numerical calculations and the asymptotic structure of the natural oscillations is observed in the other regimes considered in Sec. 1.

We note that the asymptotic analysis gives a graphic description of the properties of the natural oscillations in the shock layer. The dispersion relations obtained can be used as the initial approximation for numerically calculating the stability characteristics of the various modes with allowance for viscosity.

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MOTION OF A DUSTY GAS AT THE ENTRANCE TO A FLAT CHANNEL AND A CIRCULAR PIPE

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It is shown that at large Reynolds numbers, calculated from the entrance velocity and the half-width of the channel, four characteristic flow regions are formed in the entrance section. The equations describing the motion of the mixture in each of these regions are constructed by the method of matched expansions. An expression relating the particle concentration distribution at points remote from the entrance section to the particle concentration distribution in the boundary layer on a flat plate at points remote from the beginning of the plate is obtained. The dependence of the dispersed-phase concentration profile formed on the governing parameters is studied on the basis of a numerical solution. It is shown that as the contribution of the Saffman force to the interphase momentum transfer increases, the rise in particle concentration in the direction of the wall is replaced by a fall. A qualitative correspondence between the calculated particle concentration profiles and certain known experimental data is noted.

It is proposed to investigate the transition from a uniform to a nonuniform inertial particle distribution over the cross section for the motion of a gas suspension along the entrance length of a channel (pipe) using a two-continuum model of a dusty gas [1]. The Saffman force [2], which causes the transverse migration of the particles, is taken into account in the expression for the interphase momentum transfer.

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An extensive literature (see, in particular, [3—5]) has been devoted to the investigation of the general laws of channel motion of gases transporting liquid and solid particles. Although in most cases of practical importance the motion is turbulent, here we will consider laminar flow. This makes it possible to obtain an asymptotic solution for the particle concentration profile at points remote from the channel entrance without introducing the empirical hypotheses necessary for describing turbulent particle transport. For sufficiently large particles that do not participate in the fluctuating motion the qualitative conclusions remain valid for turbulent motion of the gas also.

1. Formulation of the Problem and Asymptotic Solution Near the Channel Entrance Section

Our object is to clarify the general laws of transition from a uniform to a nonuniform particle distribution over the cross section of a channel (pipe) as a result of the inertial lag of the particles relative to the carrier phase and the two-dimensionality of the flow on the entrance length. As usual [1], it is assumed that the particles are spheres with the same radius σ and mass m. The volume particle concentration is negligibly small, Brownian motion is unimportant, and the carrier phase is incompressible. The transverse migration of particles due to the local shear character of the flow over the trial particle is taken into account. Saffman's equation [2] is used for calculating the lift, and the expression for the particle drag is taken in the Klyachko form [6].

The Cartesian coordinate system x, y moves with the entrance section of the channel, the x axis being directed along the lower wall. Then the force exerted by the carrier phase on unit mass of the trial particle takes the form:

$$\mathbf{F}=\frac{6\pi\sigma\mu}{m}(\mathbf{v}-\mathbf{v}_s)D+\frac{6.46\sigma^2\sqrt{\mu\rho}}{m}\sqrt{\frac{\partial u}{\partial y}}(u-u_s)\mathbf{j},\quad D=1+\frac{1}{6}\mathrm{Re}_s^{2/3},\qquad \mathrm{Re}_s=\frac{2\sigma|\mathbf{v}-\mathbf{v}_s|\rho}{\mu}$$

Here, μ and ρ are the viscosity and density of the carrier phase, $\mathbf{j}$ is the unit vector along the ordinate axis, and the subscript s relates to the parameters of the particle medium.

At the channel entrance the flow of the gas suspension is homogeneous. The dimensional parameters at the entrance are denoted by the subscript ∞. We assume that the Reynolds number, calculated from the entrance velocity and the half-width of the channel a, is large, i.e., $\mu/U_\infty a\rho = \varepsilon \ll 1$. We will construct the asymptotic solution of the problem of the flow of the gas suspension on the entrance length of the channel as $\varepsilon \to 0$, using the procedure of the method of matched expansions [7]. In what follows we will employ two substantially different length scales: $\ell = mU_\infty/6\pi\sigma\mu$ and $L = (1 + \alpha = a^2U_\infty\rho/\mu$, where ℓ is the characteristic phase velocity relaxation length, L is the characteristic closing length of the boundary layers building up on the channel walls, and $\alpha = \rho_{s\infty}/\rho$ is the mass particle concentration. The most important case is that corresponding to $\alpha \sim O(1)$, $\lambda = a/\ell \sim O(1)$.

We will write out the system of equations of a dusty gas for the length scale ℓ in dimensionless form, using the following scales for making the parameters dimensionless: for the pressure ρU_∞^2, for the density of the particle medium $\rho_{s\infty}$, for the phase velocity components U_∞, and for the coordinates ℓ. Then the system of equations of motion of the dusty gas [1] takes the form:

$$\operatorname{div}\mathbf{v}=0,\ \operatorname{div}\rho_s\mathbf{v}_s=0,\ (\mathbf{v}\nabla)\mathbf{v}+\nabla p+\alpha\rho_s\left[(\mathbf{v}-\mathbf{v}_s)D+\varkappa(u-u_s)\sqrt{\frac{\partial u}{\partial y}}\mathbf{j}\right]=\varepsilon\lambda\Delta\mathbf{v}$$

$$(\mathbf{v}_s\nabla)\mathbf{v}_s=(\mathbf{v}-\mathbf{v}_s)D+\varkappa(u-u_s)\sqrt{\frac{\partial u}{\partial y}}j \tag{1.1}$$

$$D=1+\frac{1}{6}\mathrm{Re}_{s0}^{2/3}|\mathbf{v}-\mathbf{v}_s|^{2/3},\qquad \mathrm{Re}_{s0}=\frac{2\sigma\rho U_\infty}{\mu},\qquad \varkappa=\frac{6.46\sigma}{6\pi l\sqrt{\varepsilon\lambda}}$$

$$x=0:\ u=u_s=\rho_s=1;\ y=0.2\lambda:\ u=v=0$$

The construction of the asymptotic solution of problem (1.1) as $\varepsilon \to 0$ is analogous to the solution of the problem of a two-phase boundary layer on a flat plate [8]. The exterior solution ($\varepsilon \to 0$; x, y fixed) is obvious: $u = u_s = \rho_s = 1$, $v = v_s = 0$, $p = \text{const}$. In order to construct the boundary layer equations we introduce the stretched coordinate $\eta = y/\sqrt{\varepsilon\lambda}$. Because of the symmetry we will consider only the boundary layer on the lower wall of the channel. We seek the interior solution in the form:

$$u_0(x,\eta)+\ldots,\ u_{s0}(x,\eta)+\ldots,\ p_0(x,\eta)+\ldots,\ \rho_{s0}(x,\eta)+\ldots,\ \sqrt{\varepsilon\lambda}v_0(x,\eta)+\ldots,\ \sqrt{\varepsilon\lambda}v_{s0}(x,\eta)+\ldots$$

From (1.1) we obtain the boundary layer equations and matching conditions

$$\frac{\partial u_0}{\partial x}+\frac{\partial v_0}{\partial \eta}=0,\quad \frac{\partial \rho_{s0}u_{s0}}{\partial x}+\frac{\partial \rho_{s0}v_{s0}}{\partial \eta}=0,\quad \frac{\partial p_0}{\partial \eta}=0,\quad u_0\frac{\partial u_0}{\partial x}+v_0\frac{\partial u_0}{\partial \eta}+\alpha\rho_{s0}(u_0-u_{s0})D_0=\frac{\partial^2 u_0}{\partial \eta^2}$$

$$u_{s0}\frac{\partial u_{s0}}{\partial x}+v_{s0}\frac{\partial u_{s0}}{\partial \eta}=(u_0-u_{s0})D_0,\quad u_{s0}\frac{\partial v_{s0}}{\partial x}+v_{s0}\frac{\partial v_{s0}}{\partial \eta}=(v_0-v_{s0})D_0+\varkappa_0(u_0-u_{s0})\sqrt{\frac{\partial u_0}{\partial \eta}} \tag{1.2}$$

$$D_0=1+\frac{1}{6}\mathrm{Re}_{s0}^{*}(u_0-u_{s0})^{2/3},\qquad \varkappa_0=\frac{6.46\sigma}{6\pi}\sqrt{\frac{U_\infty}{\nu l}}$$

$$\eta=0:\ u_0=v_0=0;\ \eta\to\infty:\ u_0\to 1,\ x=0:\ u_{s0}=\rho_{s0}=1,\ v_{s0}=0$$

We will consider certain characteristic values of the dimensionless parameters for real particle sizes, particle material, flow velocities, etc. Let the half-width of the channel be 1 cm, $\nu = 0.15$ cm^2/sec, $U_\infty = 10^3$ cm/sec, and $\rho_s^\circ/\rho = 10^4$ (here, ρ_s° is the density of the particle material); in this case $\varepsilon = 1.5\cdot 10^{-4}$, and the other dimensionless parameters for three particle sizes are given below:

σ, cm	10^{-4}	10^{-3}	10^{-2}
l, cm	$1.48\cdot 10^{-1}$	14,8	$1.48\cdot 10^{3}$
Re_{s0}	1,33	13,3	133
$\varkappa_0$	1,28	40,47	1280

As the particle size increases, the contribution of the Saffman force (the quantity $\varkappa_0$) to the interphase momentum exchange in the boundary layer rapidly increases.

2. Asymptotic Solution for the Boundary Layer Closing Scale

We introduce the following new scales: for the longitudinal coordinate $L = (1 + \alpha)a^2U_\infty/\nu$, for the transverse coordinate a, for the longitudinal velocity components U_∞, for the transverse velocity components aU_∞/L, for the pressure $\rho(1 + \alpha)U_\infty^2$, and for the density of the particle medium $\rho_{s\infty}$. In this domain the dimensionless coordinates are denoted by X, Y; the unknown dimensionless functions are denoted by the subscript 1. The system of equations of motion takes the form:

$$\frac{\partial u_1}{\partial X}+\frac{\partial v_1}{\partial Y}=0,\quad \frac{\partial \rho_{s1}u_{s1}}{\partial X}+\frac{\partial \rho_{s1}v_{s1}}{\partial Y}=0$$

$$u_1\frac{\partial u_1}{\partial X}+v_1\frac{\partial u_1}{\partial Y}+\alpha\rho_{s1}\left(u_{s1}\frac{\partial u_{s1}}{\partial X}+v_{s1}\frac{\partial u_{s1}}{\partial Y}\right)+\frac{\partial p_1(1+\alpha)}{\partial X}=\frac{\varepsilon^2}{1+\alpha}\frac{\partial^2 u_1}{\partial X^2}+(1+\alpha)\frac{\partial^2 u_1}{\partial Y^2}$$

$$u_1\frac{\partial v_1}{\partial X}+v_1\frac{\partial v_1}{\partial Y}+\alpha\rho_{s1}\left(u_{s1}\frac{\partial v_{s1}}{\partial X}+v_{s1}\frac{\partial v_{s1}}{\partial Y}\right)+\frac{(1+\alpha)^3}{\varepsilon^2}\frac{\partial p_1}{\partial Y}=\frac{\varepsilon^2}{1+\alpha}\frac{\partial^2 v_1}{\partial X^2}+(1+\alpha)\frac{\partial^2 v_1}{\partial Y^2} \tag{2.1}$$

$$u_{s1}\frac{\partial u_{s1}}{\partial X}+v_{s1}\frac{\partial u_{s1}}{\partial Y}=\frac{\lambda(1+\alpha)}{\varepsilon}(u_1-u_{s1})$$

$$u_{s1}\frac{\partial v_{s1}}{\partial X}+v_{s1}\frac{\partial v_{s1}}{\partial Y}=\frac{\lambda(1+\alpha)}{\varepsilon}\left[(v_1-v_{s1})+\varkappa_0(1+\alpha)\left(\frac{\lambda}{\varepsilon}\right)^{1/4}\sqrt{\frac{\partial u_1}{\partial Y}}(u_1-u_{s1})\right]$$

$$Y=0,2:\ u_1=v_1=0;\ X=0:\ \rho_{s1}=u_{s1}=u_1=1$$

As $\varepsilon \to 0$ from (2.1) we obtain the equations for the leading terms of the expansion (for simplicity we have not introduced new notation)

$$\frac{\partial u_1}{\partial X}+\frac{\partial v_1}{\partial Y}=0, \quad u_1\frac{\partial \rho_{s1}}{\partial X}+v_1\frac{\partial \rho_{s1}}{\partial Y}=0, \quad \frac{\partial p_1}{\partial Y}=0, \quad u_1=u_{s1}, \quad v_1=v_{s1}$$

$$u_1\frac{\partial u_1}{\partial X}+v_1\frac{\partial u_1}{\partial Y}+\alpha\rho_{s1}\left(u_1\frac{\partial u_1}{\partial X}+v_1\frac{\partial u_1}{\partial Y}\right)+(1+\alpha)\frac{\partial p_1}{\partial X}=(1+\alpha)\frac{\partial^2 u_1}{\partial Y^2} \tag{2.2}$$

The boundary conditions at the channel entrance are obtained from the condition of matching with the uniform homogeneous flow for $X \to 0$ and fixed Y. Then from the continuity equation for the particle medium we obtain $\rho_{s1} = 1$ and system (2.2) can be considerably simplified:

$$\frac{\partial u_1}{\partial X}+\frac{\partial v_1}{\partial Y}=0, \quad u_1\frac{\partial u_1}{\partial X}+v_1\frac{\partial u_1}{\partial Y}+\frac{dp_1}{dX}=\frac{\partial^2 u_1}{\partial Y^2}, \quad u_{s1}=u_1, \; v_{s1}=v_1$$

$$X=0:\; u_1=1; \; Y=0.2:\; u_1=v_1=0 \tag{2.3}$$

System (2.3) completely coincides with the formulation of the problem of the motion of a homogeneous viscous fluid on the entrance length of a channel, for which an approximate solution was constructed by Schlichting [9]. This solution has the form (for details see [9]):

$$u_1(X, Y)=f_0'+\sqrt{X}f_1' \; (X<X_0), \quad u_1(X, Y)=1.5(2Y-Y^2)-0.3485g_0'\exp(-18.75X) \; (X\geqslant X_0) \tag{2.4}$$

Here, $\zeta = Y/\sqrt{X}$, and the functions $f_0(\zeta)$, $f_1(\zeta)$, and $g_0(Y)$ satisfy the following boundary-value problems:

$$f_0f_0''+2f_0'''=0; \; \zeta=0:\; f_0=f_0'=0; \; f_0'(\infty)=1$$

$$2f_1'''+f_0f_1''-f_0'f_1'+2f_0''f_1=-1.72; \; \zeta=0:\; f_1=f_1'=0; \; f_1'(\infty)=1.72$$

$$g_0^{\prime\sim}+3k[(Y-0.5Y^2)g_0''+g_0]=0; \; Y=0:\; g_0=g_0'=0; \; Y=1:\; g_0=g_0''=0$$

The eigenvalue k = 18.75, and the coordinate $X_0 \simeq 0.16$ [9].

The solution (2.4) and $\rho_{s1} = 1$ are not a uniformly applicable asymptotic solution of the system (2.1), since thin zones in which the particle concentration differs from unity occur near the channel walls. This follows from the solution of Eqs. (1.2) described in Sec. 3, from which it is clear that as the relaxation of the phase velocities is completed a layer of inhomogeneous particle concentration is formed in the boundary layer near the wall. Asymptotically, this layer is thinner than the half-width of the channel.

We will construct the equations describing the motion of the mixture in this layer, which we shall call the lower sublayer. We introduce the new stretched variables (scales selected from the conditions of matching with the solution in the other regions)

$$u_1=\varepsilon_1^{1/4}u_2(X, z)+\ldots, \; u_{s1}=\varepsilon_1^{1/4}u_{s2}(X, z)+\ldots, \; v_1=\varepsilon_1^{1/2}v_2(X, z)+\ldots, \; v_{s1}=\varepsilon_1^{1/2}v_{s2}(X, z)+\ldots$$

$$\rho_{s1}=\rho_{s2}(X,z)+\ldots, \quad \varepsilon_1=\frac{\varepsilon}{(1+\alpha)\lambda}, \quad z=\frac{Y}{\varepsilon_1^{1/4}} \tag{2.5}$$

Here and in what follows the subscript 2 denotes the flow parameters in the lower sublayer. Substituting expansions (2.5) in (2.1) and retaining the leading terms, we obtain the equations describing the flow in the lower sublayer

$$\frac{\partial u_2}{\partial X}+\frac{\partial v_2}{\partial z}=0, \quad u_2\frac{\partial \rho_{s2}}{\partial X}+v_2\frac{\partial \rho_{s2}}{\partial z}=0, \quad \frac{\partial^2 u_2}{\partial z^2}=0, \; u_{s2}=u_2, \; v_{s2}=v_2 \tag{2.6}$$

The solution of (2.6) has the form $u_2 = G(X)z$, $v_2 = -G'z^2/2$. The function G(X) is found from the condition of matching u_2 with u_1 in the region of linear growth of u_1:

$$G(X)=\left.\frac{\partial u_1}{\partial Y}\right|_{Y=0}$$

The density value ρ_{s2} in the lower sublayer is transferred along the carrier phase streamline $\omega = z\sqrt{G(X)}$ = const. The form of the function $\rho_{s2}(\omega)$ must be found from the condition of matching with the solution for the density of the particle medium obtained from (1.2) as $x \to \infty$. From the choice of scales for the nondimensionalization of the coordinates it follows that

$$\frac{\eta}{x^{1/4}}=\frac{z}{X^{1/4}(1+\alpha)^{1/2}}=\theta \tag{2.7}$$

Here, θ is the notation for the coordinate combination in question. From (2.7) it follows that the solutions for the particle density obtained from (1.2) and (2.6) can conveniently be matched in the variables x, θ and X, θ for fixed θ and $x \to \infty$ and $X \to 0$, respectively. In this case X plays the part of exterior, and x that of interior coordinate, since $x = \varepsilon X/[\lambda(1 + \alpha)]$. From (2.4) it follows that for small X we have $G(X) \sim f_0''(0)/\sqrt{X}$, where $f_0''(0) = 0.33206$ [9]. Consequently, for small X the relation ω = const can be written in the form $(z\sqrt{0.33206})X^{-1/4}$ = const. Hence the particle density matching condition can be represented in the form:

$$\rho_{s2}(\omega)=\rho_{s0}^{\lim}(\theta), \quad \omega=\theta\sqrt{0.33206(1+\alpha)} \tag{2.8}$$

Here, $\rho_{s0}^{\lim}$ is the limit of the density of the particle medium as $x \to \infty$, which must be found from the solution of Eqs. (1.2) in the variables x, θ.

For large values of X, from (2.4) we have $G(X) \to 3$, and the relation ω = const takes the form $z\sqrt{3}$ = const. The condition of conservation of ρ_{s2} in the lower sublayer for fixed ω enables us, knowing $\rho_{s0}^{\lim}(\theta)$ from the solution of (1.2), to find the density distribution $\rho_{s3}(Y)$ at large values of X corresponding to a Poiseuille velocity profile. Using the relation between z and Y (2.5), from (2.8) for large X we have

$$\rho_{s3}(Y)=\rho_{s0}^{\lim}(\theta), \quad Y=\theta\left[\frac{\varepsilon(1+\alpha)}{\lambda}\right]^{1/4}\sqrt{\frac{0.33206}{3}} \tag{2.9}$$

Thus, the problem of determining the particle concentration profile $\rho_{s3}(Y)$ in the region of flow independent of the longitudinal coordinate has been reduced to finding the limit $\rho_{s0}^{\lim}(\theta)$ for large x from the solution of the two-phase boundary layer equations (1.2).

3. Determination of the Density of the Particle Medium in the Boundary Layer

In [8] Eqs. (1.2) were solved for Stokes flow over the particles ($Re_{s0} = 0$) and without allowance for the Saffman force ($\varkappa_0=0$). Moreover, it was shown that, qualitatively, the form of the particle concentration profile at points remote from the entrance does not depend on the mass particle concentration α. Therefore we will set $\alpha = 0$, which considerably simplifies the problem (1.2), and investigate the effect of the finiteness of $\varkappa_0$ and Re_{s0} on the dispersed-phase concentration distribution. When the particles do not affect the motion of the carrier phase ($\alpha = 0$), the carrier-phase velocity field in the boundary layer is determined from the solution of the Blasius problem and takes the form [9]:

$$u_0(x,\eta)=\varphi', \quad v_0(x,\eta)=\frac{1}{2\sqrt{x}}\left(\frac{\eta\varphi'}{\sqrt{x}}-\varphi\right)$$

Here, the function $\varphi(\eta/\sqrt{x})$ satisfies the boundary-value problem

$$2\varphi'''+\varphi\varphi''=0, \; \varphi(0)=\varphi'(0)=0, \; \varphi'(\infty)=1$$

The equations of motion and continuity of the particle medium, considered on a fixed particle trajectory, can be reduced to a system of ordinary differential equations. For this purpose, as the independent variables we introduce the dimensionless time of particle motion along the trajectory t and the Lagrangian coordinate η_0, the ordinate of the origin of the particle trajectory at x = 0. For fixed η_0 the equations of motion of the particle medium and the boundary conditions take the form:

$$\frac{dx}{dt}=u_{s0}, \quad \frac{d\eta}{dt}=v_{s0}, \quad \frac{du_{s0}}{dt}=(u_0-u_{s0})D_0$$
$$\frac{dv_{s0}}{dt}=(v_0-v_{s0})D_0+\varkappa_0(u_0-u_{s0})\sqrt{\frac{\partial u_0}{\partial\eta}}, \quad t=0\colon\; x=0,\; \eta=\eta_0,\; u_{s0}=1,\; v_{s0}=0 \tag{3.1}$$

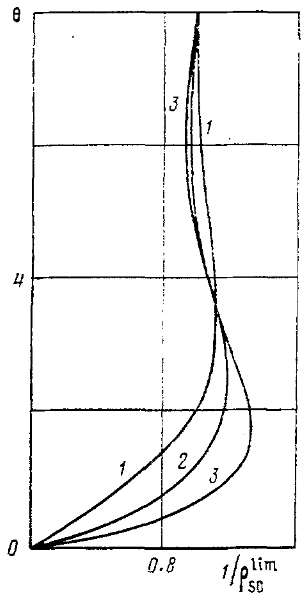

Fig. 1

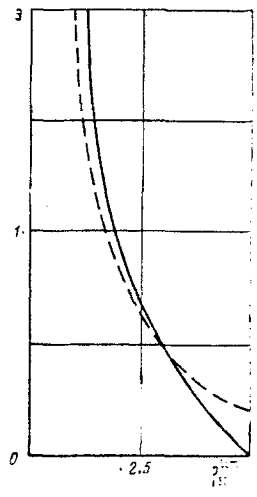

Fig. 2

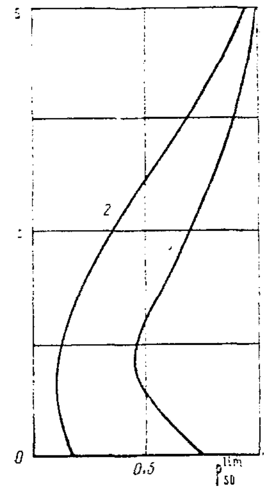

Fig. 3

In the Lagrangian coordinates selected the continuity equation has the form:

$$\frac{1}{\rho_{s0}(\eta_0,t)} = u_{s0}g(\eta_0,t) - v_{s0}e(\eta_0,t), \quad e=\frac{\partial x(\eta_0,t)}{\partial \eta_0}, \quad g=\frac{\partial \eta(\eta_0,t)}{\partial \eta_0} \tag{3.2}$$

In order to find ρ_{s0} from (3.2) along the particle trajectory it is necessary to know e and g. In order to determine these quantities we differentiate (3.1) with respect to η_0 and obtain the equations and boundary conditions

$$\frac{de}{dt}=f, \quad \frac{df}{dt}=A\left[1+\frac{5}{18}\mathrm{Re}_{s0}^{2/3}(u_0-u_{s0})^{2/3}\right], \quad \frac{dg}{dt}=h$$

$$\frac{dh}{dt}=BD_0+\frac{\mathrm{Re}_{s0}^{2/3}(v_0-v_{s0})A}{9(u_0-u_{s0})^{1/3}}+\varkappa_0\left[A+\frac{C(u_0-u_{s0})}{4}\right]$$

$$A=e\frac{\partial u_0}{\partial x}+g\frac{\partial u_0}{\partial \eta}-f, \quad B=e\frac{\partial v_0}{\partial x}+g\frac{\partial v_0}{\partial \eta}-h, \quad f=\frac{\partial u_{s0}(\eta_0,t)}{\partial \eta_0} \tag{3.3}$$

$$C=2\left(e\frac{\partial^2 u_0}{\partial x \partial \eta}+g\frac{\partial^2 u_0}{\partial \eta^2}\right)\frac{\partial u_0}{\partial \eta}, \quad h=\frac{\partial v_{s0}(\eta_0,t)}{\partial \eta_0}, \quad t=0:\ e=0,\ f=0,\ g=1,\ h=0$$

In preparation for the numerical calculations we introduced the new independent variable x; the system of equations obtained was integrated numerically by the Kutta–Merson method. For calculating the velocity components of the carrier phase and their derivatives with respect to the coordinates we used cubic interpolation of the tabulated values of the function φ from [9].

We investigated the dependence of the particle concentration profile formed at points remote from the entrance on the parameters Re_{s0} and $\varkappa_0$. From the results of the numerical calculations it follows that as x increases the particle concentration becomes a function of the single variable θ, which confirms the asymptotic flow structure described above. The distance from the leading edge at which a self-similar concentration profile is formed depends to a considerable extent on $\varkappa_0$; thus when $\varkappa_0=0$ $x \gtrsim 20$, and when $\varkappa_0=300$ $x \geqslant 200$.

In Fig. 1 we have plotted $1/\rho_{s0}^{\lim}$ (limiting distribution for large x) as a function of θ without allowance for the Saffman force ($\varkappa_0=0$); the numbers 1–3 denote the curves corresponding to Re_{s0} = 0, 100, and 1000, respectively. Qualitatively, it is clear that in the stabilized flow zone the particle concentration distribution depends only weakly on the maximum Reynolds number. The dependence of the limiting particle concentration profile on the contribution of the Saffman force is demonstrated in Figs. 2 and 3. It was assumed that $Re_{s0} = 0$. The broken curve in Fig. 2 corresponds to the case $\varkappa_0=0$, the continuous curve to the case $\varkappa_0=10$. In Fig. 3 curves 1 and 2 correspond to the cases $\varkappa_0=50$ and 300. Clearly, with increase in the contribution of the Saffman force to the interphase momentum exchange the particle concentration profile undergoes a qualitative restructuring; the region of high particle concentration near the wall disappears.

It should be kept in mind that in the mathematical model employed particles reflected from the wall are not taken into account. Thus, it was assumed that the particles carried to the wall by the Saffman force remained within a thin layer of insignificant thickness.

Moreover, in the case of small particles ($\varkappa_0=0$) in the solution obtained when x = 1 the particle concentration at the wall increases without bound, and, as shown in [10], the resulting concentration singularity is not integrable. This means that, in fact, near x = 1 the small particles become concentrated and a particle "hillock," in which the particle content approaches close packing, develops. Behind the "hillock" a particle-free zone should form. Consequently, in the real small-particle concentration distribution on the stabilized flow interval instead of an unbounded increase in concentration (Fig. 1) near the wall there should be a narrow zone in which the particle concentration falls sharply to zero, i.e., the concentration maximum is reached at a finite distance from the wall.

By simple conversion of the argument, using (2.9) we can find the particle concentration distribution over the channel cross section from the calculated relations $\rho_{s0}^{\lim}(\theta)$ (Figs. 1–3).

The monograph [5] gives five typical particle concentration distributions obtained in various experiments on vertical pipes and channels, in particular in [11]. Qualitatively, these distributions fall into two types: a) the particle concentration maximum lies close to the walls; b) concentration maximum on the channel axis. The solution described above explains the formation of these profiles and the transition from a) to b) with increase in the contribution of the Saffman force to the interphase momentum exchange.

4. Flow on the Entrance Length of a Circular Pipe

The asymptotic structure of the two-phase flow described above is preserved in the case of a circular pipe. In fact, on the length scale a the equations of the boundary layer on the walls coincide with the equations of the boundary layer on a plate. Likewise, the curvature of the walls does not affect the equations describing the flow in the region of the lower sublayer (because it is thin as compared with a). Only the asymptotic equations describing the equilibrium flow of the mixture on the longitudinal L and transverse a scales change. However, for determining the particle concentration distribution it is necessary to know only the function G(X), and for finding the particle concentration distribution in the stabilized flow zone only the asymptotic form of G(X) as $X \to \infty$ is required. We will find this expression: at large X the velocity profile tends to the Poiseuille profile [9] (r is the dimensional distance from the pipe axis)

$$u\left(\frac{r}{a}\right)=2\left(1-\frac{r^2}{a^2}\right), \quad \lim_{X\to\infty} G(X)=4$$

We can write the relation between $\rho_{s0}^{\lim}(\theta)$ and the distribution $\rho_{s3}(Y)$ in the stabilized flow zone (by analogy with (2.9))

$$\rho_{s3}(Y)=\rho_{s0}^{\lim}(\theta), \quad Y=\theta\left[\frac{\varepsilon(1+\alpha)}{\lambda}\right]^{1/4}\sqrt{\frac{0.33206}{4}}$$

In conclusion, we note that the Saffman expression for the lift force is applicable only at small Reynolds numbers and low shear flow velocities [2]. On the entrance length of the channel or pipe there are zones in which these conditions may not be satisfied; accordingly, it is only possible to draw a qualitative comparison between the particle concentration distributions obtained above and the experimental data.

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KINEMATICS OF FRAGMENTS FROM DISINTEGRATING PRESSURE VESSELS

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The disintegration of an elastoplastic vessel filled with high-pressure gas is experimentally investigated and a model of the process is proposed.

The most important characteristics of the fragments generated by the accidental explosion of industrial plant are their maximum velocity, shape and ejection angle. These are the input parameters for external ballistic calculations, the main purpose of which is to determine the range of the fragments and their momentum along the trajectory. The behavior of the fragments from a pressure vessel made of elastoplastic material is a problem of practical importance. Usually, as a result of the disintegration of such vessels only a small number of fragments is formed. Existing models [1—3] make it possible to estimate the fragment velocity, but exclude from consideration the separation characteristics of the fragment and its rotation about the center of mass, although they may have an important influence on the trajectory and penetrating power. These characteristics can be taken into account if the acceleration of the fragment is considered in relation to its separation dynamics.

1. Experimental Apparatus and Results of Measurements

The experimental apparatus is shown schematically in Fig. 1. A diaphragm assembly 1, similar to those used in shock tubes, is mounted in a stand 2 and used to clamp the edges of a flat disk-shaped diaphragm 108 mm in diameter made of annealed copper foil or some other elastoplastic material 0.05—0.5 mm thick. Under the pressure exerted by gas entering the diaphragm assembly from the mains 3, the diaphragm is deformed into a segmental shape 4 and at the moment of disintegration is almost hemispherical (nonsphericality coefficient 0.7). Thus, the device described makes it possible to model the disintegration of a pressure vessel with an elastoplastic shell using a minimum of

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