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CONTROL OVER LINEAR PULSE SYSTEMS

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Rank conditions for control of linear pulse systems are established. The Pontryagin maximum principle is obtained in sufficient form. An example of control synthesis in a problem for linear pulse systems is given.

In [1, 2], a method was suggested for solving problems of control over linear systems based on the normal solvability of boundary-value problems. A similar approach was given in [3]. In the present paper, we develop these ideas for pulse systems.

Let us fix real numbers $\alpha, \beta \in R$, $\alpha < \beta$, and integer numbers $r > 0, p > 0$. We denote the set of square integrable functions $\varphi\colon [\alpha, \beta] \to R^r$ by $L_2[\alpha, \beta]$, and the set of sequences $\xi_i \in R^r$, $i = \overline{1, p}$, by $D^r[\overline{1, p}]$. Let us construct the space $\Pi_p^r = L_2^r \times D^r$, denote its elements by $\{\varphi, \xi\}$, and define a scalar product in it as follows:

$$\langle \{\varphi, \xi\}, \{w, \nu\} \rangle = \int\limits_\alpha^\beta (\varphi, w)\, dt + \sum_{i=1}^p (\xi_i, \nu_i),$$

where $(\,,\,)$ is a scalar product in R^r.

Consider the pulse system

$$\frac{dx}{dt} = A(t)x + C(t)u(t) + f(t), \quad t \neq t_i,$$

$$\Delta x|_{t=t_i} = B_i x + D_i v_i + I_i \tag{1}$$

with the boundary condition

$$x(\alpha) = a, \quad x(\beta) = b. \tag{2}$$

Here, $x \in R^n$, A and C are $n \times n$ and $n \times m$ matrices, respectively, with elements from $L_2^1[\alpha, \beta]$, B_i and D_i are constant $n \times n$ and $n \times m$ matrices, respectively, t_i, $i = \overline{1, p}$, is a sequence strictly ordered in the interval $]\alpha, \beta[$. We also set $\det(E + B_i) \neq 0$, $i = \overline{1, p}$. The solutions of system (1) are functions absolutely continuous in each interval $[\alpha, t_i]$, $]t_i, t_{i+p}]$, $i = \overline{1, p-1}$, $]t_p, \beta]$.

We say that the control problem I is solvable if, for any $\{f, I\} \in \Pi_p^n$ and any $a, b \in R^n$, there exists $\{u, v\} \in \Pi_p^m$ for which the boundary-value problem (1), (2) has a solution.

Parallel with problem (1), (2), we consider system (1) with the boundary condition $x(\alpha) = x(\beta) = 0$, which is called the control problem II.

By analogy with [2], one can check that control problems I and II are simultaneously solvable.

Let

$$\frac{dx}{dt} = A(t)x, \ t \neq t_i, \quad \Delta x|_{t=t_i} = B_i x \tag{3}$$

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be the system corresponding to (1), and let

$$\frac{dy}{dt} = -A^T(t)y, \ t \neq t_i, \qquad \Delta y|_{t=t_i} = -(E+B_i^T)^{-1}B_i^T y \tag{4}$$

be the system adjoint to (3). Here, T denotes transposition.

In what follows, we denote by $\{\varphi, \varphi\}$ an element $\{\varphi(t), \varphi(t_i)\}$ of the space Π_p^r for an arbitrary function $\varphi\colon [\alpha, \beta] \to R^r$.

The validity of the following statement can be verified similarly to the proof of Theorem 19.2 from [4]:

Lemma 1. *Let* $\{F, V\} \in \Pi_p^n$. *Then the boundary-value problem*

$$\frac{dx}{dt} = A(t)x + F(t), \quad t \neq t_i, \qquad \Delta x|_{t=t_i} = B_i x + V_i,$$
$$x(\alpha) = x(\beta) = 0 \tag{5}$$

is solvable if and only if, for any solution of the system (4), the following relation holds:

$$\langle\{F, V\}, \{y, y\}\rangle = 0. \tag{6}$$

The theorem below allows one to prove all the further statements.

Theorem 1. *The control problem II is solvable if and only if the trivial solution of Eq. (4) is the only solution satisfying the condition*

$$\langle\{Cu, Dv\}, \{y, y\}\rangle = 0, \quad \forall\, \{u, v\} \in \Pi_p^m. \tag{7}$$

Proof. *Sufficiency.* Let $Y(t) = (y_1, y_2, \ldots, y_n)$ be a fundamental matrix of solution of system (4), $c \in R^n$. According to the condition of the theorem, the infinite system of equations

$$\langle\{Yc, Yc\}, \{Cu, Dv\}\rangle = 0, \quad \forall\, \{u, v\} \in \Pi_p^m \tag{8}$$

admits only the trivial solution $C = 0$. Let us show the existence of h elements $\{u^k, v^k\} \in \Pi_p^m$, $k = \overline{1, n}$, for which the matrix

$$N = (\langle\{y_j, y_j\}, \{Cu^k, Dv^k\}\rangle)_{jk}$$

is nondegenerate.

Assume the contrary. Without loss of generality, we can assume that the last row of the matrix N linearly depends on the other rows. Denote by $C = C^*$ a nontrivial solution of the system

$$\langle\{Yc, Yc\}, \{Cu^k, Dv^k\}\rangle = 0, \qquad k = \overline{1, n-1}. \tag{9}$$

Since, for any $\{u, v\} \in \Pi_p^m$, there exist constants μ_k for which the equality

$$\langle\{y_j, y_j\}, \{Cu, Dv\}\rangle = \sum_{k=1}^{n-1}\mu_k\langle\{y_j, y_j\}, \{Cu^k, Dv^k\}\rangle$$

is true, it follows from (9) that equality (8) holds for the nonzero vector $C = C^*$. Hence, the matrix N is nondegenerate.

Let us now consider the boundary-value problem

$$\frac{dx}{dt} = A(t)x + f(t) - C(t)\sum_{k=1}^{n} c_k n^k(t), \quad t \neq t_i,$$

$$\Delta x|_{t=t_i} = B_i x + I_i - D_i\sum_{k=1}^{n} c_k v_i^k,$$

$$x(\alpha) = x(\beta) = 0,$$

where $\{u^k, v^k\}$ are the elements of Π_p^m defined above. By virtue of Lemma 1, for the solvability of this problem, it suffices that there exist solutions of the system

$$\sum_{k=1}^{n}\langle\{y_j, y_j\}, \{Cu^k, Dv^k\}\rangle c_k = \langle\{y_j, y_j\}, \{f, I\}\rangle, \quad j = \overline{1, n},$$

which is true due to the nondegeneracy of the matrix N.

Necessity. Assume the contrary. The control problem II is solvable, and there exists a nontrivial solution of Eq. (4) that satisfies condition (7). It is easy to show that there exists $\{f, I\} \in \Pi_p^n$ for which the relation $\langle\{f, I\}, \{y, y\}\rangle \neq 0$ is true. Let us fix this element. Then by adding the last inequality to relation (7), we get $\langle\{y, y\}, \{Cu + f, Dv + I\}\rangle \neq 0$, which contradicts the criterion of the existence of a solution of the boundary-value problem. The theorem is proved.

The following statement is a corollary of Theorem 1.

Theorem 2. *The control problem I is solvable if and only if, for any* $t \in [\alpha, \beta]$ *and* $i = \overline{1, p}$, *the following relations hold:*

$$\det(C^T(t)Y(t)) \neq 0, \qquad \det(D_i^T Y(t_i)) \neq 0. \tag{10}$$

This theorem can be used to prove the Kalman criterion in the solution of the problem of control over pulse systems.

Consider the system with constant matrix of coefficients

$$\frac{dx}{dt} = Ax + Cu + f(t), \quad t \neq t_i,$$

$$\Delta x|_{t=t_i} = Du_i + I_i, \tag{11}$$

$$x(\alpha) = a, \quad x(\beta) = b.$$

Here, the matrices A, C, and D are constant. Expressions (10) allow one to prove the validity of the following theorem:

Theorem 3. *In order that the control problem I in the case of system (11) be solvable, it is necessary and sufficient that the rank of the matrix*

$$(C, AC, \dots, A^{n-1}C, D, AD, \dots, A^{n-1}D)$$

be equal to n.

Let us formulate one more criterion of the solvability of the control problem I which is a consequence of the theorems given above.

Let Γ be the Gram matrix for the system of elements $\{C^T y_i, D^T y_i\}$, $j = \overline{1, n}$.

Theorem 4. *In order that the control problem I be solvable, it is necessary and sufficient that the Gram matrix* Γ *be nondegenerate.*

Let us construct the solving control for the control problem I.

Theorem 5. *If the control problem I is solvable, then the control calculated by the formulas*

$$U(t) = C^T(t)Y(t)\Gamma^{-1}\left[Y^T(\beta)b - Y^T(\alpha)a - \int_\alpha^\beta Y^T(t)f(t)dt + \sum_{i=1}^{p} Y^T(t_i)I_i \right],$$

$$W_i = D_i^T Y^T(t_i)\Gamma^{-1}\left[Y^T(\beta)b - Y^T(\alpha)a - \int_\alpha^\beta Y^T(t)f(t)dt + \sum_{i=1}^{p} Y^T(t_i)I_i \right] \tag{12}$$

is the solving control.

Proof. Let us change the variables $x = z + \varphi(t)$ in the boundary-value problem I. Here, φ is an arbitrary function continuous with its derivatives and satisfying the boundary conditions of problem I, i.e., $\varphi(\alpha) = a$, $\varphi(\beta) = b$, and the conditions $\varphi(t_i) = 0$, $i = \overline{1, p}$. Such a function can easily be constructed. For instance, as φ, one can take the Lagrange polynomial.

After the change of variables, we obtain the control problem II in the form

$$\frac{dz}{dt} = A(t)z + C(t)u + f(t) + [\dot{\varphi}(t) - A(t)\varphi(t)], \quad t \neq t_i,$$

$$\Delta z|_{t=t_i} = B_i z - D_i v_i + I_i, \quad z(\alpha) = z(\beta) = 0.$$

By virtue of Lemma 1, for the solvability of this problem it is necessary and sufficient that the conditions

$$\int_\alpha^\beta Y^T(t)[C(t)u(t) + f(t)]\,dt + \sum_{i=1}^{p} Y^T(t_i)[D_i u_i + I_i] = \int_\alpha^\beta Y^T(t)[\dot{\varphi}(t) - A(t)\varphi(t)]\,dt, \quad \forall \{u, v\} \in \Pi_p^m \tag{13}$$

be satisfied.

Let y_k be the kth column of the matrix $Y(t)$. Integrating by parts, we obtain

$$\int_\alpha^\beta (y_k, \dot\varphi)\,dt = (y_k(\beta), \varphi(\beta)) - (y_k(\alpha), \varphi(\alpha)) - \int_\alpha^\beta (\dot y_k, \varphi)\,dt.$$

From this and (13), we conclude that the validity of the relation

$$\int_\alpha^\beta Y^T(t)[C(t)u(t)+f(t)]\,dt + \sum_{i=1}^p Y^T(t_i)[D_i u_i + I_i] = Y^T(\beta)b - Y^T(\alpha)a \tag{14}$$

is the necessary and sufficient condition of solvability of the control problem I.

By substituting the expressions

$$U = C^T(t)Y(t)c, \quad W_i = D_i^T Y(t_i)c \tag{15}$$

into (14), we obtain a system of linear equations with respect to the vector c. Substituting the solutions of this system into (15), we get expressions (12). The theorem is proved.

The solving control (12) allows one to describe the entire set of solving controls of problem I. In order that $\{u, v\}$ be a solving control for the problem, it is necessary and sufficient for it to be representable in the form $u = U + \xi$, $u_i = W_i + \nu_i$ where $\{\xi, \nu\}$ is an element of the space Π_p^m which is orthogonal to all pairs of the form $\{C^T y_k, D^T y_k\}$, $k = \overline{1, n}$. Moreover, the following condition is satisfied: $\langle\{U, W\}, \{\xi, \nu\}\rangle = 0$.

Let us introduce the norm $\| u, v \|_m = \langle\{u, v\}, \{u, v\}\rangle^{1/2}$ in the space Π_p^m.

By analogy with [5], one can show that the control $\{U, W\}$ defined by formulas (12) has the smallest norm in Π_p^m among all solving controls of the control problem I.

By using the results obtained above, we now consider the problem of fast response for linear pulse systems. Similar problem was studied earlier in [6] in the general case.

Consider the boundary-value problem

$$\frac{dx}{dt} = A(t)x + C(t)u + f(t), \quad t \neq t_i,$$

$$\Delta z|_{t=t_i} = B_i x - D_i v_i + I_i, \tag{16}$$

$$x(0) = a, \quad x(\beta) = b,$$

where A and C are defined for any time $t \geq 0$, B_i and D_i are bounded sequences, and the final time $\beta > 0$ is arbitrary. Let us define the space-product $\Pi_p^r(\beta) = L_2^r[0, \beta] \times D^r[\overline{1, p}]$ for which t_i, $i = \overline{1, p}$, are points of discontinuity of functions from $L_2^r[0, \beta]$, which form the ordered sequence in the interval $(0, \beta)$.

We assume that the control $\{u, v\}$ can be chosen only from the bounded set $\Delta \times \Delta' \subset \Pi_p^m(\beta)$, $\beta > 0$.

The problem of fast response is to find, by using a given element $\{f, I\}$ belonging to the space $\Pi_p^m(\beta)$ for any $\beta > 0$, the control $\{u, v\}$ that solves problem (15) in the shortest time.

Let us fix a positive number β. We say that a control $\{u, v\}$ with vector $c = c_0$ in the domain $\Delta \times \Delta'$ satisfies the Pontryagin condition if this control provides in this set the maximum for the expression $c_0^T Y^T(t) \times C(t)u(t)$ for almost all $t \in [0, \beta]$ and the maximum for the expressions $c_0^T Y^T(t_i) D_i v_i$, $i = \overline{1, p}$.

Theorem 6. *Let the control* $\{u, v\}$ *solve the control problem for system (16) for time* $\beta > 0$, *let it satisfy the Pontryagin condition for some vector* $c = c_0$ *in the domain* $\Delta \times \Delta'$. *Suppose that the expression* $c_0^T Y(t) \times [C(t)u(t) + f(t) + A(t)b]$ *is positive for almost all* $t \in [0, \beta]$, *and the numbers* $c_0^T Y^T(t_i)[D_i v_i + I_i + B_i(E + B_i)^{-1} b]$ *are positive for all* $i = \overline{1, p}$. *Then the control* $\{u, v\}$ *and trajectory corresponding to it are optimal in the sense of fast response.*

Proof. According to (14), we have

$$\int_{\alpha}^{\beta} Y^T(t)[C(t)u(t) + f(t)]\,dt + \sum_{0<t_i<\beta} Y^T(t_i)[D_i v_i + I_i] = Y^T(\beta)b - Y^T(0)a. \tag{17}$$

The fundamental system of solutions of the adjoint equation (4) satisfies the equality [4]

$$Y(\beta) = Y(0) - \int_0^{\beta} A^T(t)Y(t)\,dt - \sum_{0<t_i<\beta} (E + B_i^T)^{-1} B_i^T Y(t_i).$$

Substituting this equality into (17), carrying out the transposition, and assuming that $Y(0) = E$, we get

$$\int_{\alpha}^{\beta} Y^T(t)[C(t)u(t) + f(t) + A(t)b]\,dt + \sum_{0<t_i<\beta} Y^T(t_i)[D_i v_i + I_i + B_i(E + B_i)^{-1} b] = b - a. \tag{18}$$

Assume the contrary, i.e., that there exists the control $\{\bar{u}, \bar{v}\}$ which transfers the point $x(0) = a$ into position $x(\tau) = b$ at the time $\tau < \beta$. Then

$$\int_0^{\tau} c_0^T Y^T(t)[C(t)\bar{u}(t) + f(t) + A(t)b]\,dt + \sum_{0<t_i<\tau} c_0^T Y^T(t_i)[D_i \bar{v}_i + I_i + B_i(E + B_i)^{-1} b] = c_0^T(b - a).$$

Subtracting the last equality termwise from (18) multiplied by the vector c_0^T, we get

$$\int_0^{\tau} c_0^T Y^T(t)C(t)[u(t) - \bar{u}(t)]\,dt + \int_0^{\beta} c_0^T Y^T(t)[C(t)u(t) + f(t) + A(t)b]\,dt +$$

$$+ \sum_{0<t_i<\tau} c_0^T Y^T(t_i) D_i [v_i - \bar{v}_i(t)] + \sum_{\tau<t_i<\beta} c_0^T Y^T(t_i)[D_i v_i + I_i + B_i(E + B_i)^{-1} b] = 0.$$

By virtue of the conditions of the theorem, the first and third terms in the last equality are nonnegative, while the second and fourth ones are positive. The contradiction obtained proves the theorem.

Corollary. *Suppose that the conditions of Theorem 6 are satisfied, and* $x(t)$ *is the optimal trajectory connecting points* $x(0) = a$ *and* $x(\beta) = b$. *Then any part of this trajectory which connects points* $x(t_1)$ *and* $x(t_2)$, $0 \le t_1 \le t_2 \le \beta$, *is also the optimal trajectory.*

Let us now apply Theorem 6 and its corollary for the synthesis of the optimal control in the fast-response sense.

Example. Let $x = \operatorname{colon}(x_1, x_2)$. We consider the system of differential equations with pulse action at points $t = i$ with i being integers, which has the form

$$\frac{dx}{dt} = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} x + \begin{pmatrix} 0 \\ 1 \end{pmatrix} u, \quad t \neq i, \qquad \Delta x|_{t=i} = \begin{pmatrix} 0 \\ 1 \end{pmatrix} v_i. \tag{19}$$

Here, the function u and the sequence v take values in R, $|u| \leq 1$, and $|v_i| \leq 1$.

Let us consider the problem of fast response in the case where $x(0)$ is an arbitrary point of the plane Ox_1x_2 and $x(\beta) = 0$ is the origin.

Comparing (19) and (11), we find that the sufficiency conditions for the controllability which are determined by Theorem 3, are satisfied because the following equality is valid:

$$\operatorname{rank}(C, AC, D, AD) = \operatorname{rank}\begin{pmatrix} 0 & 1 & 0 & 1 \\ 1 & 0 & 0 & 1 \end{pmatrix} = 2.$$

The fundamental matrix of solutions of the system $dy/dt = -A^T y$ is equal to

$$Y(t) = \begin{pmatrix} 1 & 0 \\ -t & 1 \end{pmatrix}.$$

Using this matrix, we conclude, by virtue of Theorem 6, that the conditions of optimal control $\{u, v\}$ take the form

$$ktu = \min_{\bar{u} \in \Delta} kt\bar{u}, \qquad kiv_i = \min_{\bar{v} \in \Delta'} ki\bar{v}_i, \tag{20}$$

$$ktu < 0, \qquad kiv_i < 0, \tag{21}$$

where k is a real number, $t \geq 0$ and $i > 0$.

Consider the cases $k > 0$ and $k < 0$.

1. Let $k > 0$. Then it follows from (20) and (21) that the optimal control is equal to $\{u, v\} = \{-1, -1\}$ and system (19) has the form

$$\frac{dx}{dt} = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} x + \begin{pmatrix} 0 \\ -1 \end{pmatrix}, \quad t \neq i, \qquad \Delta x|_{t=i} = \begin{pmatrix} 0 \\ -1 \end{pmatrix}. \tag{22}$$

2. Let $k < 0$. In this case, the optimal control is equal to $\{u, v\} = \{1, 1\}$ and Eq. (19) can be written in the form

$$\frac{dx}{dt} = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} x + \begin{pmatrix} 0 \\ 1 \end{pmatrix}, \quad t \neq i, \qquad \Delta x|_{t=i} = \begin{pmatrix} 0 \\ 1 \end{pmatrix}. \tag{23}$$

Let us denote by L' the set which is symmetrical to L with respect to the origin. Let $\bar{L} = L \cup L'$. The set $\bar{L}$ is a figure on the plane Ox_1x_2 which divides it into two parts, the "lower" and "upper" ones. From an arbitrary point of the "lower" part one can get into the origin following the optimal trajectory with the switching on the figure L only. Until the switching the representative point moves according to Eq. (23). The switching occurs when the boundary point of the set L is reached. Below we describe the mechanism of switching. The following cases can be realized:

(a) The representative point meets the set L at $t=\tau$ in the point which belongs to an arc of a phase trajectory connecting points $\bar{x}(i+)$ and $\bar{x}(i+1)$, where $\bar{x}$ is a solution of system (22). Moreover, the point $x(\tau)$ can coincide with the point $\bar{x}(i+)$. The case $x(\tau)=\bar{x}(i+1)$ is considered below.

Then, the following movement will be realized along the trajectory of the solution $\bar{\bar{x}}(t)$ of system (22) for which $\bar{\bar{x}}(1-\bar{x}_2(i+1)-x_2(\tau))=x(\tau)$. Thus, if the point $\bar{\bar{x}}(t)$ coincides with the point $x=0$ at time $t=\tau_1$ then the optimal time equals $t_{\text{opt}} = \tau+\tau_1-1+\bar{x}_2(i+1)-x_2(\tau)$.

(b) Let the trajectory which begins in the "lower" part, at the time $t=\tau$, meet the boundary of the set L at the point which belongs to the interval connecting the points $\bar{x}(i)$ and $\bar{x}(i+)$ and which does not coincide with the point $\bar{x}(i+)$, $i=1,2,\dots$.

Then, applying the supplementary pulse of the strength $\bar{v}=\bar{x}_2(i+)-x_2(\tau)$, we can carry out the transition of the point $x(\tau)$ in the position $\bar{x}(i+)$.

As a result, the movement will be realized along the trajectory of the solution $\bar{\bar{x}}(t)$ of Eq. (22) which satisfies the initial condition $\bar{\bar{x}}(0) = \bar{x}(i+)$ and gets of the initial position without a jump.

Let us note that Eqs. (22) and (23), in contrast to similar systems for the control without pulses [7], are not autonomous, which complicates the synthesis of the optimal control.

Since the phase portraits (22) and (23) are symmetrical with respect to the origin, it is obvious that it is sufficient to consider only one of the cases indicated. Let $k>0$.

First, let us describe the optimal trajectories which get into the origin without switching. This description will be based on the solutions which begin at the time $t=0$, have first discontinuity at the point $t=1$, and get into the origin at the times $t=1,2,3,\dots$. The initial points of these solutions form a denumerable set and have the coordinates

$$x_1 = -\sum_{j=0}^{i}(2j+1/2), \quad x_2 = 2i+1, \quad i=0,1,2,\dots.$$

The phase trajectories of these solutions belong to the same curve, which we denote by l. Let us now select the solutions which go out from the point x_0 at the time $t=0$ without a jump and which get into the origin at the time τ such that $\{\tau\}=\alpha$. Here $\{\cdot\}$ denotes a fractional part of the variable. These are the solutions whose representative point hits the arc Δl of the curve l, contained between the points $(0,0)$ and $(1/2,1)$, at some time $t=i$ and subsequently moves to the origin along the curve l. With α changing in the interval $[0,1]$, the phase trajectory of these solutions takes positions from the curve which is the plot of the function $x_2(x_1)$ with discontinuities of the first kind at points

$$x_1 = -\sum_{j=1}^{i}(2j-1/2),$$

until the curve l.

Let us join all the phase trajectories into a set L. Moreover, we shall include in this set the vertical intervals connecting the points $x(i)$ and $x(i+)$ of the indicated trajectories. If $\bar{\bar{x}}(\tau_1)=0$ then the optimal time equals $t_{\text{opt}}=\tau+\tau_1$.

Remarks. *1.* The boundary of the set L is given by a system of algebraic equations and inequalities. When investigating the latter together with the analytical expression for the solution $x(t)$ of system (22), it is useful to make the switching.

2. Since $|\bar{v}|<1$, the supplementary control pulse $\bar{v}$, used in the case (b), completely satisfies the initial conditions of the problem.

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