Eq. (12) we obtain $\bar{I}(x_0, z) \approx 1$. Thus, a developed turbulent AID region in general does not change the energy characteristics of passing short wave radiation.

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THEORY FOR THE FORMATION OF RESONANCE STRUCTURE IN THE SPECTRUM OF ATMOSPHERIC ELECTROMAGNETIC BACKGROUND NOISE IN THE RANGE OF SHORT-PERIOD GEOMAGNETIC PULSATIONS

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We have developed a theory for the formation of resonance structure in the spectrum (RSS) of ultra-low-frequency (ULF) magnetic field which has been observed in the frequency range F = (0.1-10) Hz, and which manifests itself in the form of alternating maxima and minima in the spectrum with a frequency interval ΔF = (0.5-2.5) Hz. In the theoretical model, we allow for the sphericity of the earth, and also for anisotropy and inhomogeneity in the ionosphere. We show that since the admittance of the ionosphere has a resonant dependence on frequency (due to the excitation of oscillations in the ionospheric Alfvenic resonator), this leads to RSS. We have calculated the planetary distribution of the tangential components of the magnetic fields which are generated by a vertical lightning discharge. We show that the resonance structure in the spectrum has a magnetic field component corresponding to the source direction along a great circle arc. The RSS parameters are determined by the structure of the ionosphere in the region of the observing point (localization). The theory which we have developed enables us to explain the principal experimental facts.

1. In our earlier papers [1-3], we described the results of experimental investigations into the structure of the spectrum of the regular noise background in the magnetic component of the electromagnetic field in the frequency range F = (0.1-10) Hz. The major result in those papers was the discovery and detailed investigation of resonance structure in the averaged spectra. (The averages were performed over roughly 100 independent spectral realizations.) This structure, together with the Schumann resonance [6], is a fundamental characteristic of ultra-low-frequency (ULF) electromagnetic noise. The resonance structure which

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we have discovered in the spectra (RSS) manifests itself in the form of alternating maxima and minima in the spectrum with a frequency interval $\Delta F \approx (0.5\text{-}2.5)$ Hz. The depth of modulation in the spectrum may be as large as 50%. We can formulate the major results of the experimental investigations of RSS parameters:

— RSS is observed regularly in the tangential components of the magnetic field, at least under night-time conditions;

— the RSS frequency interval ΔF has a characteristic diurnal behavior: ΔF is maximum during the night-time hours, and it decreases sharply during the morning and evening;

— the diurnal behavior of ΔF is closely correlated with the diurnal behavior of f_0^{-1}, where f_0 is the critical frequency of the F-layer in the region of the observing point (RSS localization);

— the depth of modulation of the spectrum is greatest during the night-time hours: there is a sharp reduction in the morning and evening, and RSS is observed only very rarely during the day;

— RSS is observed as a rule in one of the two orthogonal components of the tangential field;

— the regular background noise which contains RSS apparently originates in lightning.

It was mentioned in [2] that the most natural and probable cause for the formation of RSS is the effect of the ionospheric Alfvenic resonator (IAR) [4, 5] on the propagation of electromagnetic fields from lightning flashes. The separation between the IAR eigenfrequencies (and their diurnal behavior) correspond to the magnitude (and diurnal behavior) of the RSS frequency interval ΔF [2].

The aim of the present paper is to develop a theoretical model for the formation of RSS, allowing for the sphericity of the earth, the anisotropy and inhomogeneity of the ionosphere, and also the finite conductivity of the earth.

2. From the point of view of the theory of electromagnetic wave propagation in the frequency range $F = (0.1\text{-}10)$ Hz, the most important qualitative feature of the medium around the earth is the existence of three ranges of altitude where geometric optics break down: at the surface of the earth, in the lower ionosphere, and in the region above the ionospheric F-layer maximum (where the electron density is falling off). This feature determines the existence of the earth-ionosphere waveguide (resonator) in the cavity between the earth and the ionosphere, and at the same time, it also determines the existence of an ionospheric Alfvenic resonator (IAR) and an ionospheric magnetosonic waveguide (IMW) (in the upper ionosphere). Let us consider a plane-parallel model for the medium, with vertical magnetic field directed along the z-axis. The earth's surface lies at the level $z = 0$; for $0 < z < h$, there is a vacuum gap ($h \approx 60$ km); and for $h < z < h_1$, we have the lower ionosphere ($h_1 \approx 200$ km). We will model the lower ionosphere in the form of a disk with a tensor surface conductivity $\hat{\Sigma}$:

$$\hat{\Sigma} = \begin{pmatrix} \Sigma_P & \Sigma_H \\ -\Sigma_H & \Sigma_P \end{pmatrix}. \tag{1}$$

Here, Σ_P and Σ_H are integrated Pedersen and Hall conductivities. This sort of representation can be justified when the optical depth τ of the lower ionosphere is less than unity. Estimates show that in the conditions which exist in the mid-latitude night-time ionosphere, $\tau < 1$ for $F < 20$ Hz, while for the daytime ionosphere, $\tau < 1$ for $F < 0.2$ Hz. We recall that RSS is usually observed at night.

We will describe electromagnetic waves in the upper ionosphere ($z > h_1$) in the framework of magnetohydrodynamics. We prescribe the following model for the Alfvenic refractive index:

$$n^2 = n_A^2\,\{\varepsilon^2 + \exp[-2(z - h_2)/L]\}, \quad z > h_2,$$
$$n^2 = n_A^2\,(1 + \varepsilon^2), \quad h_1 < z < h_2. \tag{2}$$

Here $n_A(1 + \varepsilon^2)$ is the Alfvenic refractive index at the maximum of the ionospheric F-layer: $n_A \approx (1\text{-}2)\cdot 10^3$, $h_2 \approx (300\text{-}400)$ km, $L \approx (100\text{-}300)$ km, and $\varepsilon^2 \approx 10^{-2}\text{-}10^{-3}$. In [5], the reflection coefficient was calculated for an Alfven wave incident from below on the half-space $z > h_2$. According to [5],

$$R = |R|e^{i\varphi}, \quad |R| = 1 - \pi\varepsilon k_A L, \quad \varphi = 2(k_A L - \pi/4). \tag{3}$$

Here, $k_A = n_A k_0$, and $k_0 = 2\pi F/c$. Eq. (3) is valid when the following inequalities are satisfied:

$$k_A L \gg 1, \qquad (\pi/2)\varepsilon k_A L \ll 1. \tag{4}$$

The impedance of the half-space $z > h_2$ is related to the reflection coefficient (3) by the relationship

$$Z = \frac{Z_0}{n_A}\frac{1+R}{1-R} = Y^{-1}. \tag{5}$$

Here, Z_0 is the impedance of vacuum, and Y is the admittance. The input impedance of the half-space $z > h_1$ can be calculated using (2) and (5), and the impedance conversion formula [8]. At the lower boundary of the range we are considering, where the optical thickness of the layer $h_1 < z < h_2$ is small, we can use Eq. (5) for estimating the impedance of the half-space $z > h_1$. In what follows, we shall be interested mainly in the spectrum of ULF background noise at the earth's surface which is created by distant lightning (thousands of kilometers or more). In this case, the characteristic horizontal scale of that part of the field which is associated with propagation in the earth-ionosphere gap is of the same order as the distance to the source: it is substantially greater than the wavelength in the ionosphere. In this case, obviously, the impedances of the upper ionosphere for Alfven waves and for magnetosonic waves coincide with Eq. (5). This statement is valid for the case of a vertical magnetic field. If the magnetic field is inclined, wave transformation is possible upon reflection, and the impedance of the upper ionosphere takes on a matrix form. This matter requires special investigation.

Let us now consider the effects of the lower ionosphere. Assuming that the horizontal electrical field is continuous in the optically thin lower ionosphere, we will have that the following relationships are valid for the magnetic field:

$$H_x(h) - H_x(h_1) = \Sigma_H E_x - \Sigma_P E_y,$$

$$H_y(h) - H_y(h_1) = \Sigma_P E_x + \Sigma_H E_y.$$

In the case which we are considering (vertical magnetic field), we may without loss of generality consider that the field depends on the horizontal coordinate as $\exp(ik_x x)$.

The boundary conditions in the upper ionosphere (in the half-space $z > h_1$) have the following form in the impedance approximation:

$$\frac{E_x(h_1)}{H_y(h_1)} = -\frac{E_y(h_1)}{H_x(h_1)} = Z = Y^{-1}. \tag{7}$$

In view of (7), the relationships in (6) take on the form

$$H_x(h) = Y_2 E_x(h) - Y_1 E_y(h), \quad H_y(h) = Y_1 E_x(h) + Y_2 E_y(h). \tag{8}$$

Here, $Y_2 = \Sigma_H, \quad Y_1 = \Sigma_P + Y, \quad \Sigma_P \simeq \Sigma_H.$

3. Wait [7] constructed a solution of the problem of electromagnetic wave propagation in spherical Earth-ionosphere cavity, when the cavity is excited by a vertical electric dipole. In [7], the ionosphere and the Earth were modeled by spherically symmetric impedance walls: the anisotropy of the ionosphere was allowed for by introducing a matrix admittance. By using a representation of the solution in the form of a series over normal modes, Wait [7] obtained explicit analytic results in an application to the VLF range. In this section, we consider an analogous problem, except that we consider a frequency range which lies below the first Schumann resonance ($F < 8$ Hz): in this range, the electromagnetic field in the Earth-ionosphere cavity is quasistatic in nature. As will be obvious in what follows, this feature

enables us to advance significantly farther than in [7]. We consider a spherically symmetric model consisting of a perfectly conducting Earth ($r < a$), a vacuum gap ($a < r < d$), and an ionosphere ($r > d$). We use a spherical coordinate system (r, θ, φ). At the point $\theta = 0$, $r = b$ (where $a < b < d$), we place a vertical electric dipole with moment $I\ell$. By analogy with [7], we introduce the potentials U and V, corresponding to the radial components of the electric and magnetic Hertz vector. In the case of azimuthal symmetry $\left(\frac{\partial}{\partial\varphi}=0\right)$, the electromagnetic fields in the cavity $a < r < d$ are related to the potentials U and V by the relationships

$$
\begin{aligned}
E_r &= \left(\frac{\partial^2}{\partial r^2}+k_0^2\right)(Ur), \qquad H_r=\left(\frac{\partial^2}{\partial r^2}+k_0^2\right)(Vr),\\
E_\theta &= \frac{1}{r}\frac{\partial^2}{\partial\theta\partial r}(Ur), \qquad H_\theta=\frac{1}{r}\frac{\partial^2}{\partial\theta\partial r}(Vr),\\
E_\varphi &= -\frac{i\omega\mu_0}{r}\frac{\partial}{\partial\theta}(Vr), \qquad H_\varphi=\frac{i\varepsilon_0\omega}{r}\frac{\partial}{\partial\theta}(Ur).
\end{aligned} \tag{9}
$$

In the quasistatic limit ($k_0 a \ll 1$), the potentials U and V in the cavity ($a < r < d$) should satisfy the equation

$$
\Delta U=\frac{I l\delta(r-b)\delta(\theta)}{2\pi i\omega\varepsilon_0 b r^2\sin\theta}, \qquad \Delta V=0. \tag{10}
$$

The boundary conditions at the perfectly conducting Earth ($r = a$) are

$$
V|_{r=a}=0, \qquad \left.\frac{\partial(Ur)}{\partial r}\right|_{r=a}=0. \tag{11}
$$

In the ionosphere, we prescribe the matrix for the surface admittance:

$$
\begin{aligned}
H_\theta &= Y_{\theta\theta}E_\theta+Y_{\theta\varphi}E_\varphi,\\
H_\varphi &= Y_{\varphi\theta}E_\theta+Y_{\varphi\varphi}E_\varphi.
\end{aligned} \tag{12}
$$

Initially we shall assume that the components of the surface admittance matrix are independent of θ and φ (spherical symmetry). Since the vertical dimension of the IAF is substantially smaller than the radius of the earth, we shall use the results of the previous section to estimate the components of the admittance. In particular, for the model in which the magnetic field has radial spherical symmetry,

$$
Y_{\theta\theta}=Y_{\varphi\varphi}=Y_2, \quad Y_{\varphi\theta}=-Y_{\theta\varphi}=Y_1. \tag{13}
$$

Here, Y_1 and Y_2 are determined from Eqs. (8).

In view of Eq. (9), the boundary conditions in the ionosphere (12) can be written in the form

$$
\begin{aligned}
\frac{\partial}{\partial r}(Vr) &= Y_{\theta\theta}\frac{\partial}{\partial r}(Ur)-Y_{\theta\varphi}i\mu_0\omega(Vr),\\
i\varepsilon_0\omega(Ur) &= Y_{\varphi\theta}\frac{\partial}{\partial r}(Ur)-Y_{\varphi\varphi}i\mu_0\omega(Vr).
\end{aligned} \tag{14}
$$

We can write down the general solution of the homogeneous equations (10) [9]:

$$
U_0=\sum_{n=0}^{\infty}\left[A_n\left(\frac{r}{a}\right)^n+B_n\left(\frac{r}{a}\right)^{-n-1}\right]P_n(\cos\theta), \tag{15}
$$

$$V_0 = \sum_{n=0}^{\infty}\left[M_n\left(\frac{r}{a}\right)^n + N_n\left(\frac{r}{a}\right)^{-n-1}\right]P_n(\cos\theta). \tag{15}$$

Here, $P_n(\cos\vartheta)$ are orthogonal Legendre polynomials. The particular solution of the inhomogeneous Eq. (10) which has the requisite properties at the source is [7]

$$U_H = -\frac{Il}{4\pi i\varepsilon_0\omega}\frac{1}{bR} = C\frac{b}{R}. \tag{16}$$

Here, $R^2 = r^2 + b^2 - 2rb\cos\vartheta$. The general solution is $U = U_0 + U_H$. Furthermore, if we use the representation [9]

$$\frac{1}{R} = \begin{cases} \dfrac{1}{b}\sum\limits_{n=0}^{\infty}\left(\dfrac{r}{b}\right)^n P_n(\cos\theta), & r < b \\ \dfrac{1}{r}\sum\limits_{n=0}^{\infty}\left(\dfrac{b}{r}\right)^n P_n(\cos\theta), & r > b \end{cases}, \tag{17}$$

and the solutions of Laplace's equation (15) and (16), plus the boundary conditions (11) and (14), as well as the orthogonality of the Legendre polynomials, we can obtain a system of linear equations from which we can derive the constants A_n, B_n, M_n, and N_n. Omitting the cumbersome intermediate calculations, we can immediately write down approximate solutions of this system of equations:

$$\begin{aligned} B_n &= CD^{-1}[(1+ik_0hY'_{\theta\varphi})(ik_0h - Y'_{\varphi\theta}) + ik_0hY'_{\theta\theta}Y'_{\varphi\varphi}], \\ M_n &= -CD^{-1}Y'_{\theta\theta}\frac{ik_0a}{n+1}, \\ A_n &= \frac{n}{n+1}B_n - C, \quad N_n = -M_n, \\ D &= (1+ik_0hY'_{\theta\varphi})\left(\frac{ik_0a}{n+1} - Y'_{\varphi\theta}\frac{nh}{a}\right) + ik_0hY'_{\theta\theta}Y'_{\varphi\varphi}nha^{-1}. \end{aligned} \tag{18}$$

Here, and in what follows, $Y' = Z_0Y$. In obtaining (18), we have made use of the obvious inequality $h \ll a$. If the following inequalities are satisfied

$$\left|\frac{k_0a^2}{hY'_{\varphi\theta}}\right| \ll 1, \quad \left|\frac{Y'_{\varphi\theta}(1+ik_0hY'_{\theta\varphi})}{k_0hY'_{\theta\theta}Y'_{\varphi\varphi}}\right| \gg 1 \tag{19}$$

Eqs. (18) become simplified so much that the series for the electromagnetic fields (9), (15), and (18) can be summed in individual cases. In particular, the tangential components of the magnetic field at the earth can be expressed by the formulas

$$H_\varphi = -\frac{Il}{4\pi ah}\frac{\partial}{\partial\theta}\left[\sum_{n=1}^{\infty}\frac{2n+1}{n(n+1)}P_n(\cos\theta)\right] = \frac{Il}{4\pi ah}\operatorname{ctg}\frac{\theta}{2}; \tag{20}$$

$$H_\theta = \frac{Y_{\theta\theta}H_\varphi}{Y_{\varphi\theta}(1+ik_0hY'_{\theta\varphi})}. \tag{21}$$

In the formula for H_φ, the components of the ionospheric admittance matrix do not enter. Moreover, it can be shown that Eq. (20) coincides with the expression for H_φ in the case where the ionosphere is a perfectly conducting wall. In the quasistatic limit $\omega \to 0$, or more precisely, when the following inequality is satisfied

$$k_0hY'_{\theta\varphi} \ll 1 \tag{22}$$

the ratio H_ϑ/H is equal to $Y_{\vartheta\vartheta}Y_{\varphi\vartheta}$. Inequality (22) is more stringent than the second of the inequalities in (19): The reason is that, according to (8) and (13), $|Y_1| > |Y_2|$. It is interesting to note that the quasistatic limit of the ratio H_ϑ/H_φ follows directly from the boundary conditions in the ionosphere (12) if we neglect the rotational electric field

E_φ. When inequality (19) is satisfied, and when $\vartheta \ll 1$, Eqs. (20) and (21) go over into the corresponding formulas for a plane-parallel model of the ionosphere [2].

4. Let us consider the physical meaning of the inequalities in (19) by means of which we have obtained the simple analytic formulas (20) and (21) for the magnetic field of lightning discharges in the quasistatic approximation ($k_0 a < 1$). To do this, let us write down the impedance Z_{TH} and Z_{TE} for the half-space $z < h$ for the fields of different types of polarization (TH, TE). In the plane-parallel approximation, using the form $\exp(ik_\perp x)$ for the field as a function of the horizontal coordinates, it is easy to derive

$$Z_{TH} = Z_0 \frac{i\varkappa}{k_0} \operatorname{tg} \varkappa h, \quad Z_{TE} = -Z_0 \frac{ik_0}{\varkappa} \operatorname{tg} \varkappa h. \tag{23}$$

Here $\kappa = \sqrt{k_0^2 - k_\perp^2}$, and $Z_0 = 120\pi\ \Omega$ is the impedance of vacuum. Setting $k_\perp \sim a^{-1}$, and using $h \ll a$, we can see that by comparing the first inequality in (19) with (23), it may be rewritten in the form

$$(Y_{\varphi\theta})^{-1} \ll Z_{TH}. \tag{24}$$

This inequality has an obvious physical meaning. For a field with TH-polarization, the ionosphere is to a first approximation a perfectly metallic wall, since its impedance $(Y_{\varphi\vartheta})^{-1}$ is much less than the impedance of the half-space $z < h$. By further analogy, we may also find that $Z_{TE} \approx -ik_0 h Z_0$, i.e., $E_\varphi = -ik_0 h Z_0 H_\vartheta$. Substituting this relationship into the boundary conditions (12), we find that the second inequality in (19) can be written in the form

$$Y_{\varphi\varphi} E_\varphi \ll Y_{\varphi\theta} E_\theta. \tag{25}$$

This inequality indicates that we can neglect the transformation of the field of TE-polarization into the field of TH-polarization. Generally speaking, the inverse transformation in this case may be significant. The physical meaning of this fact can apparently be understood as follows. The components of the field E_ϑ and H_φ are the quasielectrostatic portion of the field, whereas the components E_φ, and H_ϑ are quasimagnetostatic, i.e., $E_\varphi/H_\vartheta \to 0$ as $\omega \to 0$. When inequality (22) is satisfied, we can neglect all terms in the boundary conditions (12) which contain the rotational electric field E_φ. From this, a simple physical picture follows for the formation of magnetic field from a lightning discharge. The discharge generates an electromagnetic field with TH-polarization (H_φ, E_ϑ, E_r), and for this field, the ionosphere can be considered as a perfectly metallic wall when inequality (19) is satisfied. Due to Hall currents, the gyrotropic ionosphere transforms this field into a quasimagnetostatic field with TE-polarization: roughly speaking, the latter contains only a magnetic component. When inequality (22) is satisfied, all currents flowing in the ionosphere (both rotational and potential) are determined solely by the component of the quasielectrostatic field E_ϑ.

Now let us admit that the ionosphere is horizontally inhomogeneous. In order for the impedance boundary conditions (12) to be valid locally, the horizontal scale of the inhomogeneities ℓ_1 must be substantially smaller than the wavelength in the ionosphere $\lambda_A(\lambda = \lambda/2\pi)$. In this case, it is obvious that if the inequalities in (19) are valid over the entire ionosphere, then the field H_φ will be determined, as before, by Eq. (20). Furthermore, if we use the relationship

$$E_\varphi = -ik_0 h Z_0 H_\theta, \tag{26}$$

(which is valid for $\ell_1 \gg h$), and if we apply the boundary conditions (12) as well as inequalities (19), we will obtain Eq. (21) for H_ϑ at the level $z = h$. Now we can rescale this field to the earth ($z = 0$ according to the magnetostatic formula, although it is obvious that there will be practically no change in H_ϑ if $\ell_1 \gg h$.

5. The ideas which we have developed in the present work enable us to explain the basic properties of resonance structure in the spectrum. We can now make some estimates. For purposes of estimating, we omit the second term in the denominator of Eq. (21): The role of this term decreases as the frequency decreases. In this case, using (5), (8), and (13), the ratio H_ϑ/H_φ can be written in the form

$$\frac{H_\theta}{H_\varphi}=\frac{\Sigma_H/\Sigma_W}{\Sigma_P/\Sigma_W+(1-R)/(1+R)}. \qquad (27)$$

Here $\Sigma_W = n_A/Z_0$ is the wave conductivity of the F-layer. In the night-time ionosphere, $\Sigma_P/\Sigma_W \simeq 0.1$, and $\Sigma_H/\Sigma_P \simeq (1-3)$. the function $(1 - R)/(1 + R)$ takes on a resonant character due to the effects of IAR: according to (3), this function varies over the range

$$\frac{\pi\varepsilon k_A L}{2} \leqslant \left|\frac{1-R}{1+R}\right| \leqslant \frac{2}{\pi\varepsilon k_A L}. \qquad (28)$$

as a function of Φ. Setting $n_A = 10^3$, $L = 100$ km, $\varepsilon = 10^{-1}$, and $F = 1$ Hz, we find $0.3 \leq |(1 - R)/(1 + R)| \leq 3$. With $\Sigma_H/\Sigma_P = 2$ this corresponds to variations of H_ϑ/H from 0.08 to 0.5. The separation between maxima (minima) of the resonance structure, ΔF, is determined by the condition $\Delta\varphi = 2\pi$, where φ is the phase of the reflection coefficient R (3). From this we find $\Delta F = c(2n_A L)^{-1} \approx 1.5$ Hz. These estimates correspond to the characteristics of the resonance structure which have been observed experimentally.

With increasing electron density N_e in the upper ionosphere, n_A increases as $\sqrt{N_e}$, i.e., ΔF varies as $(N_e)^{-1/2}$. From this, it follows that ΔF varies as f_0^{-1} (where f_0 is the critical frequency of the ionospheric F-layer): This agrees with the experimental data on the diurnal behavior of ΔF. We emphasize that, according to the ideas which we have developed, resonance structure should be observed only in the H_ϑ-component of the horizontal magnetic field (along the direction to the source). If we suppose that there is only a single source, then according to the above estimates, very large modulation of the spectrum is possible (practically up to 100%) for special orientation of the receiving antenna. Actually, there are always several effective sources (centers of lightning), and there will be a mixture of various field components at the receiving antenna, some with resonance structure, some without. (In the terminology which was used above, these would be H_ϑ and H_φ, respectively.) In the experiments which have been performed [1-3], one magnetic antenna was oriented with its axis toward the African lightning center, while the second (orthogonal to the first), was oriented toward the American and Australian centers. If we consider the fields from different lightning centers as statistically independent sources at a particular antenna, they will combine as $\sqrt{H_1^2 + H_2^2 + \ldots}$ then it becomes understandable why RSS is observed experimentally only in one of the two orthogonal components of the horizontal magnetic field. As an example, let us consider an antenna which has been oriented toward the African source: The magnetic field has RSS at the antenna in question. When other sources are measured by this antenna, they should have smooth spectra. If RSS is to be observed in the resultant spectrum, the amplitude of the field from the African center must be substantially greater than the amplitude of the fields from the other sources (according to the summation law). If the opposite is true, RSS will be observed in the orthogonal antenna.

Let us consider the range of applicability of Eqs. (20) and (21), which are determined by inequalities (19). According to the above estimates of the quantity $(1 - R)/(1 + R)$ and the definition (5), the component of ionospheric admittance Y_1 varies across the spectrum in the range

$$Y_1' = Y_1 Z_0 \sim (0.3-3)\cdot 10^3. \qquad (29)$$

From this, setting $h = 60$ km and $a \approx 6\cdot 10^3$ km, we obtain the following estimate from the first of the inequalities in (19):

$$\left|\frac{k_0 a^2}{h Y'_{\varphi\theta}}\right| \leqslant 4F(\text{Hz})\cdot 10^{-2}. \qquad (30)$$

We recall that the model which we are using for the ionosphere is suitable only in night-time conditions. In order to estimate the admittance of the daytime ionosphere, we can use an exponential model for the lower ionosphere [10]. According to this,

$$Y_1^{-1} \simeq -k_0 l_2 Z_0 \ln(k_0 l_2). \qquad (31)$$

Here, ℓ_2 is a height scale for inhomogeneities in the lower ionosphere ($\ell_2 \approx 15$ km). Setting $F = 2$ Hz, we find from this that $|Y_1'| \approx 2.5\cdot 10^2$, i.e., on the same order as the smallest night-time value. Thus, in the frequency regime below the first Schumann resonance, the first of inequalities (19) is satisfied not only during the day but also in the night-time ionosphere. For the second inequality in (19) to be satisfied, it is sufficient to have

$$|Y_2^2| \ll |Y_1^2|, \quad |k_0 h Y_1'| \gtrsim 1. \tag{32}$$

In an analogous manner to what we did above for these inequalities, we can obtain the estimate $|Y_2{}^2|/|Y_1{}^2| \leq 0.07$, and $k_0 h Y_1' \approx (0.3\text{-}3)F$ (Hz). Thus, in the frequency range which we are considering, all of the necessary inequalities are satisfied with a margin of a factor of roughly ten.

And now a few words about the approximation of a perfectly conducting earth. Since the first inequality in (19) characterizes the possibility of using a model of the ionosphere as a perfectly conducting wall for fields of TH-polarization, then obviously if we wish to use a mode with a perfectly conducting earth, an analogous inequality should be satisfied with an appropriate substitution for the admittance. This inequality is satisfied with an even larger margin, since the typical admittance of the earth is at least an order of magnitude greater than the admittance of the ionosphere.

We ought to have some reservations about using a model for the earth's magnetic field $\boldsymbol{H}_0$ which does not satisfy the equation $\operatorname{div} \boldsymbol{H}_0 = 0$. The only reason this model is realistic is that the effect is localized. The fact that it is localized means that the component of the field H_α which has the resonance structure is determined by the magnitude and orientation of $\boldsymbol{H}_0$ only in the vicinity of the observing point. RSS owes its very existence to the fact of localization, since the IAR eigenfrequencies are a function of the distribution of electron density in the ionosphere, and of the inclination of the earth's magnetic field: These may be quite different at different points on the earth's surface. A fundamental point about localization is as follows. We have shown in the present paper that the ULF field H_φ is to a first approximation independent of the state of the ionosphere: For the H_φ calculations, the ionosphere may be regarded as a perfectly metallic wall. In this case, the tangential component of the electric field E_ϑ with the appropriate polarization (TH) in the lower ionosphere is determined by the local value of the admittance $Y_{\varphi\theta}$. But it is precisely this component of E_ϑ which (as a result of the Hall conductivity) acts as a source for a ULF field with TE-polarization: In particular, it is a source for H_ϑ, which contains RSS.

According to the estimates made above, the second term in the denominator of Eq. (21) is substantial. From a qualitative point of view, this should lead to the appearance of double resonance structure with frequency intervals of ΔF and $\Delta F/2$. This is observed experimentally [3].

For a more detailed comparison between theory and experiment, the very least we need is to have numerical calculations of the admittance matrix for a realistic ionosphere.

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