or does not have) (cf. Lewis 1986, pp. 53–54). Not all propositions in this general sense turn out to be nice ones, i.e., can be considered as meaningful from the point of view of quantum theory. Rather, we have to restrict our attention to closed propositions, that is, sets X of worlds that satisfy $X = X^{**}$.

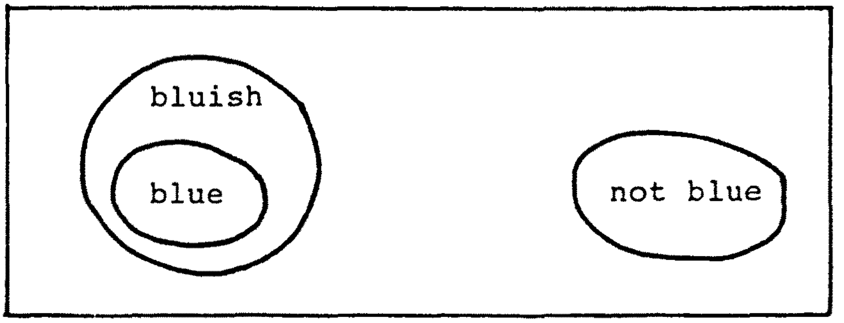

Fig. 6.

The set of closed propositions is closed under set-theoretical intersection and orthocomplementation '*'. It defines a so-called 'minimal quantum logic', MQL, based on the conceptual space (U, R) (see dalla Chiara 1986, pp. 433ff.).

Thus, in the case of quantum logic the problem of distinguishing natural from non-natural predicates is solved with the help of the closure operator **. We shall see that in the case of colour predicates we can tackle this problem in a quite similar manner.

Refutative and Affirmative Properties. Most properties encountered in nature are instantiated to varying degree of typicality. For example, concerning the property 'is a bird' most people consider that property being instantiated typically by a robin but not by, say, a penguin. Somewhat more generally, we may assume that for each property we have a class of definite and typical instantiations, and a class of not so typical or *borderline cases*. These considerations are illustrated in the figure above.

They are made precise in the framework of topology as follows.

DEFINITION 3.4: (i) An open neighbourhood U(x) of x is an open set containing x. As is well known a set U of a topological space X is open iff it contains for each of its elements an open neighbourhood.
(ii) Let V be a subset of the topological space X. The closure cl(V) of V is the smallest closed set that contains V (cf. Davey and Priestley 1990, p. 36):

$$\mathrm{cl}(V) := \bigcap \{W: W \text{ closed and } V \subseteq W\}$$

(iii) x is a boundary point of V iff x cl(V) − V. The set of boundary points of V is denoted by bd(V). Evidently bd(V) = ∅ iff V is closed.

The following lemma is well known.

LEMMA 3.5: Let W be a subset of the topological space X. x is a boundary point of W iff every open neighbourhood U(x) of x has a non-empty intersection with W:

$$x \in bd(V) \Leftrightarrow \text{for all } U(x): U(x) \cap W \neq \emptyset$$

The element of an open neighbourhood U(x) of x may be considered as elements that are (more or less) similar or near to x. Or, in other words, y ∈ U(x) may be considered as a (more or less small) variation of x.

Expressed informally, x is a boundary point of U if x does not belong to U but any slight variation of x yields an element of U. Thus, if we consider U as the extension of some property, we may consider the boundary points of U as borderline cases of U. The topological concept of a boundary point gives rise to the following two extreme strategies to deal with the problem of borderline cases.

(1) We may consider the assertion 'a has property P' as true for all borderline cases. Then the subset P ⊆ C, which represents that property, satisfies cl(P) = P, i.e., P is closed. In this way an assertion 'a has property P' can be definitively *refuted* by showing that a does not belong to the set P. For example, if we take this option and we find something that is *not* a bird, i.e., is located outside of P, we may vary it slightly thereby still staying outside P.

(2) The other extreme strategy is to consider the assertion 'a has property P' as false for all borderline cases. Then the representing subset P ⊆ C is considered to be open since for its complement CP we get CP = cl(CP). In this way an assertion 'a has property P' can be definitively *confirmed* by showing that a does belong to the set P. If we opt for this choice and find something that is a bird, i.e., belongs to P, we may vary it slightly, and it still remains belonging to P. This is expressed topologically by asserting that the set representing P is open (or, equivalently, that the complement CP is closed).

In this way we get the following topologically based classification of properties:

(a) *Refutative* properties, represented by *closed* subsets of C.
(b) *Affirmative* properties, represented by *open* subsets of C.
(c) Properties that are neither refutative nor affirmative, represented by subsets neither closed nor open.[4]

Following a Popperian methodology, according to which the refutability of (scientific) assertions is to be considered as a virtue, we would consider refutative properties as natural. On the other hand it can be shown that for a "logic of finite observations" (cf. Vickers 1989) the class of affirmable properties should be considered as natural.

It does not seem advisable to insist dogmatically that once and for all we have to embrace *one* class of predicates as natural ones and oppose the other as non-natural. Rather, we may say that it depends on the case in question which class should be chosen.

Thus, the closure structure (or, more generally, any other convenient structure) of a conceptual space does not uniquely determine which predicates are to be considered as natural. Rather, it enriches our language enabling us to distinguish between several classes of natural predicates. In a second step we have to find out their respective advantages and drawbacks.

Natural Colour Predicates. Let C be a conceptual space with a product structure, i.e., $C = C_1 \times C_2$, for example, $C_1 = \{\text{blue, green, } \ldots\}$ and $C_2 = T$ (time), as Gärdenfors's colour cylinder. Then a closure structure on C is defined as follows.

Let $Z \subseteq C_1 \times C_2$, and let M_Z be the set of all $A \times B$ with $Z \subseteq A \times B$, $A \subseteq C_1$, $B \subseteq C_2$. Label the elements of M_Z by an index set I, and:

$$(3.6) \qquad M_Z := \{A_i \times B_i; i \in I\}$$

Now we can define a closure structure on C depending on the specific product structure $C_1 \times C_2$ as follows:

$$(3.7) \qquad J(Z) := \cap M_Z = \cap A_i \times B_i = \cap A_i \times \cap B_i := A_Z \times B_Z$$

One can easily check that the following holds.

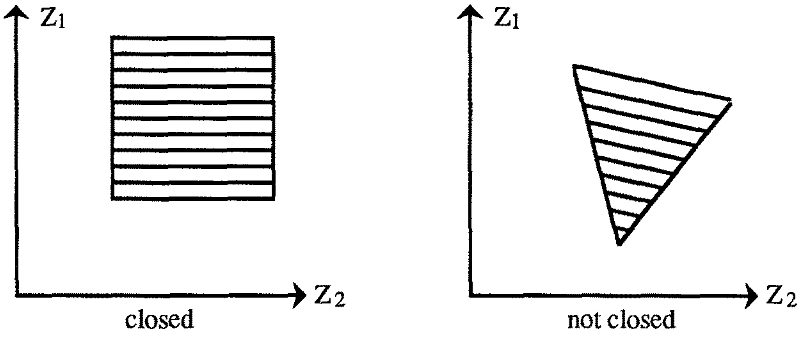

Fig. 7.

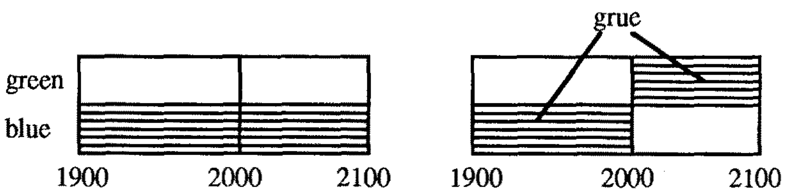

Fig. 8.

LEMMA 3.8: On C the operator (3.7) defines a closure structure in the sense of 3.1.

A set $Z \subseteq C$ is closed with respect to J iff it is a 'rectangle parallel to the axes C_1 and C_2 of the coordinate system', i.e., $Z = Z_1 \times Z_2, Z_i \subseteq C_i$.

Now we are in a position to prove that the notion of closedness may be used to distinguish between 'natural' and 'not-so-natural' predicates.

PROPOSITION 3.9: In the colour cylinder the predicates 'grue' and 'bleen' are represented by non-closed sets, whereas 'blue' and 'green' are represented by closed sets.

Proof: We just display the relevant parts of the colour cylinder C (cf. Gärdenfors 1990, p. 89).

However, one may doubt whether 3.9 can really be considered as a 'solution' to the grue–bleen paradox. Defining a 'Goodmanian' colour cylinder in the obvious way we get the following.

PROPOSITION 3.10: Let $C_1' = \{\text{bleen}, \text{grue}, \ldots\}$ and $C_2 = T(\text{time})$. Define the 'Goodmanian' colour cylinder by $C' = C_1' \times C_2$. Then the following holds: if C' is provided with its natural closure structure, 'grue' and 'bleen' are represented by closed subsets of C', whereas 'blue' and 'green' correspond to non-closed ones.

Since the selection of the conceptual space seems to be arbitrary and so "everybody can pick his own conceptual space thereby making his favorite predicates to come out as natural" (Gärdenfors 1990, p. 91), it seems we have gained nothing by moving from one's favourite pairs of predicates (('blue', 'green') or ('grue', 'bleen')) to one's favourite conceptual space (C or C').[5] However, this is not the case: the shift from predicates to conceptual spaces is definitively a turn to the better, and this is what I would like to argue in the following sections.[6]

The appeal of Goodman's riddle largely lies in the fact that the pairs (blue, green) and (grue, bleen) on the one hand appear to be totally symmetric, whereas on the other hand we want to give them a totally different epistemological status (cf. Mulhall 1989, p. 169). In the following section, I want to argue that the reformulation of the riddle in the framework of conceptual spaces enables us to elucidate this apparent paradox in a new way not accessible if we remain on the level of predicates. For this purpose we connect the topic of projectible predicates with the problem of geometric conventionalism. Thus, even if the framework of conceptual spaces does not lead to a definitive 'solution' to the problem of projectibility (it seems doubtful that problems of this kind have one), it points to some interesting connections with other important philosophical problems.

4. STRUCTURAL CONVENTIONALISM

It is not the aim of this paper to go into the matters of geometric conventionalism in any greater depth (for this compare Grünbaum (1973), Putnam (1975), and McKie (1988)). For our purposes it is sufficient to describe a special kind of *geometric conventionalism* favoured especially Grünbaum as the thesis that the metrical structure of physical space is a matter of convention, i.e., physical space is *metrically* *amorphous* and may be metrically structured in many different ways, and all these metrical structures have equal rights. I want to characterize *structural conventionalism* as the corresponding generalized thesis concerning the structure of conceptual spaces. Structural conventionalism claims that conceptual spaces, e.g., those of the colour language, are structurally amorphous, i.e., it is a matter of convention which structure is imposed on them.

Geometric conventionalism has been vigorously criticized by Putnam (cf. Putnam 1975, pp. 164–65; or McKie 1988, p. 85).

The aim of this section is to provide a formal basis for a similar line of criticism of structural conventionalism, thereby resolving or at least defusing the destructive scepticism spread by Goodmanian and other non-natural predicates.

For this purpose we have to work out further our formal apparatus, which we have developed this far, in order to prove the following proposition.

PROPOSITION 4.1: Let C be the 'natural' and C′ be the 'Goodmanian' colour cylinders defined in the previous section. Concerning C and C′ the following hold:

(i) C and C′ are isomorphic as sets.

(ii) C and C′ are not isomorphic as closures spaces, i.e., (C, J) and (C′, J′) are not isomorphic.

(iii) There is a conceptual space C_0 with two non-equivalent closure structures J_0 and J_0' defined on it such that (C_0, J_0) and (C, J) are isomorphic (as closure spaces) and (C_0, J_0') and (C, J′) are isomorphic (as closure spaces).

Proof: ad (i): This claim is essentially nothing but the assertion that the pairs (blue, green) and (grue, bleen) are interdefinable. Explicitly, an isomorphism from C to C′ is defined as follows:

$$(4.2.)\qquad (\text{blue}, t) \xrightarrow{U} \begin{cases} (\text{bleen}, t), & t < 2000 \\ (\text{grue}, t), & t \geqslant 2000 \end{cases}$$

$$(\text{green}, t) \xrightarrow{U} \begin{cases} (\text{grue}, t), & t < 2000 \\ (\text{bleen}, t), & t \geqslant 2000 \end{cases}$$

The inverse V: $C' \to C$ is defined in an analogous manner, interchanging 'natural' and 'Goodmanian' terms in (4.2).
ad (ii): The set-theoretical isomorphism U does not preserve the closure structures of C and C' since {blue × T} is closed in (C, J) but U({blue × T}) is not closed in (C', J').
ad (iii): Define the conceptual space C_0 by the following equivalent definitions:

$$(4.3) \qquad C_0 := \{\{x, U(x)\}; x \in C\}, \qquad C_0 := \{\{y, V(y)\}, y \in C'\}$$

Since U and V are isomorphism, obviously C_0 is isomorphic to C and C', by the following natural isomorphisms:

$$(4.4) \qquad \begin{array}{ll} C \xleftarrow{M} C_0 \xrightarrow{K} C', & C \xrightarrow{N} C_0 \xleftarrow{L} C' \\ M(\{x, U(x)\}) := x, & N(x) := \{x, U(x)\} \\ K(\{x, U(x)\}) := V(x), & L(y) := \{y, V(y)\} \end{array}$$

Define operators J_0 and J_0' on PC_0 as follows ($A \subseteq C_0$):

$$(4.5) \qquad J_0(A) := N(J(M(A))), \qquad J_0'(A) := L(J'(K(A)))$$

When we check the definitions we obtain the following:

(i) J_0 and J_0' are closure operators on C_0.
$(ii)_1$ (C_0, J_0) and (C, J) are isomorphic as closure spaces.
$(ii)_2$ (C_0, J_0') and (C', J') are isomorphic as closure spaces.
$(ii)_3$ (C_0, J_0) and (C', J'), respectively (C_0, J_0') and (C, J), are not isomorphic as closure spaces since (C, J) and (C', J') are not isomorphic.

This proves Proposition 4.1.[7]

It may be elucidating to consider the analogue of 4.1 in the case of conceptual spaces (frames) of possible world semantics: suppose we use a frame (U, R) to explain our modal intuitions. For example, for a proposition p we may explain the truth conditions of the proposition 'It is possible that p' by stating that 'It is possible that p' is true iff there is an (R-)accessible world i' to the actual world i_0 where p is true.

Now suppose somebody comes along and proposes a new frame (U', R') interdefinable with U in the sense that U and U' are isomorphic as sets. According to 4.1 we may construct frames and (U_0, R_0) and (U_0, R_0') such that (U, R) and (U_0, R_0) as well as (U', R') and $(U_0,$

R_0') are isomorphic as frames. Moreover, U, U′, and U_0 are isomorphic as sets. Considering propositions as sets of possible worlds, the isomorphisms between U, U′, and U_0 provide us with faithful translations. Thus one might be tempted to consider the conceptual systems based on (U_0, R_0) and (U_0, R_0') as equivalent. But this contention would probably turn out to be wrong: the truth values of the proposition 'It is possible that p' would differ in (U, R_0) and (U, R_0') since we disregarded the accessibility relations R_0 and R_0'. Hence the frames (U_0, R_0) and (U_0, R_0') are non-equivalent notwithstanding the fact that both are based on the same set of possible worlds U_0.[8]

That frames (U, R) and (U, R′) with different accessibility relations R and R′ are to be treated as different, is of course, a truism of possible world semantics. In the case of modal discourse, nobody would subscribe to fully fledged structural conventionalism that affirms that the logical space U of possible worlds is 'amorphous' with respect to accessibility relations R in such a way that frames (U, R) and (U, R′) with different accessibility relations could be considered as equivalent.

5. NATURAL PREDICATES AND STRUCTURES OF CONCEPTUAL SPACES

Let us evaluate what we have achieved so far with respect to the task of distinguishing natural from not-so-natural predicates: we have constructed a conceptual space C_0 that can be considered as a common background space.[9] On C_0 we can define closure or topological structures such that natural predicates are represented by subsets that are invariant or – more generally – natural with respect to these structures. A given predicate may be natural with respect to a structure \$ and non-natural ('Goodmanian') with respect to another structure \$′. Thus we are left with the problem of choosing the 'right' structure on C_0.

The thesis that everybody can choose his own conceptual structure (to make his preferred predicates come out as natural) can be reformulated as the radical conventionalist thesis asserting that all structures on C_0 are on an equal footing, i.e., it is a matter of convention if we choose \$ or \$′ just as it is a matter of convention if we use the unit 'metre' or 'yard' for measuring length.

Putnam has directed a powerful criticism against this kind of geometric conventionalism:

The conventionalist fails precisely because of an insight of Quine's. That is the insight that *meaning*, in the sense of reference, is a function of theory, and that the enterprise of trying to list the sentences containing a term which are true by virtue of its meaning, let alone to give a list of statements which *exhaust* its meaning is a futile one. (Putnam 1975, pp. 164–65)

Thus in the case of conceptual spaces the meaning of terms like 'naturalness' of predicates, defined on C_0 by the structures \$ (or \$′), is *not* exhausted by a short list of axioms (in our case the axioms J1–J3) of a closure structure, rather it is a function of an extended net of empirical knowledge: that is, we do *not* fix the reference of the term 'natural predicate of C_0' by convention but by coherence.

The fixation of projectibility by coherence means that the task of determining which of the subsets of C_0 represent natural predicates is not achieved by defining simple structures like closure or topological structures on C_0, rather it involves large parts of scientific (and cultural) background knowledge. To make this claim plausible let us take the concept 'metric of physical space' that was considered by Putnam (1975).

The coherential fixation of the term 'metric of physical space' proceeds in a series of approximations. A first step for the fixation of a physically meaningful metric of physical space is to impose the condition that a measuring rod is to remain the same length when transported. This condition is not sufficient to determine the metric of physical space uniquely, but at least it excludes certain contrived candidates that satisfy the metrical axioms but can hardly count as physically meaningful. Further steps of the approximation process may take into account constraints concerning the form of physical theories, e.g., invariance principles and the relations with other conceptual systems (theories) based on other conceptual spaces.

Now let us consider an example of a coherentist fixing of the concept of projectibility in the case of colour theories. For this purpose we may consider the neurobiological theory of colour vision that is based on the fact that the human eye possesses three different types of visual cells (let us call them cells of type B, G, and O) specifically adapted for the colours blue, green, and orange, respectively, such that every physiological colour is realized by a mixed stimulus of these three cell types (cf. Hubel 1988, Chap. 8).

The relation between this neurobiological theory of colour vision and the various 'common-sense' colour theories based on conceptual spaces, like the colour circle, the colour disks, etc., may be quite intricate, but in a first approximation it may be described by 'bridge principles' of the following kind. Let x be a 'colour event':

> x is (seen as) blue iff x preferably stimulates cells of type B.
> x is (seen as) green iff x preferably stimulates cells of type G.

For the Goodmanian predicates we get analogous assertions but more complicated disjunctive assertions:

> x is (seen as) bleen iff x preferably stimulates cells of type B when $t < 2000$,
> or x preferably stimulates cells of type G when $t \geqslant 2000$.

Since according to the neurological theory of colour vision there are no Goodmanian cells (preferably stimulated by bleen or grue, respectively), the predicates 'bleen' and 'grue' are certainly more complicated, and are to be considered as contrived. In this way the symmetry between traditional and Goodmanian predicates is broken. Hence, if we rely on a coherentist approach for fixing the meaning of 'natural', this counts as strong evidence against the Goodmanian predicates – the traditional ones are far better entrenched in the global system of our conceptual framework.

Of course this argument is not to be understood in the dogmatic sense that a set of traditional predicates is beyond any doubt – under certain circumstances we may be forced or, at least, be inclined to revise our conceptual system thereby changing what we regard as 'natural' and what we do not. Perhaps, contrary to the case of metrical structure of physical space, the problem of selecting natural predicates for a given conceptual space generally does not possess a clear-cut solution. Evidence for this conjecture is the fact that up to now many structurally very different colour spaces are in use and different spaces select different predicates as natural ones. As was pointed out in Section 2 this already occurs for the quite elementary spaces of the colour circle and the colour line.

Moreover, as is shown by the example of refutative and affirmable predicates, the definition of naturalness may be characterized by disjunctive criteria of the form:

> $P \subseteq C$ represents a natural predicate iff P satisfies either condition A or condition B.

It may be the case that some predicates satisfy both conditions. Thus, some predicates turn out to be more natural than others. This would be in line with David Lewis's remark "that the distinction between natural properties and others admits of degrees" (cf. Lewis 1986, p. 61).

In other words, a kind of approximation process has to take place if we want to fix the term 'natural' with respect to a conceptual space. For example, for the colour cylinder C_0 ($C_{01} \times C_{02}$) it is obvious that closedness in the sense of 3.9 may be a necessary condition that a subset Z is to represent a natural property. Surely it is not a sufficient one since it does not impose any restriction at all on the factors Z_1, Z_2 of $Z = Z_1 \times Z_2$. In fact, condition 3.9 may be satisfied by arbitrarily 'ugly' sets Z_1 and Z_2 for which we would never be prepared to accept $Z_1 \times Z_2$ as natural. Thus, closedness is not the solution to the problem: in order to sieve out the natural predicates it must be supported by further structural restrictions on the factors C_{01} and C_{02}. As was shown in Section 2 plausible restrictions are that the factors Z_1 and Z_2 of $Z_1 \times Z_2$ are open, closed, connected, or otherwise topologically well-behaved sets with respect to suitable factor topologies on C_{01} and C_{02}.

In any case the problems of distinguishing natural and non-natural predicates cannot be solved by a priori considerations or formal constructions. What is needed is a detailed structural description of the conceptual space in question and its relations with other conceptual spaces since projectibility is not – so to speak – in the possession of an insulated conceptual space but is specified by the whole of our conceptual apparatus, which is based on a great many number of conceptual spaces. Thus, the following approach of 'structural enrichment' to the problem of projectible predicates seems to be promising: conceptual spaces are *structured* sets, geometrically, topologically, or otherwise – the more structure we impose on them the better we can distinguish natural from not-so-natural predicates.

Even if we cannot exhaust the meaning of 'natural' by imposing an all-embracing final structure on the conceptual space in question, we can approximate it step by step, thereby eliminating more and more Goodmanian predicates of various degrees of sophistication.

This structural approach leads to a kind of 'mathematical epistemology' based not on the unspecific and general framework of set theory but on specific frameworks of appropriate mathematical theories, e.g., topological and geometrical ones.

NOTES

[1] For the role of symmetries and invariances of models in science, see van Fraassen (1989, Part III).

[2] This result should not be interpreted as the sweeping thesis that convexity is of no use whatsoever for the distinction of natural and non-natural predicates. It may well be the case that for certain conceptual spaces, e.g., the conceptual spaces of states in quantum mechanics (cf. Beltrametti and Cassinelli 1981, Chap. 9), convexity is useful. It is, however, to be understood as an argument for the plurality of naturality criteria.

[3] It is easy to see that Gärdenfors's approach, based on the criterion of convexity, is just a special case of the topological approach: given a set M of a conceptual space C endowed with a convex structure we can form the convex hull cv(M) defined as the intersection of all convex subsets of C that contain M. cv(M) is the smallest convex set containing M. As is well known the operation of forming the convex hull is a closure operator in the sense of Definition 3.1.

[4] It can be noted that there may be properties that are refutative as well as affirmative. These are represented by subsets of C that are closed *and* open ('clopen'). Although for the usual topologies of metric spaces there are no non-trivial 'clopen' sets, for many topologies, especially adapted for logics and theoretical computer science, there are non-trivial clopen sets (cf. Vickers 1989).

[5] To avoid the resulting relativism we can rely on the following evolutionary argument: projectible predicates are the basic ingredients of valid inductions. In order to survive we must have been able to make valid inductions (at least more often than not). Thus, we have only a rather limited freedom in choosing the structure of our conceptual spaces.

I do not think that this sweeping argument is fundamentally wrong, but of course it already works on the predicate's level ((blue, green) vs. (grue, bleen)). It does not justify the step from the level of predicates to the level of conceptual spaces.

[6] Evidently the latter are more holistic concepts and following Quine's "third milestone of empiricism" (Quine 1981, pp. 70ff.), it could be argued that the shift to more holistic concepts in general provides an epistemological improvement.

[7] We may interpret 4.1 as the assertion that the traditional and the Goodmanian frameworks are analogous to *as well as* different from each other: they are analogous since they are set-theoretically isomorphic, and they are different with respect to their closure structure.

In Mulhall (1989) the author points out that Goodman's riddle can arise only if the conceptual framework based on the predicates 'grue' and 'bleen' is "analogous to *as well as* different from the framework of our colour concepts" (Mulhall 1989, p. 169); he claims that this is impossible and therefore Goodman's riddle is no riddle at all (ibidem, p. 172). This thesis is not supported by our formal reconstruction 4.1.

[8] An analogous thesis holds of course for orthoframes (U, R) and (U, R′) of quantum logics with the same logical space U but different accessibility relations R and R′.
[9] In our case, C_0 is simply a set but generally it may well be a *structured* set. The important thing is that C_0 may be endowed with *further* structure.

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Institut für Philosophie
Freie Universität Berlin
Habelschwerdter Allee 30
D-1000 Berlin 33
Germany