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A NEW THEORY OF CONTENT I: BASIC CONTENT

ABSTRACT. Philosophers of science as divergent as the inductivist Carnap and the deductivist Popper share the notion that the (logical) content of a proposition is given by its consequence class. I claim that this notion of content is (a) unintuitive and (b) inappropriate for many of the formal needs of philosophers of science. The basic problem is that given this notion of content, for any arbitrary p and q, $\ulcorner(p \vee q)\urcorner$ will count as part of the content of both p and q. In other words, any arbitrary p and q share some common content. This notion of content has disastrous effects on, for instance, Carnap's attempts to explicate the notion of confirmation in terms of probabilistic favorable relevance, and Popper's attempts to define verisimilitude. After briefly reviewing some of the problems of the traditional notion of content I present an alternative notion of (basic) content which (a) better fits our intuitions about content and (b) better serves the formal needs of philosophers of science.

1. INTRODUCTION

In Salmon [1970], p. 55, we read

> it seems reasonable to think of the content of a statement p in terms of the statements p entails.

The idea that a statement's content[1] is captured by the class of its logical consequences is common to philosophers as divergent as Carnap and Popper.[2] Indeed it is by far the dominant notion of content among philosophers of science and logicians. In the following I propose a new notion of basic content. It will be briefly argued that this new notion (a) better fits our intuitions about content and (b) better serves technical philosophical purposes than does the traditional notion of content. However, the bulk of this paper will concern definitions of this new notion of basic content for a formal propositional language, and investigation of some of the properties of those definitions. In later sections extensions of these definitions covering basic content for more complex formal languages will be briefly considered. In as much as the current paper only really achieves full results (proof of transitivity, mechanical

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decision procedure, etc.) for a simple non-quantificational language it may be viewed as an invitation for further suggestions for the treatment of more complex languages. In a follow-up paper, "A New Theory of Content II: Full Content", the account of basic content introduced here for propositional languages will be extended to an account of full content for such languages and applications to more complex languages will be further investigated.

2. SOME PROBLEMS FOR THE TRADITIONAL NOTION OF CONTENT

We start our investigations by considering a generic propositional language $L_{\&v\sim\rightarrow}$, hereafter simply L. The well formed formulae (wffs) of L are the atomic wffs 'p', 'q', 'r', etc., and their logical compounds formed with the logical connectives '&', '$\vee$', '$\sim$' and '$\rightarrow$', and grouping indicators '(' and ')' in the usual ways. The notion of derivable consequences ($\vdash$), logical consequence ($\models$), contradiction and tautology are defined as usual.[3] Hereafter we shall use the term 'logically contingent,' or, more simply, 'contingent,' to refer to non-tautologous, non-contradictory wffs. We shall use the Greek letters 'α', 'β', 'σ', 'μ' and 'ϕ' as meta-variables ranging over wffs of L and other subsequently defined languages. Occasionally we will take 'β' to range over both wffs and sets of wffs.

According to the traditional concept of content in its simplest version,

T.C. α is part of the content of β iff $\beta \vdash \alpha$.

Traditionally, T.C. is seen as giving the content of both single wffs and theories, where a theory is seen as any set of wffs.[4]

More complex versions favored by Carnap and Popper usually exclude tautologies from having or being content parts.[5] Arguably, the same caveat is applicable to contradictions.[6] Thus we have the more complex definition,

T.C.1 α is part of the content of β iff α and β are contingent and $\beta \vdash \alpha$.[7]

We shall now consider several drawbacks of T.C.1 applied to the formal language L occasionally noting how analogous problems arise when T.C.1 is applied to more complicated formal and informal languages.

According to T.C.1, for any wffs α and β, as long as α and β are contingent and $\ulcorner \sim \alpha \urcorner$ does not entail β, α and β have at least one common content part, namely $\ulcorner (\alpha \vee \beta) \urcorner$. So according to T.C.1, and contra our intuitive judgment, the L atomic wffs 'p' and 'q' share common content.[8] Where T.C.1 is taken to give the content of theories it has the result that for any (contingent) theories T and T' if there are sentences α and β such that α is a contingent consequence of T and β is a contingent consequence of T' and $\ulcorner \sim \alpha \urcorner$ does not entail β, T and T' have common content. So given a broad application of T.C.1, not only do Relativity theory and Newtonian mechanics share common content but also so do Relativity theory and your favorite crackpot theory, say, Dianetics.

According to T.C.1, where (contingent) α is a true wff inconsistent with (contingent) wff β, and there is some true wff σ such that σ is not a consequence of α and $\ulcorner \sim \beta \urcorner$ does not entail σ, β will have some true content not shared by α, namely $\ulcorner (\beta \vee \sigma) \urcorner$. For instance, where the L atomics 'p' and 'q' are true, then the false '$\sim p$' will have some true content not shared by 'p,' for instance '$(\sim p \vee q)$'. It is this type of consequence of the traditional notion of content that has continually subverted attempts to define the notion of verisimilitude. For instance, Popper's first definitions of verisimilitude claimed that for α to have more verisimilitude than β all of the truth content of β must occur in α, that is to say, every true consequence of β must be a consequence of α.[9] So Popper's early definitions have the consequence, where the L atomic wffs 'p' and 'q' of L are true, the true statement 'p' does not have more verisimilitude that its negation!

A related problem affects attempts to define the notion of partial truth. In a world where both the L atomics 'p' and 'q' are true, we would intuitively say that while '$(p \,\&\, \sim q)$' is partially true, '$(\sim p \,\&\, \sim q)$' is not even partially true. Yet if we identify partial truth with having a true content part, then, where any contingent consequence counts as a content part, '$(\sim p \,\&\, \sim q)$' is partially true, since it has such true consequences as '$(\sim p \vee q)$'. Indeed, under this notion of content, any non-tautologous statement is partially true.

Again a related problem affects traditional conceptions of hypothetico-deductive confirmation. According to hypothetico-deductivism observational O confirms T if O is a true observational consequence of T. Given the concept of content as contingent consequence, that is to say, O confirms T if O is a true observational content part T. Now suppose T is some non-observational theory, say Newton's second law of motion, and O is some unrelated observation sentence, say O is the claim that Sydney has a harbor bridge. Then according to traditional hypothetico-deductivism we can confirm T by *observing* the Sydney Harbor Bridge and thus noting that T's consequence '$(T \vee O)$' is true! That this result is unacceptable is shown by the fact that our observation of the Sydney harbor Bridge would equally allow us to confirm the negation of T, say '$\sim T$', by noting that its consequence '$(\sim T \vee O)$' is true.[10]

Finally, T.C.1 has unintuitive consequences for any attempt to define confirmation in terms of probabilistic favorable relevance *a la* Carnap.[11] According to Carnap's notion of confirmation as favorable relevance, α confirms β if and only if the posterior probability of β given α is greater than the prior probability of β. That is, α confirms β if and only if $P(\beta/\alpha) > P(\beta)$. Now it follows from the probability calculus that for any statements α and β, where $P(\beta/\alpha) < 1$ and $P(\beta/\sim\alpha) < 1$, there will always be some content parts σ and μ of β such that α confirms σ (i.e. $P(\sigma/\alpha) > P(\sigma)$) and disconfirms μ (i.e. $P(\mu/\alpha) < P(\mu)$). In particular, where $P(\beta/\alpha) < 1$ and $P(\beta/\sim\alpha) < 1$, α will confirm β's content part $\ulcorner(\beta \vee \alpha)\urcorner$ and α will disconfirm β's content part $\ulcorner(\beta \vee \sim\alpha)\urcorner$.[12] So, applying T.C.1 to English indicative sentences, and assuming P(All ravens are black/Raven *a* is white) < 1 and P(All ravens are black/It is not the case that Raven *a* is white) < 1, we get the result that 'Raven *a* is white' probabilistically confirms part of 'All ravens are black'! Similarly, assuming P(All ravens are black/Raven *a* is black) < 1 and P(All ravens are black/It is not the case that Raven *a* is black) < 1 yields the result that 'Raven *a* is black' probabilistically disconfirms part of 'All ravens are black'!

While I will not seek to fully demonstrate the point here these last results have dire consequences for attempts to explicate the every day intuitive notion of confirmation or evidential support in terms of the probabilistic notion of confirmation as favorable relevance. Briefly, the basic problem is that intuitively evidence *e* only really supports hypothesis h if e is favorably relevant to every part of h. But as long as $P(h/e) < 1$, there will always be some part of h, namely $\ulcorner(h \vee \sim e)\urcorner$, such that e does not probabilistically confirm that part. It is just this type of result that leads to the notorious Popper–Miller "refutation" of probabilistic inductivism in Popper–Miller [1983]. There Popper and Miller argue that for inductive arguments e to h, e counter-supports the untested part of h relative to e, which they identify as $\ulcorner(\sim e \vee h)\urcorner$. Of course, intuitively, $\ulcorner(\sim e \vee h)\urcorner$ is not even part of the content of h, let alone the untested part of h relative to e.[13]

If the ordinary notions of confirmation and inductive support are to be analyzed in terms of the probabilistic notion of favorable relevance we need a notion of content that allows for the possibility that there are cases where e does not entail h yet e is favorably relevant to every content part of h.

While this by no means exhausts the list of infirmities of T.C.1 I will presume it suffices to motivate the consideration of a notion of content which does not labor under such handicaps. It is perhaps worth adding that in a series of forthcoming papers it will be shown that with the new notion of basic content advanced below we can achieve notable progress in solving many traditional problems in the philosophy of science. These include the formulation of definitions, for instance, definitions of hypothetico-deductivism, verisimilitude, probabilistic confirmation, bootstrapping confirmation, and the notion of a natural axiomatization, not open to well known counter-examples affecting previous formulations.[14]

Before proceeding further we need to make a caveat. This essay has so far been cast in terms of the problem of using the notion of logical consequence to define the notion of content. In fact, the crucial point is that many of the needs of philosophers, especially philosophers of science, are better served by using a non-classical notion of consequence. In particular, we need a notion of consequence that does not automatically count '$(p \vee q)$' as a consequence of p. I have called the new notion of consequence defined below a notion of content because nearly all philosophers of science, perhaps following the lead of Carnap and Popper, have associated the term 'content' with the notion of logical consequence.[15] In the final analysis we could proceed by dropping the talk of content. Talk of the infelicities of the traditional notion of content could then be substantially replaced by talk of the infelicities of the standard notion of logical consequence.

3. A NEW NOTION OF CONTENT

The above infelicities of the traditional notion of content all result because where content is identified with consequence class, for any α and β, as long as $\ulcorner \sim \alpha \urcorner$ does not entail β, there will always be a content part of α that "includes" β, namely $\ulcorner(\alpha \vee \beta)\urcorner$. The problem with the traditional notion of content is that it allows us to form content parts by arbitrary disjunctions. What we want of our new notion of content is that, for instance, the atomics 'p' and 'q' should count as parts of '$(p \,\&\, q)$' while '$(p \vee r)$' and '$(p \vee \sim q)$' do not.

The natural thought that one might at this point turn to relevance logic is stymied by the realization that typically relevance logics take α to entail $\ulcorner(\alpha \vee \beta)\urcorner$.[16] There are some relevance type logics, following the pioneering work of W. T. Parry,[17] which do not invariably allow the derivation of $\ulcorner(\alpha \vee \beta)\urcorner$ from α. Typically, they only allow such a derivation where all the non-logical vocabulary in β occurs in α. Such logics are not suitable for our purposes for at least two reasons.

First, they would lead to the result that statements which are logically equivalent according to classical logic can have different content parts. For instance, according to Parry's notion of analytic implication '$(p \vee q)$' is an analytically implicate of '$(p \,\&\, (p \vee q))$' but not of its classical equivalent 'p'. While this is not an insurmountable difficulty – one might after all simply abandon the notion of classical equivalence – embracing a notion of content that entails an abandonment of classical equivalence would make that notion of content difficult to use for many projects in the philosophy of science which carry a commitment to classical equivalence. For instance, suppose we defined a relation of confirmation between evidence e and hypothesis h as a function of the relations between e and various content parts of h. If we then allowed that classically equivalent hypotheses could have different content parts we would allow that classically equivalent hypotheses could bear different confirmation relationships to the same evidence. This would fly in the face of a fundamental tenet of confirmation theory, namely that, since classically equivalent hypotheses share the same truth conditions, any evidence that confirms a hypothesis, that is, confirms it as being true, should equally confirm any hypothesis true under the same conditions, that is, any classical equivalent. In Hempel's seminal paper "Studies in the Logic of confirmation" this principle is enshrined as the "Equivalence Condition", and it has long been regarded as an inviolable condition of adequacy for any account of confirmation.[18]

Second, and most importantly, the use of a Parry-type notion of implication to define the notion of content part would wreak havoc on our attempt to reach a notion of content which allows for cases where α does not entail β but nevertheless is favorably relevant to every content part of β. For example, it would in most cases preclude α confirming every content part of $\ulcorner(\beta \,\&\, \alpha)\urcorner$ since $\ulcorner(\beta \vee \sim \alpha)\urcorner$ would typically still count as a content part of $\ulcorner(\beta \,\&\, \alpha)\urcorner$.

When we concentrate on the type of examples mentioned at the beginning of this section an obvious suggestion comes to mind: The problem is with addition! Simplification, whereby we break down a conjunction into its component parts, is, *prima facie*, a paradigm method of extracting content parts from a compound statement such as '$(p \,\&\, q)$'. Thus we obtain the content parts 'p' and 'q' from '$(p \,\&\, q)$' by simplification. However, in determining the content parts of a statement we should not be allowed to add on irrelevant or "inimical" disjuncts as in the cases of '$(p \vee r)$' and '$(p \vee \sim q)$'. Perhaps then we may construct the appropriate partition by proof-theoretic means, developing a content part calculus by accepting only a subset of the normal rules by which we determine logical consequences.

Yet here already we have strayed into murky waters. For consider the following proposal:

P1 Statements which are logically equivalent (according to classical logic) should have the same content parts.

This proposal does not sit well with our suggestion for finding content parts by proof-theoretic means, where those means are restricted to merely pruning addition from the rules by which we determine consequences.[19] To see this we need only recall that by applying simplification to (classical) logical equivalents of any statement we may derive these consequences of it we would ordinarily obtain by addition. For instance,

'p' is (classically) equivalent to '$((p \vee q) \,\&\, p)$'. By simplification we obtain the undesired consequence '$(p \vee q)$'. More generally, for any wffs α and β we may obtain the disjunction of α and β from α's (classical) logical equivalent $\ulcorner(\alpha \,\&\, (\alpha \vee \beta))\urcorner$ by simplification.

In our brief discussion of Parry type relevance logics we noted some of the drawbacks associated with the rejection of P1. Rather than rejecting P1 we will strive to avoid the unwanted effects of addition without rejecting addition or any other rule of the propositional calculus.

What does addition allow us to do to a statement? One thing it allows is the weakening of a statement. Clearly the reason '$(p \vee \sim q)$' follows from '$(p \,\&\, q)$' is that it contains 'p' as a disjunct. In respect of this entailment the second disjunct '$\sim q$' is not doing any work. It is what we might call a "free rider". The free rider merely serves, when disjoined to the working disjunct, to form a weaker whole. The whole, in this case '$(p \vee \sim q)$', is weaker than the working disjunct, in this case 'p', in the sense that while the working disjunct entails the whole, the whole does not entail the working disjunct. This suggests a method of eliminating disjunctions containing such free riders. When looking for the content parts of a statement β should not merely look at consequences of β. We should also make sure that for any candidate content part α (of β), there is no stronger consequence (of β) that can be constructed using just those atomic wffs occurring in α. This would serve to rule out '$(p \vee \sim q)$' as a content part of '$(p \,\&\, q)$' since, for example, 'p' is stronger than '$(p \vee \sim q)$' yet contains only atomic wffs that occur in '$(p \vee \sim q)$'. Here then is a first attempt at a definition of content parts of wffs of L:

CPL* α is a content part of β iff (i) α and β are contingent, (ii) α is a consequence of β and (iii) there is no consequence σ of β such that σ is stronger than α and every atomic wff that occurs in σ occurs in α.

The first two clauses of this account ensure that content parts of a sentence are limited to contingent consequences of the sentence. The third clause ensures that for any content part there is no stronger consequence constructible using the relevant atomic wffs. Clause (iii) demands that for any candidate content part α there is no *defeater* of α, that is, a statement containing only atomic wffs which occur in α, which is a consequence of the original formula yet is stronger than α.

Definition CPL* captures some of our paradigm intuitions about content parts. It tells us that 'p' and 'q' are both content parts of '$(p \& q)$.' It tells us that '$(p \vee r)$' is not a content part of 'p' or of '$(p \& q)$'. It is not a content part because in both cases it is defeated by the stronger consequences 'p'.

According to this definition of content it is not the case that for any two contingent statements α and β, if $\ulcorner \sim \alpha \urcorner$ does not entail β then α and β share some common content. For instance, the L atomics 'p' and 'q' share no common content.

Moreover, it does not follow from this notion of content that for any α and β, provided $P(\beta/\alpha) < 1$ and $P(\beta/\sim\alpha) < 1$, α confirms part of β and α disconfirms part of β. It does not follow since it is not the case that for any α and β meeting the relevant conditions $\ulcorner(\beta \vee \alpha)\urcorner$ and $\ulcorner(\beta \vee \sim\alpha)\urcorner$ are part of the content of β.

Despite these virtues CPL* has at least two minor and one major drawbacks.

First the minor drawbacks.

CPL* does not have the, arguably, desirable consequence that where σ is a content part of β and μ is a content part of β then so is their disjunction, $\ulcorner(\sigma \vee \mu)\urcorner$. For instance, according to CPL*,

(1) $p \vee q$

is not a content part of

(2) $p \& q$

though both 'p' and 'q' are content parts of (2). (1) is not a content part of (2) since (2) itself is a stronger than (1) and contains only atomic wffs that occur in (1).

CPL* does not have the, arguably, desirable consequence that where σ is a content part of β and μ is a content part of β then so is their conjunction, $\ulcorner(\sigma \& \mu)\urcorner$. For instance, according to CPL*,

(3) $(p \vee q) \& (p \vee r)$

is not a content part of

(4) $(p \vee q) \& (p \vee r) \& (q \vee r)$

though both

(1) $(p \vee q)$

and

(5) $(p \vee r)$

are content parts of (4). (3) is not a content part of (4) since (4) itself is a stronger than (3) and contains only atomic wffs that occur in (3).

Now for the major drawback.

Consider the following intuitively plausible proposal

P2 The set of content parts of a formula should be closed under the relationship of (classical) logical equivalence.

After all, if two propositions are logically equivalent they have the same content and hence if one is a content part of a third statement then so is the other. Now our present definition does not preserve P2. Thus consider the following three L wffs,

(2) $p \,\&\, q$
(6) p

and

(7) $(p \vee q) \,\&\, p.$

By our definition (6) is a content part of (2), though its logical equivalent (7) is not. (7) is defeated by the stronger formula (2) which is, of course, a consequence of (2) and only contains atomic wffs that occur in (7).

To avoid our two minor drawbacks of CPL* we shall reconceptualize our definition of content parts as giving only the *basic* content parts of wffs, hence our definitions will henceforth include a subscript 'b'. Should one wish to count disjunctions and conjunctions of content parts as content parts then one could appropriately complicate the definitions presented below by adding suitable recursion clauses. Since all the major applications for which this new notions of content is intended to serve do not in any way depend on whether such conjunctions or disjunctions are included this limitation will be of little import.

On the other hand, if our new notion of content were to involve a rejection of P2, besides flying in the face of a strong intuition, it would have dire consequences for many of the intended applications. So to preserve P2 we will now make the necessary amendments to CPL*.

Here then is our final definition for basic content parts of wffs of L. BCPL1 gives a definition of the basic content parts of arbitrary wff β of language L, hence we use '$<_b$' to indicate the relationship of being a basic content part:

BCPL1 $\alpha <_b \beta =_{\text{df}}$ (i) both α and β are contingent, (ii) α is a consequence of β, and (iii) for some μ, μ is equivalent to α and there is no σ such that σ is stronger than μ, σ is a consequence of β and every atomic wff that occurs in σ occurs in μ.

While this definition has been framed for the specific propositional language L, we should note that it is equally applicable to other propositional languages lacking an identity operator. For instance, let L' be like L save that L' contains an infinite stock of individual constants, a_1, a_2, etc., and a finite stock of predicate letters of varying degrees, F, G, H, etc. In place of L's atomics wffs, p, q, r, etc., the atomics wffs of L' are all subject predicate wffs of the form $\ulcorner Pi_1 \ldots i_n \urcorner$ where P is an n-adic predicate letter of L' and $\ulcorner i_1 \ldots i_n \urcorner$ is a n membered series of individual constants of L'. Then, the definition above gives the basic content parts of the wffs of L'. This fact will come in handy when we consider defining content part relationships for quantificational languages in Sections 8 and 9 below.

4. THE DNF APPROACH AND A MECHANICAL DECISION PROCEDURE

One desideratum for an account of content for a propositional language such as L is decidability. Where content is defined in terms of classical consequence it is clearly decidable for propositional languages such as L. We will now show that on the above definition of basic content parts there are effective procedures for determining content. To prove this we first give an alternative characterization of the basic-content-part relation. We then show (i) that this alternative characterization gives an effective procedure for determining for arbitrary wffs α and β whether α

is a basic content part of β, and (ii) that it captures the same relation as that captured by BCPL1.

Suppose α is a multiple disjunction, that is α is $\ulcorner(\alpha_1 \vee \ldots \vee \alpha_n)\urcorner$. (We ignore parentheses since it does not matter where they go). We then say that β is a proper sub-disjunction of α iff β is a disjunction of some, but not all, of the disjuncts of α.

Let $\ulcorner\mathrm{At}(\alpha)\urcorner$ designate the set of all atomics wffs occurring in α. Atomic wff a_j is a member of $\mathrm{EAt}(\alpha)$, that is the set of atomics occurring essentially in contingent α, iff where $\langle a_1, \ldots, a_n\rangle$ is an ordered n-tuple containing each of, and only, the n members of $\mathrm{At}(\alpha)$, there is a n-membered conjunction $\ulcorner(*a_1 \,\&\, \ldots \,\&\, *a_n)\urcorner$ such that each conjunct $*a_k$, $1 \leq k \leq n$, is either a_k or $\ulcorner\sim a_k\urcorner$ and $(*a_1 \,\&\, \ldots \,\&\, *a_n) \vdash \alpha$ and there is some conjunction c that differs from $\ulcorner(*a_1 \,\&\, \ldots \,\&\, *a_n)\urcorner$ only in that c contains $\ulcorner\sim *a_j\urcorner$ where $\ulcorner(*a_1 \,\&\, \ldots \,\&\, *a_n)\urcorner$ contains $*a_j$ and $c \nvdash \alpha$.

Now we can define a disjunctive normal form, α_{dnf}, for arbitrary contingent sentence α, that is, a canonical disjunctive normal form of contingent α in α's essential vocabulary unique up to order and grouping of conjuncts and disjuncts.

D1 For any contingent α, $\alpha_{\mathrm{dnf}} =_{\mathrm{df}}$
A disjunction of conjunctions $\ulcorner(c_1 \vee \ldots \vee c_m)\urcorner$ consisting of 1 or more disjuncts such that where $\langle a_1, \ldots, a_n\rangle$ is an ordered n-tuple containing each of, and only, the n members of $\mathrm{EAt}(\alpha)$, the set of the essential atomic wffs occurring in α, each disjunct c_k is a n membered conjunction $\ulcorner(*a_1 \,\&\, \ldots \,\&\, *a_n)\urcorner$ such that each conjunct $*a_k$ is either a_k or $\ulcorner\sim a_k\urcorner$, and for each disjunct c_k, $c_k \vdash \alpha$, and no two disjuncts are the same, and $(c_1 \vee \ldots \vee c_m) \dashv\vdash \alpha$.

We can now determine whether arbitrary wff α is a basic content part of arbitrary wff β in the following manner. We first check to see if both α and β are contingent. If either is not then α is not a basic content part of β. We then check to see if $\beta \vdash \alpha$; if not then α is not a basic content part of β. We then form α_{dnf} and check to see if β classically entails any proper sub-disjunction of α_{dnf}. If it does, α is not a basic content part of β and if it does not then α is a basic content part of β. In short,

BCPL2 $\alpha \ll_b \beta =_{\mathrm{df}}$ (i) α and β are contingent, (ii) α is a consequence of β and (iii) there is no μ such that μ is a proper sub-disjunction of α_{dnf} and μ is a consequence of β.[20]

[Note that '$\ll_b$' is used to denote the basic content part relationship defined by BCPL2 while '$<_b$' is used to denote the basic content part relationship defined by BCPL1 above.]

The idea behind BCPL2 is quite close to our original one of requiring that there be no defeater of α. We simplify things a bit in this case, however, by only having to look at potential defeaters that result from dropping some disjuncts from α_{dnf}. The dropping of a disjunct amounts to a strengthening of α. Since (i) the notion of classical consequence, (ii) the contingency of α and β, (iii) the procedure for forming α_{dnf}, and (iv) the checking for a proper sub-disjunction of α_{dnf} that is a consequence of β are decidable, $\alpha \ll_b \beta$ is also decidable. All that remains to be shown in order to show that $\alpha <_b$ is decidable is that $\alpha <_b \beta$ iff $\alpha \ll_b \beta$.

The characterizations of the notions of essential vocabulary and the canonical disjunctive normal form, α_{dnf}, for arbitrary wff α, given above are syntactic. Now since there is a well established equivalence between semantic and syntactic properties for such propositional languages as L we know that there are semantic specifications of the notions of essential vocabulary and disjunctive normal form, α_{dnf}, equivalent to the above syntactic specifications. Sometimes in proving theorems concerning our new content parts relationships it will be both more concise and more perspicuous to deal with semantic surrogates of our definitions of essential vocabulary and α_{dnf}.

First let us define a semantic analog of the notion of essential vocabulary. Let T be a standard full truth table for arbitrary wff α. So each row of T is an assignment of truth values to each of the sub formulas of α. An atomic wff σ is in the essential vocabulary of α just in case in α's truth table T there are two assignments of truth values to all the atomic wffs that occur in α that agree on every atomic wff except σ and give different values for α. Nothing else is part of the essential vocabulary of α.

We now define a disjunctive normal form, α_{dnf}, for contingent α unique up to order and grouping of conjuncts and disjuncts. It consists of a disjunction of conjunctions, exactly one conjunction for each of the non-zero but finitely many assignments of values to the essential vocabulary of α that gives α the value T. The conjunction corresponding to a given such assignment consists of one conjunct for each member of the (finite) essential vocabulary: the member itself if on the given assignment the member has the value T, and otherwise the negation of the member.

5. THE EQUIVALENCE OF '$<_b$' AND '$\ll_b$'

Before proceeding to demonstrate the equivalence of the basic content part relationships defined by BCPL1 and BCPL2 above we need to consider the following lemma:

LEMMA 1.1. *If $\beta \dashv\vdash \beta'$ and β is contingent then $\beta_{\mathrm{dnf}} = \beta'_{\mathrm{dnf}}$.*
Proof. Cf. the construction of β_{dnf} and β'_{dnf}.

Now we are in a position to prove

THEOREM 1. *$\alpha <_b \beta$ iff $\alpha \ll_b \beta$*
Proof. We assume α and β are contingent and that $\beta \vdash \alpha$, otherwise the case is trivial.

Now we show that, given this assumption, if $\alpha <_b \beta$ then $\alpha \ll_b \beta$ by proving its contrapositive. Assume that $\alpha \not\ll_b \beta$. Then for some proper sub-disjunction μ of α_{dnf}, $\beta \vdash \mu$. Since μ is a proper sub-disjunction of α_{dnf} and α_{dnf} contains no redundant disjuncts, $\mu \vdash \alpha_{\mathrm{dnf}}$, $\alpha_{\mathrm{dnf}} \nvdash \mu$ and $\mathrm{At}(\mu) \subseteq \mathrm{At}(\alpha_{\mathrm{dnf}})$. So μ is a defeater for α_{dnf} as a content part of β. Now consider any α' such that $\alpha' \dashv\vdash \alpha_{\mathrm{dnf}}$. Since $\mu \vdash \alpha_{\mathrm{dnf}}$, $\mu \vdash \alpha'$. Since $\alpha_{\mathrm{dnf}} \nvdash \mu$, $\alpha' \nvdash \mu$. Clearly $\mathrm{At}(\alpha_{\mathrm{dnf}}) \subseteq \mathrm{At}(\alpha')$, so $\mathrm{At}(\mu) \subseteq \mathrm{At}(\alpha')$. So μ is a defeater of all such α', including α itself. So $\alpha \not<_b \beta$.

Now we show that, given our initial assumption, if $\alpha \ll_b \beta$ then $\alpha <_b \beta$ by proving its contrapositive. Assume $\alpha \not<_b \beta$. Then for any $\alpha' \dashv\vdash \alpha$ there is a σ such that $\beta \vdash \sigma \vdash \alpha'$ and $\alpha' \nvdash \sigma$ and $\mathrm{At}(\sigma) \subseteq \mathrm{At}(\alpha')$. In particular, since $\alpha_{\mathrm{dnf}} \dashv\vdash \alpha$, there is a σ such that $\beta \vdash \sigma \vdash \alpha_{\mathrm{dnf}}$ and $\alpha_{\mathrm{dnf}} \nvdash \sigma$ and $\mathrm{At}(\sigma) \subseteq \mathrm{At}(\alpha_{\mathrm{dnf}})$. Now put σ into disjunctive normal form, σ_{df}, in the full vocabulary of α_{dnf}. Since $\sigma \vdash \alpha_{\mathrm{dnf}}$ and $\alpha_{\mathrm{dnf}} \nvdash \sigma$ every assignment to the atomic wffs of α_{dnf} that makes σ true also makes α_{dnf} true but there is at least one assignment that makes α_{dnf} true but not σ true. So σ_{df} contains only disjuncts from

α_{dnf} but not all of α_{dnf}'s disjuncts. So σ_{df} is a proper sub-disjunction of α_{dnf} and since $\beta \vdash \sigma$ and $\sigma_{\mathrm{df}} \dashv\vdash \sigma$, $\beta \vdash \sigma_{\mathrm{df}}$. So $\alpha \not<\!\not<_b \beta$. □

6. THE TRANSITIVITY OF THE BASIC CONTENT PART RELATIONSHIP FOR L

We are now almost in a position to prove that the basic content part relation is transitive. That is, if α is a basic content part of β and β is a basic content part of σ then α is a basic content part of σ. Presumably transitivity is a strong desideratum for any notion of content part.

First we need to prove a pair of lemmas:

LEMMA 2.1. *If* $\alpha \ll_b \beta$ *then* $\mathrm{At}(\alpha_{\mathrm{dnf}}) \subseteq \mathrm{At}(\beta_{\mathrm{dnf}})$.

Proof. Assume $\alpha \ll_b \beta$. Then $\beta \vdash \alpha$. So $\beta_{\mathrm{dnf}} \vdash \alpha_{\mathrm{dnf}}$. So by Craig's interpolation theorem, for some μ, $\beta_{\mathrm{dnf}} \vdash \mu \vdash \alpha_{\mathrm{dnf}}$ and $\mathrm{At}(\mu) \subseteq (\mathrm{At}(\beta_{\mathrm{dnf}}) \cap \mathrm{At}(\alpha_{\mathrm{dnf}}))$. Now suppose $\alpha_{\mathrm{dnf}} \nvdash \mu$. Then $\beta \vdash \mu \vdash \alpha_{\mathrm{dnf}}$, $\alpha_{\mathrm{dnf}} \nvdash \mu$ and $\mathrm{At}(\mu) \subseteq \mathrm{At}(\alpha_{\mathrm{dnf}})$. And hence, since α_{dnf} is a representation of α in its minimal vocabulary, for any $\alpha' \dashv\vdash \alpha$, $\beta \vdash \mu \vdash \alpha'$, $\alpha' \nvdash \mu$ and $\mathrm{At}(\mu) \subseteq \mathrm{At}(\alpha')$. So $\alpha \not< \beta$ and hence $\alpha \not<\!\not<_b \beta$, contra our initial assumption. So $\alpha_{\mathrm{dnf}} \vdash \mu$. So $\alpha_{\mathrm{dnf}} \dashv\vdash \mu$. Now clearly $\mathrm{At}(\alpha_{\mathrm{dnf}}) \subseteq \mathrm{At}(\mu)$. And hence, since $\mathrm{At}(\mu) \subseteq \mathrm{At}(\beta_{\mathrm{dnf}})$, $\mathrm{At}(\alpha_{\mathrm{dnf}}) \subseteq \mathrm{At}(\beta_{\mathrm{dnf}})$.

LEMMA 2.2. *If* $\mathrm{At}(\alpha_{\mathrm{dnf}}) \subseteq \mathrm{At}(\beta_{\mathrm{dnf}})$ *then if* β_j *is a disjunct of* β_{dnf} *then* $\mathrm{At}(\alpha_{\mathrm{dnf}}) \subseteq \mathrm{At}(\beta_j)$.

Proof. Cf. the construction of β_{dnf}. This ensures that for any disjunct β_j, $\mathrm{At}(\beta_j) = \mathrm{At}(\beta_{\mathrm{dnf}})$.

THEOREM 2. *If* $\alpha \ll_b \beta$ *and* $\beta \ll_b \sigma$ *then* $\alpha \ll_b \sigma$.

Proof. Assume $\alpha \ll_b \beta$ and $\beta \ll_b \sigma$. Let $\ulcorner(\beta_1 \vee \ldots \vee \beta_n)\urcorner$ be β_{dnf} and $\ulcorner(\alpha_1 \vee \ldots \vee \alpha_m)\urcorner$ be α_{dnf}. Assume $\alpha \not<\!\not<_b \sigma$. Therefore for some proper sub-disjunction $(\alpha_s \vee \ldots \vee \alpha_t)$ of α_{dnf}, $\sigma \vdash (\alpha_s \vee \ldots \vee \alpha_t)$. Clearly for some β_j, $1 \leq j \leq n$, $\beta_j \nvdash (\alpha_s \vee \ldots \vee \alpha_t)$. For if for every such β_j, $\beta_j \vdash (\alpha_s \vee \ldots \vee \alpha_t)$ then $(\beta_1 \vee \ldots \vee \beta_n) \vdash (\alpha_s \vee \ldots \vee \alpha_t)$, in which case $\alpha \not<\!\not<_b \beta$, contra our initial assumption. Now since $\alpha \ll_b \beta$, $\mathrm{At}(\alpha_1 \vee \ldots \vee \alpha_m) \subseteq \mathrm{At}(\beta_1 \vee \ldots \vee \beta_n)$ – Cf Lemma 2.1. And since β_j is a disjunct of $(\beta_1 \vee \ldots \vee \beta_n)$, $\mathrm{At}(\alpha_1 \vee \ldots \vee \alpha_m) \subseteq \mathrm{At}(\beta_j)$ – Cf. Lemma 2.2.

So, $\mathrm{At}(\alpha_s \vee \ldots \vee \alpha_t) \subseteq \mathrm{At}(\beta_j)$. Now β_j is simply a conjunction of atomic and negated atomic wffs, so β_j represents a distribution of truth values for each atomic in $(\alpha_s \vee \ldots \vee \alpha_t)$. Now since that distribution makes none of $\alpha_s, \ldots, \alpha_t$ true – otherwise $\beta_j \vdash (\alpha_s \vee \ldots \vee \alpha_t)$ – it must make each of them false, so $\beta_j \vdash \sim (\alpha_s \vee \ldots \vee \alpha_t)$. Now since $\sigma \vdash (\alpha_s \vee \ldots \vee \alpha_t)$ and $\beta_j \vdash \sim (\alpha_s \vee \ldots \vee \alpha_t)$, $\sigma \vdash \sim \beta_j$. So $\sigma \vdash (\beta_1 \vee \ldots \vee \beta_{j-1} \vee \beta_{j+1} \vee \ldots \vee \beta_n)$, that is, $\beta_1 \vee \ldots \vee \beta_n$ less the disjunct β_j. So if $\alpha \not< \not<_b \sigma$ then $\beta \not< \not<_b \sigma$ contra our initial assumption. So $\alpha \ll_b \sigma$. □

Note that since we have shown that the $<_b$ relation and the $\ll_b$ relation are equivalent the above suffices to prove that both $<_b$ and $\ll_b$ are transitive.

7. A MECHANICAL PROCEDURE FOR GENERATING ALL LOGICALLY DISTINCT CONTENT PARTS OF ARBITRARY WFF β OF LANGUAGE L

Where content is identified with the class of contingent logical consequences then for any language such as L which has an infinite number of atomic wffs, any contingent wff has an infinite number of logically distinct content parts. Under our new notion of basic content as defined above all wffs have only a finite number of logically distinct basic content parts. Indeed under our new notion of basic content for any wff β we can mechanically construct a finite set which contains one, and only one, logical equivalent of each content part of β and nothing else.

First we need to show that for any given vocabulary s and any wff β, β has at most one logically distinct basic content part in s. Then we show that where α is a basic content part of β, there is some equivalent ϕ of α such that the vocabulary of ϕ occurs within β, that is $\mathrm{At}(\phi) \subseteq \mathrm{At}(\beta)$. Then we provide a mechanical means of constructing each and every content part of β expressible is some sub-vocabulary of β.

THEOREM 3. *If* $\alpha <_b \beta$ *then for any* ϕ *if* $\phi <_b \beta$ *and* $\mathrm{At}(\phi) = \mathrm{At}(\alpha)$ *then* $\phi \dashv\vdash \alpha$.

Proof. Assume $\alpha <_b \beta$, $\phi <_b \beta$ and $\mathrm{At}(\phi) = \mathrm{At}(\alpha)$. So $\mathrm{At}(\phi \,\&\, \alpha) \subseteq \mathrm{At}(\phi)$ and $\mathrm{At}(\phi \,\&\, \alpha) \subseteq \mathrm{At}(\alpha)$. Now suppose $\phi \nvdash \alpha$. Then $\beta \vdash (\phi \,\&\, \alpha) \vdash \phi$, $\phi \nvdash (\phi \,\&\, \alpha)$ and $\mathrm{At}(\phi \,\&\, \alpha) \subseteq \mathrm{At}(\phi)$. So $\phi \not<_b \beta$ contra our initial assumption. So $\phi \vdash \alpha$. By the same reasoning, mutatis mutandis, $\alpha \vdash \phi$. So $\phi \dashv\vdash \alpha$. □

THEOREM 4. *If* $\alpha <_b \beta$ *then for some* ϕ, $\phi \dashv\vdash \alpha$ *and* $\mathrm{At}(\phi) \subseteq \mathrm{At}(\beta)$.

Proof. Assume $\alpha <_b \beta$. So $\beta \vdash \alpha$. So, by Craig's theorem, for some ϕ, $\beta \vdash \phi \vdash \alpha$ and $\mathrm{At}(\phi) \subseteq (\mathrm{At}(\beta) \cap \mathrm{At}(\alpha))$. So $\mathrm{At}(\phi) \subseteq \mathrm{At}(\beta)$ and $\mathrm{At}(\phi) \subseteq \mathrm{At}(\alpha)$. Now suppose $\alpha \nvdash \phi$. Then $\beta \vdash \phi \vdash \alpha$ and $\alpha \nvdash \phi$ and $\mathrm{At}(\phi) \subseteq \mathrm{At}(\alpha)$. So, contra our initial assumption, $\alpha \not<_b \beta$. So $\alpha \vdash \phi$. So $\phi \dashv\vdash \alpha$ and $\mathrm{At}(\phi) \subseteq \mathrm{At}(\beta)$. □

Given the above theorems we may easily construct a mechanical means for constructing all the logically distinct basic content parts of an arbitrary (contingent) wff β. To construct β's logically distinct basic content parts: (1) Construct β_{dnf}; (2) For each non-empty subset s of $\mathrm{At}(\beta_{\mathrm{dnf}})$ construct $s\beta_{\mathrm{dnf}}$ by first eliminating from each disjunct d of β_{dnf} those conjuncts containing atomics that do not occur in s, and then eliminating each redundant disjunct; (3) If $s\beta_{\mathrm{dnf}}$ is contingent then $s\beta_{\mathrm{dnf}}$ is a basic content part of β; (4) Nothing else is a logically distinct basic content part of β.

From Theorems 3 and 4 above it follows that provided each contingent $s\beta_{\mathrm{dnf}}$ is indeed a basic content part of β then the above constructions yield all the logically distinct basic content parts β.

THEOREM 5. *If* β and $s\beta_{\mathrm{dnf}}$ *are contingent,* $s\beta_{\mathrm{dnf}} \ll_b \beta$.

Proof. Assume β and $s\beta_{\mathrm{dnf}}$ are contingent. Now clearly $\beta \vdash s\beta_{\mathrm{dnf}}$ and $s\beta_{\mathrm{dnf}}$ is its own dnf form. Now suppose $s\beta_{\mathrm{dnf}} \not\ll_b \beta$. Then for some proper sub-disjunction d of $s\beta_{\mathrm{dnf}}$, $\beta \vdash d$. Let c be any of the one or more disjuncts of $s\beta_{\mathrm{dnf}}$ that do not occur in d. Now since c does not occur in d and each disjunct of $s\beta_{\mathrm{dnf}}$, and hence of d, is a unique conjunction of atomics and negated atomics, $c \nvdash d$. Now since c is a disjunct of $s\beta_{\mathrm{dnf}}$ and every disjunct of $s\beta_{\mathrm{dnf}}$ is merely a disjunct of β_{dnf} less 0 or more conjuncts, there is some disjunct c^* of β_{dnf} such that $c^* \vdash c$. Since c^* is one of β_{dnf}'s disjuncts, $c^* \vdash \beta_{\mathrm{dnf}}$. But since c^* is c with 0 or more extra conjuncts none of which contain atomics occurring in d, and $c \nvdash d$, $c^* \nvdash d$. So $c^* \vdash \beta_{\mathrm{dnf}}$ and $c^* \nvdash d$, and hence $\beta_{\mathrm{dnf}} \nvdash d$, and so $\beta \nvdash d$. So $s\beta_{\mathrm{dnf}} \ll_b \beta$. □

While this completes our investigation of this new conception of content as applied to the propositional language L we shall now briefly consider how this conception might be extended to more complex formal languages.

8. CONTENT PARTS FOR CARNAP'S LANGUAGE SYSTEMS L_N

The definitions of content provided above apply equally to the language system L_N investigated in Carnap's *Logical foundations of Probability*, Carnap [1962], provided we treat those languages as not containing the identity sign '='.[21,22] For each language L_n, $n > 0$, of the language system L_N, L_n is like the propositional language L' described at the end of Section 3, above, save that (i) where L' contains an infinite number of individual constants, a_1, a_2, etc., L_n contains only the first n constants of L' and (ii) L_n contains an infinite number of individual variables, and universal and existential quantifiers, for instance '(x)' and '$(\exists x)$', defined in the customary ways.

While the members of language systems L_N, unlike our L and L', contain quantifiers these are essentially eliminable. A simple existential formula can be replaced by a disjunction of non-quantificational formulae. Thus in L_2, which contains only the two logical constants a_1 and a_2, the simple existential formula '$(\exists x)Px$' is equivalent to the non-quantificational formula '$Pa_1 \vee Pa_2$.' A simple universal formula can be replaced by a conjunction of non-quantificational formulae. Thus the L_2 formula '$(x)Px$' is equivalent to 'Pa_1 & Pa_2.' Statements with mixed quantifiers can be translated to non-quantificational equivalents through their prenex-normal forms. Starting with the outermost quantifier and working inwards existential quantifiers and their associated variables give way to disjunctions and universal quantifiers and their associated variables are replaced by conjunctions. Thus the L_2 formula '$(x)(\exists y)Gxy$' is transformed first to '$((\exists y)Ga_1y \mathbin{\&} (\exists y)Ga_2y)$' and then to its non-quantificational equivalent '$((Ga_1a_2 \vee Ga_1a_1) \mathbin{\&} (Ga_2a_1 \vee Ga_2a_2))$.'

To determine the basic content parts of formulae of Carnap's language systems L_N we look at their non-quantificational equivalents. Where α and β are variables for formulae of $L_j, j \in N$, let $P(\alpha)$ and $P(\beta)$ be, respectively, α and β's non-quantificational (i.e. propositional) equivalents. Then we can define basic content parts for arbitrary Language L_j of Carnap's language systems L_N as follows,

$\mathrm{BCPL}_{\substack{j \\ j \in N}}$ $\alpha <_b \beta =_{\mathrm{df}} P(\alpha)$ is a content part of $P(\beta)$ by definition BCPL1.[23]

Thus we get the result that the L_2 formula '$(x)Fx$' is a basic content part of '$(x)(Fx \,\&\, Gx)$' since its non-quantificational equivalent, '$(Fa_1 \,\&\, Fa_2)$,' is a basic content part of '$(x)(Fx \,\&\, Gx)$'s non-quantificational equivalent,

(8) $(Fa_1 \,\&\, Ga_1) \,\&\, (Fa_2 \,\&\, Ga_2)$

by CPL1. On the other hand '$(x)(Fx \vee Hx)$' is not a basic content part of '$(x)(Fx \,\&\, Gx)$' in L_2. The L_2 formula $(x)(Fx \vee Hx)$ is equivalent to

(9) $(Fa_1 \vee Ha_1) \,\&\, (Fa_2 \vee Ha_2)$.

(9) is not a CPL1 basic content part of (8) since it is defeated by

(10) $Fa_1 \,\&\, Fa_2$.

(10) defeats (9) because it is stronger than (9), its atomic wffs all occur in (9) and it is also a consequence of (8).

9. BASIC CONTENT PARTS FOR A PREDICATE CALCULUS WITHOUT IDENTITY

While the languages of Carnapian language system L_N are, in an obvious sense, not genuine quantificational languages they provide a convenient stepping stone to genuine quantificational languages. We shall now consider a monadic predicate calculus before considering a relational predicate calculus.

Let the language LM be like the languages of language system L_N save that LM contains an infinite stock of individual constants and only the monadic predicates of language systems L_N. LM then is a generic monadic predicate calculus. The restriction to monadic rather than relational predicate logic is important because formulae of the former, unlike the latter, always have finite models.

In defining the basic content parts for wffs of the languages of language system L_N, we handled quantifiers by eliminating them in favor of finite conjunctions and disjunctions of quantifier free wffs. In essence the quantifiers of the language system L_N are, we might say, "translated substitutionally". This allowed our specification of the basic content parts of formulae of the language system L_N to piggy back on our specification of the content part relationship for the propositional language L. Yet since LM contains an infinite stock of individual constants quantificational statements cannot be handled in quite the same way. The problem here is that where a quantificational statement of LM such as '$(x)Fx$' is translated substitutionally it yields an infinitely long sentence.[24] Nevertheless the piggy-backing strategy is still in order. We just need to complicate things a little bit.

Following Hempel[25] we will call the development of a wff σ over a set of individuals constants I, Dev $\langle \sigma, I \rangle$, as σ with its quantifiers translated substitutionally over the union set of all those individual constants which occur in σ and/or I. Thus the development of '$(x)Fx$ & Ga_3' over $\{a_2\}$ is '(Fa_2 & Fa_3) & Ga_3'. Now to find if a wff α is part of the basic content of a wff β we need to look at the relationship between the developments of α and β for certain sets of individual constants. In particular,

BCPLM $\alpha <_b \beta =_{\text{df}}$ For any non-empty set of individual constants I such that I contains every constant occurring in α or β, Dev $\langle \alpha, I \rangle$ is a content part of $\langle$Dev $\langle \beta, I \rangle$ by BCPL1.

Consider the statements '$(x)(Fx$ & $Gx)$' and '$(x)Fx$ & Ga_1'. Let $\{a_1, a_j, \ldots, a_t\}$ be any finite set of individual constants containing the individual constant 'a_1'. Then the development of '$(x)(Fx$ & $Gx)$' over this set is '(Fa_1 & Ga_1) & (Fa_j & Ga_j) & ... & (Fa_t & Ga_t)'. The development of '$(x)(Fx)$ & Ga_1' over this set is '(Fa_j & ... Fa_t) & Ga_1'. Clearly the second is by BCPL1 a basic content part of the first.

Consider the wff '$(\exists x)(Fx$ & $Gx)$' of LM. Definition BCPLM delivers the intuitively correct result that '$(\exists x)Fx$' and '$(\exists x)Gx$' are among its content parts while '$(\exists x)Fx \vee (x) \sim Gx$' is not.

When we move to a quantificational language with relational predicates (but not identity) our situation is complicated by the fact that some formulae have no finite models. However, this extra complication can presumably be handled by extending our original definitions of content parts for propositional languages to infinitary propositional languages. Rather than continue with this analysis, which poses no inherent problems, we will now briefly turn to the more vexing question of how to handle languages containing an identity operator.

10. BASIC CONTENT PARTS FOR LANGUAGES WITH IDENTITY

Till now we have avoided working with languages, propositional or quantificational, which include an identity operator. The reason for this omission is to be explained by the fact that once identity is allowed our standard definition of basis content part in terms of the strongest consequence obtainable in a given vocabulary of atomic wffs exhibits a breakdown of transitivity. Thus suppose we augment our language L_2 of language system L_N with the standard identity operator '='. Now note according to our definitions '$a_1 = a_2 \rightarrow Fa_2$ is a content part of 'Fa_1'. Yet '$a_1 = a_2 \rightarrow Fa_2$' is not a content part of 'Fa_1 & Fa_2'. It is defeated by Fa_2 itself, since Fa_2 is stronger than '$a_1 = a_2 \rightarrow Fa_2$', is a consequence of '$Fa_1$ & Fa_2' and contains only atomic wffs that occur in '$a_1 = a_2 \rightarrow Fa_2$'. So now we have the result that 'Fa_1' is a content part of 'Fa_1 & Fa_2', yet '$a_1 = a_2 \rightarrow Fa_2$' is a content part of '$Fa_1$' but not of '$Fa_1$ & Fa_2'.

In the face of this collapse of transitivity we might seek to tighten our definitions to preclude '$a_1 = a_2 \rightarrow Fa_2$' as counting as a content part of 'Fa_1'. Is this merely an ad-hoc response? That is a difficult question to answer. While I have no overwhelming intuitions concerning this question I am slightly moved by the following consideration: Consider a world W where 'Fa_1', '$a_1 = a_2$' and 'Fa_2' are all false. Then if '$a_1 = a_2 \rightarrow Fa_2$' counts as a content part of 'Fa_1' then 'Fa_1' is partially true (i.e. has a true content part) in W. This seems a somewhat undesirable result.

Rather than pursue this thorny question I will offer two definitions of (basic) content parts for the propositional language $L' + =$, being L' – recall L' is like L save that its atomics are subject predicate wffs of the from $\ulcorner Pi_1 \ldots i_n \urcorner$ where P is an n-adic predicate and $\ulcorner i_1 \ldots i_n \urcorner$ is a n membered series of individual constants – supplemented with the identity operator '='. The first definition preserves transitivity; the second does not. The first gives what we might call the primary basic content parts of formulae.

Essentially we preclude formulae such as '$a_1 = a_2 \rightarrow Fa_2$' counting as content parts of 'Fa_1' by insisting that if α is to be a content part of β, the essential vocabulary of α must occur as part of the essential vocabulary of β or some logical equivalent of β. This rules out '$a_1 = a_2 \rightarrow Fa_2$' as a content part of '$Fa_1$' since $a_1 = a_2$ is part of the (essential) vocabulary of '$a_1 = a_2 \rightarrow Fa_2$' but not of '$Fa_1$'. The reference to a logical equivalent of β is to ensure that wffs such as '$a_1 = a_2$' count as a primary content part of '$a_1 = a_3$ & $a_3 = a_2$.' In this case while the essential vocabulary of '$a_1 = a_2$' does not occur in '$a_1 = a_3$ & $a_3 = a_2$', it does occur essentially in its logical equivalent '$a_1 = a_2$ & $a_2 = a_3$.'

Thus we have the following definition for $L+=$:

BCPL+=1 $\alpha <_{pb} \beta =_{\text{df}}$ (i) α and β are contingent, (ii) α is a consequence of β and (iii) for some σ and μ, σ is logically equivalent to α and μ is logically equivalent to β, and every atomic wff that occurs in σ occurs in the essential vocabulary of μ, and there is no ϕ such that ϕ is stronger than σ, ϕ is a consequence of β and every atomic wff that occurs in ϕ occurs in σ.

The subscript 'pb' is to indicate that this definition gives the primary basic content parts of formulae of $L+=$.

It is worth noting that BCPL+ = 1 when applied to wffs of our propositional language L considered above is extensionally equivalent to BCPL1 and BCPL2 applied to wffs of L. Since BCPL1 is extensionally equivalent to BCPL2, to show this it will suffice to show that for any wffs α and β of L, $\alpha <_{pb} \beta$, according to BCPL+ = 1, iff $\alpha <_b \beta$, according to BCPL1. Now BCPL+ = 1 is just BCPL1 with the added condition that where α is a basic content part of β, for some equivalent σ of α and some equivalent ϕ of β every member of $\text{At}(\sigma)$ occurs in the essential vocabulary of ϕ. So since BCPL+ = 1 is just BCPL1 with an extra condition it follows that if $\alpha <_{pb} \beta$, according to BCPL+ = 1, then $\alpha <_b \beta$, according to BCPL1. Now suppose $\alpha <_b \beta$, according to BCPL1. Then, $\alpha <_b \beta_{\text{dnf}}$, and hence from Theorem 4 above it follows that for some σ, $\sigma \dashv\vdash \alpha$, $\sigma <_b \beta_{\text{dnf}}$ and $\text{At}(\sigma)$ $\text{At}(\beta_{\text{dnf}})$. Now $\beta_{\text{dnf}} \dashv\vdash \beta$ and further since $\text{At}(\sigma) \subseteq \text{At}(\beta_{\text{dnf}})$ and every atomic in β_{dnf} is in the essential vocabulary of β_{dnf}, every atomic in σ occurs in the essential vocabulary of β_{dnf}. So the extra conditions of BCPL+ = 1 are met. So if $\alpha <_b \beta$ according to BCPL1 then $\alpha <_{pb} \beta$ according to BCPL+ = 1.

By the same token, BCPL+ = 1 applied to the wffs of L', that is $L+=$, without the identity operator '=', is extensionally equivalent to BCPL1 and BCPL2 applied to wffs of L'.

For the "secondary" basic content parts of formulae of $L+=$ we use our original definition:

BCPL+ =2 $\alpha <_{sb} \beta =_{\text{df}}$ (i) α and β are contingent, (ii) α is a consequence of β, and (iii) for some μ, μ is logically equivalent to α and there is no σ such that σ is stronger than μ, σ is a consequence of β and every atomic wff that occurs in σ occurs in μ.

The subscript '*sb*' is to indicate that this definition gives the secondary basic content parts of formulae of $L+=$. Note that while every primary content part counts also as a secondary content part not every secondary counts as a primary content part.

Presumably these definitions can be extended to cover quantificational languages with identity in much the same way that we extended our definitions of content parts for L to cover languages with quantifiers.[26]

NOTES

[1] Often such content is referred to as "logical content". We shall stick to the term "content" for the sake of concision (however Cf. p. 599 below). We should bear in mind that the everyday philosopher's notion of content is a frankly semantic notion and hence cannot be captured by the merely syntactic notions of content discussed in this paper. Content as discussed here is at most logical or syntactical content. In a follow-up paper we shall consider model-theoretic and possible-worlds extensions of the notion of content examined here which are yet closer to the everyday notion.

[2] In Popper [1962], p. 385, we read: "By logical content (or the consequence class of *a*) we mean the class of all statements that follow from *a*."

For a Carnap reference see Note 5 below.

[3] Of course, for languages such as L, the syntactic relation of being a derivable consequence and the relation of being a logical consequence are co-extensive, that is, $\beta \vdash \alpha$ iff $\beta \models \alpha$. However, in as much as this project is, in its broadest conception, part of an ongoing attempt to see to what extent classical problems in the philosophy of science can be handled by merely syntactic means, future use of the term 'consequence' are best conceived as referring to the notion of derivable consequence.

[4] For our purposes here it makes no difference whether we identify a theory as any set of wffs or as any deductively closed set of wffs.

[5] In Carnap [1935], p. 42, we read: "the class of non-valid consequences of a given sentence is called the *content* of this sentence."

In Popper [1959], p. 120, we read: "the *logical content* [of a statement] is defined with the help of the concept of derivability, as the class of all non-tautologous statements which are derivable from the statement in question."

[6] In fact, nothing crucial hangs on this exclusion.

[7] Where β is a set of wffs we say β is contingent iff there is some contradiction c such that $\beta \nvdash c$ and there is some s such that $s \in \beta$ and $\{/\} \nvdash s$.

[8] Note that according to T.C.1 'p' and '$\sim p$' do not share common content since they only have tautologies as common consequences. As pointed out to me by a referee from this journal one might argue that both p and '$\sim p$' share common content in that they are both "concerned with" p. Yet according to T.C.1 they do not. I do not cite this as a criticism of the traditional notion of content since it is a feature inherited by my new notion of content.

[9] Cf. Popper [1972], p. 52.

[10] In Gemes [1990] it is demonstrated that some recent attempts to reformulate various versions of hypothetico-deductivism when combined with the classical theory of content produce thoroughly unacceptable results, for instance, that A is a black swan confirms the claim that all swans are white!

[11] Cf. Carnap [1962] Chapt. VI.

[12] From the principles of the calculus of logical probability it follows that where $e \vdash h$, and $P(e) > 0$, $P(h/e) = 1$. Now where $P(\beta/\alpha) < 1$, $P(\alpha) > 0$ and so, since $\alpha \vdash (\beta \vee \alpha)$, $P(\beta \vee \alpha/\alpha) = 1$. Now where $P(\beta/\sim\alpha) < 1$, $P(\sim\beta \;\&\; \sim\alpha) > 0$ and hence $P(\beta \vee \alpha) < 1$. *So where* $P(\beta/\alpha) < 1$ *and* $P(\beta/\sim\alpha) < 1$, $P(\beta \vee \alpha/\alpha) > P(\beta \vee \alpha)$. Further, from the principles of the calculus it follows that if $h \vdash e$, $P(e) < 1$, and $P(h) > 0$, then e is favorably relevant to h, that is, $P(h/e) > P(h)$. Now where $P(\beta/\alpha) < 1$, $P(\sim\beta \;\&\; \alpha) > 0$. And, as noted above, where $P(\beta/\sim\alpha) < 1$, $P(\beta \vee \alpha) < 1$, and hence $P(\alpha) < 1$. So where $P(\beta/\alpha) < 1$ and $P(\beta/\sim\alpha) < 1$, $P(\sim\beta \;\&\; \alpha) > 0$, $P(\alpha) < 1$, and $(\sim\beta \;\&\; \alpha) \vdash \alpha$, and so $P(\sim\beta \;\&\; \alpha/\alpha) > P(\sim\beta \;\&\; \alpha)$. Further it follows from the principles of the calculus that e is favorably relevant to h if and only if e is unfavourably relevant $\ulcorner\sim h\urcorner$, that is, $P(h/e) > P(h)$ if and only if $P(\sim h/e) < P(\sim h)$. So where $P(\beta/\alpha) < 1$ and $P(\beta/\sim\alpha) < 1$, $P(\sim(\sim\beta \;\&\; \alpha)/\alpha) < P((\sim(\sim\beta \;\&\; \alpha)$ and hence, since $\sim(\sim\beta \;\&\; \alpha) \dashv\vdash (\beta \vee \sim\alpha)$, $P(\beta \vee \sim\alpha/\alpha) < P(\beta \vee \sim\alpha)$.

[13] These points are taken up in greater detail in Gemes [1993a] and Gemes [1993d].

[14] The problems with previous formulations, some of which are briefly touched on above, and the ways our new notion of content may be employed to avoid these problems will be examined in the forthcoming papers, Gemes [1993a], [1993b], [1993c], and [1993d].

[15] It is commonly thought that the identification of content with consequence class is due to Tarski. At least this attribution is suggested by p. 47 of Popper [1972]. Usually the works cited in such attributions are Tarski's essay "Foundations of the Calculus of Systems", and, occasionally, "On Some Fundamental Concepts of Metamathematics", both in Tarski [1983]. In fact, Tarski in those essays make no mention of the notion of logical content. However, he does use the symbol 'Cn' to denote the consequence class. Perhaps some readers have taken the liberty of reading 'Cn' as an abbreviation for 'content'. I have found no place where Tarski explicitly identifies the concepts of content and consequence class. Moreover, I believe that given Tarski's general rigor in separating what he took to be semantic matters from syntactic matters he would not be inclined to make such an identification.

[16] Cf., for instance, Belnap and Anderson [1975], p. 340.

[17] Cf. Parry [1933].

[18] Cf. Hempel [1965], p. 13.

[19] In "A New Theory of Content II" I will investigate a proof-theoretic means for

constructing the content part relationship. This construction will involve several non-standard rules.

[20] In Grimes [1990] the idea of using disjunctive normal forms to construct a new notion of content, or, as Grimes calls it, "narrow consequence", is also suggested. Basically, for Grimes, α is a content part of β, if where $\ulcorner(\alpha_1 \vee \ldots \vee \alpha_n)\urcorner$ is a disjunctive normal form of α, there is some α_k, $1 \leq k \leq n$, $\beta \vdash \alpha_k$. While this notion of content has the desirable consequence that, for instance, '$(p \vee q)$' is not a content part of 'q', it also has several unpalatable consequences. For instance, according to Grimes' construction '$(p \vee q)$' is not a content part of '$(p \vee q)$'. More generally, Grimes' construction has the undesirable consequence that for any disjunction $\ulcorner\alpha \vee \beta\urcorner$, if there is no non-empty set S such that for any s, if $s \in S$ then s is an atomic wff or a negated atomic wff and $\alpha \vdash s$ and $\beta \vdash s$, $\ulcorner(\alpha \vdash \beta)\urcorner$ is not a content part of itself. It also yields the result that while, for instance, 'p' is a content part of '$(p \,\&\, q)$' and 'r' is a content part of '$(r \,\&\, s)$', '$(p \vee r)$' is not a content part of '$((p \,\&\, q) \vee (r \,\&\, s))$'. Further, it has the highly undesirable consequence that '$(p \vee \sim q)$' is a content part of '$(p \,\&\, q)$'.

[21] For typographical convenience we use the symbol 'L_N' where Carnap [1962] uses '$\mathscr{L}_N$'.

[22] This is no substantial restriction since while Carnap formally includes '=' in the vocabulary of his language systems L_N he essentially negates that inclusion by stipulating in his semantics that under all interpretations sentences of the form $a_j = a_k$, where $j \neq k$, are to be assigned the value F.

[23] Note, on this definition wffs of the form $\ulcorner(\exists x)\sigma x\urcorner$ will typically not count as part of the contents of wffs of the form $\ulcorner(x)\sigma x\urcorner$. For instance, the L_2 wff '$(\exists x)Fx$' is not part of the basic content of the L_2 wff '$(x)Fx$' since, in L_2, '$P((\exists x)Fx)$' is equivalent to '$Fa \vee Fb$' and '$P((x)Fx)$' is equivalent to '$Fa \,\&\, Fb$' and '$Fa \vee Fb$' is not a content part of '$Fa \,\&\, Fb$' by BCPL1. However, where BCPL1 is supplemented with a recursive clause allowing that disjuncts of content parts count as content parts this result does not apply.

[24] Of course, we could at this point employ the resources of inifinitary propositional languages. But I prefer to forego those resources for as long as possible.

[25] Cf. Hempel [1965], p. 36.

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