AERODYNAMICS AND STATIC STABILITY OF STAR-SHAPED CONICAL BODIES IN SUPERSONIC FLOW

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The aerodynamics of conical bodies with a star-shaped cross section have been experimentally investigated over a broad range of variation of the parameters determining their geometry at a free-stream Mach number M=6. The position of the center of pressure of star-shaped bodies with an optimum trailing edge shape is investigated in relation to similarity parameters previously obtained theoretically. A correspondence is established between the derivatives of the normal force with respect to the angle of attack for pyramidal star-shaped bodies and bodies with the optimum trailing edge.

1. AERODYNAMIC CHARACTERISTICS OF PYRAMIDAL BODIES AT M=6

Theoretical and experimental research on supersonic and hypersonic flow past conical bodies with a star-shaped cross section and their elements (for a bibliography see [1, 2]) has yielded extensive material on the structure of the flow around such bodies and on their aerodynamics. Much of this work has been devoted to the study of the aerodynamic drag of star-shaped bodies at low and intermediate supersonic speeds and an angle of attack equal to zero, which has been determined by the search for the principal parameter giving a reduction in the drag as compared with a circular cone of equivalent length L and maximum cross-sectional area S_M. This parameter [3] was found to be the ratio of the minimum radius of the transverse contour of the star-shaped body to the radius of the cross section of the equivalent cone measured at the base: $r'=r/R_0$ (Fig. 1, where the continuous lines represent one of the cycles of the star-shaped body and the broken lines the equivalent cone). The number of cycles n of the star-shaped body is not of fundamental importance for the minimization of the aerodynamic drag, if r' is given (in what follows the prime has been omitted).

At the same time, data on the aerodynamic characteristics of star-shaped bodies as a function of the angles of attack and roll are available only for $M \le 3$ [3] on a limited interval of variation of the aspect ratio of the equivalent cone $\lambda=L/(2R_0)$ and are lacking for high supersonic speeds.

The aerodynamic drag force F_τ, the normal force F_n and the moment were investigated experimentally for a class of models of star-shaped bodies with plane faces having the same maximum cross-sectional area S_M. For the purpose of comparison with the aerodynamic characteristics of equivalent bodies of revolution we also tested the equivalent circular cones with a base section diameter of 30 mm and lengths L=39, 60, and 75 mm, which correspond to aspect ratios λ=1.3, 2, and 2.5. The star-shaped bodies of each aspect ratio consisted of three series differing with respect to the value of the parameter r=0.4, 0.5, and 0.6. In each series we varied the number of cycles of the star-shaped body: n=3, 4, and 6.

The longitudinal force F_τ, the normal force F_n and the transverse moment acting on the model were measured at M=6 and a unit Reynolds number $Re=10^8$ m^{-1} using a strain-gauge balance mounted in a special automated apparatus which controlled various sequences of flow regimes on the range of angles of attack $\alpha \in (-10°, +10°)$ with a given interval.

During the experiments information was recorded from the following measuring channels: strain balance components, angle of attack of the model, pressure in the settling chamber and static pressure in the flow, stagnation temperature of the flow and the strain balance. A measuring and computing system based on a AT-286 PC and KAMAK equipment was used to calibrate the measuring channels, control the position of the model in the flow, and record and process the information. The system was programmed in a MS DOS operating environment using a specialized library for controlling the KAMAK equipment.

In order to obtain reliable values of the load components acting on the model we made multiple measurements and specified the necessary number of interrogations of the measuring channels for each position of the model in the flow. The information was recorded serially, a single interrogation cycle taking ~10 msec. The strain balance calibration interval was chosen and the amplifying equipment adjusted with reference to the maximum possible loads on each model. This made it possible to ensure that the accuracy of the results obtained was ≤ 2 without allowance for possible technological errors in constructing the experimental models. As a test we compared the experimental and theoretical [4] values of the position of the center of pressure for circular cones.

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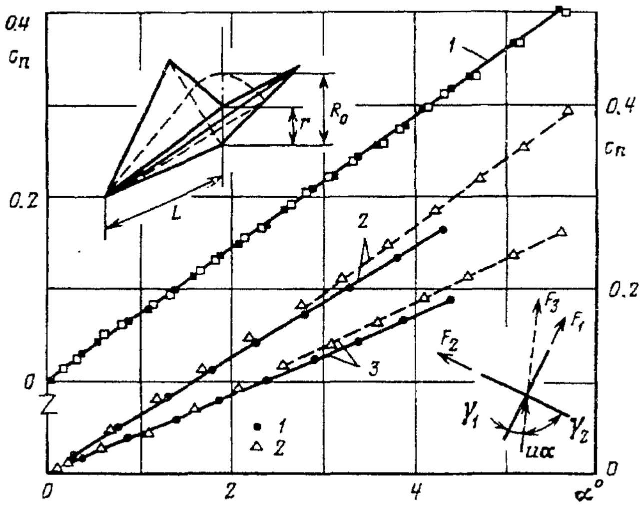

Fig 1

The results of the balance experiments on models of star-shaped bodies and the equivalent cones can be represented by the coefficients of the longitudinal force and the force normal to the body axis $c_\tau=F_\tau/(qS_M)$ and $c_n=F_n/(qS_M)$ and by the derivative of the latter with respect to the angle of attack c_n^α (q is the dynamic head). The quantity c_n^α was taken equal to the coefficient in the least-squares linear approximation of the experimental values $c_n(\alpha)$ over the entire interval of variation of the angle of attack for the axisymmetric models ($n=4$, 6) and on subintervals for the nonaxisymmetric models ($n=3$).

One important question is the effect of the angle of roll on the normal force acting on an axisymmetric body with a star-shaped cross section. We carried out experiments in which the angle of attack was varied in the plane of symmetry of the cycle (Fig. 1) and in the plane of symmetry of a lobe of the star-shaped body. As an example, in Fig. 1 (relation 1) we have plotted the data on $c_n(\alpha)$ for a star-shaped model with the parameters $\lambda=2.5$, $r=0.4$, and $n=4$. The different points represent the values of c_n corresponding to the planes of variation of the angle of attack indicated above. Clearly, the data for the normal force coefficient coincide in these model flow regimes, which differ from the gas dynamics standpoint, and can be approximated by a single linear dependence over a fairly wide interval of variation of the angle of attack.

This important qualitative result can be explained fairly simply using the principle of superposition of solutions of linear problems, if we make the reasonable assumption that friction does not affect the magnitude and direction of the normal force at small angles of attack. As an illustration, in Fig. 1 we have shown the flow diagram for a star-shaped body with $n=4$ cycles in the transverse plane.

Let a transverse velocity perturbation equal to $u\alpha$, where u is the modulus of the free-stream velocity, form angles γ_1 and γ_2 with the planes of symmetry of the lobes of the star-shaped body in the corresponding cycle ($\gamma_1+\gamma_2=2\pi/n$). Having decomposed the velocity perturbation, in accordance with the superposition principle, into two components parallel to the above-mentioned planes of symmetry: $u\alpha\sin\gamma_2/\sin(2\pi/n)$ and $u\alpha\sin\gamma_1/\sin(2\pi/n)$, we find that they cause perturbations of the normal force $F_1=A\alpha\sin\gamma_2/\sin(2\pi/n)$ and $F_2=A\alpha\sin\gamma_1/\sin(2\pi/n)$, respectively, where A is a coefficient determined as a result of solving the linear problem of flow at a small angle of attack past a star-shaped body whose plane of variation coincides with the plane of symmetry of the lobe, when the basic undisturbed solution ($\alpha=0$) is known. In accordance with the parallelogram rule, the resultant F_3 of the forces F_1 and F_2 will have the same direction as the transverse velocity perturbation $u\alpha$ (Fig. 1) and will be equal to $A\alpha$.

Thus, if the region of applicability of the linear theory is broad enough, the normal force will lie in the plane of the angle of attack and will have equal values not only for equal values of the angle of attack as it varies in the different planes of symmetry of the star-shaped body, as observed in the experiments on axisymmetric bodies ($n=4$, 6), but also for an arbitrary position of the plane of variation of the angle of attack. In the region of validity of the linear theory the derivative of the normal force with respect to the angle of attack c_n^α does not depend on the angle of roll of the star-shaped body.

It is easy to establish that this reasoning will hold for any cyclically symmetric body ($n\geq 3$), i.e., for nonaxisymmetric star-shaped bodies also. This result was first obtained in [5] within the framework of the linearized local-interaction model. However, as our experimental investigation of nonaxisymmetric star-shaped bodies ($n=3$) showed, the interval of variation of the angle of attack containing the point $\alpha=0$, on which $c_n(\alpha)$ can be approximated by a linear function, is much smaller than for axisymmetric bodies. Therefore, if the aerodynamic characteristics are measured using a fairly large step with respect to the angle of attack [3] the fairly small neighborhood of the point $\alpha=0$ in which the function $c_n(\alpha)$ is described by the linear theory may remain unnoticed.

In Fig. 1 the points 1 and 2 represent the experimental values of $c_n(\alpha)$ for two models of star-shaped bodies with $n=3$

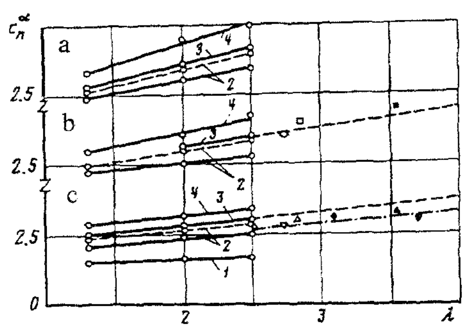

Fig 2

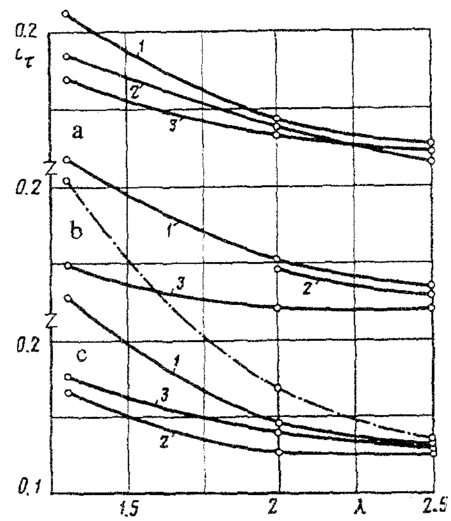

Fig 3

cycles and $r=0.4$. The points 1 and 2 correspond to positions of the star-shaped body in the flow such that its cycle, shown in Fig. 1, is on the leeward and windward sides, respectively. The relations 2 and 3 correspond to the values $\lambda=2.5$ and 1.3. In both cases the experimental points 1 and 2 can be approximated by a single straight line only when $\alpha \lesssim 2$—$2.5°$. Over the entire range of angles of attack investigated the points 1 can be very accurately approximated by a linear relation (continuous straight lines 2 and 3), whereas points 2 can be so approximated only for $\alpha \gtrsim 3°$ (broken straight lines 2 and 3). In what follows the slopes of these continuous and broken straight lines will be taken as the derivative of the normal force for nonaxisymmetric star-shaped bodies when two essentially different flow regimes are realized, taking into account the smallness of the "transition interval" in the neighborhood of $\alpha=0$.

The experimental data on the longitudinal force coefficient c_τ indicate that, as distinct from the above-mentioned effect on $c_n(\alpha)$, the asymmetry of the star-shaped bodies ($n=3$) has almost no effect on the range of angles of attack investigated.

In Fig. 2 for $\lambda \in [1.3; 2.5]$ we have reproduced the results for the derivative of the normal force with respect to the angle of attack c_n^α for star-shaped bodies (relations 2—4 correspond to $n=3$, 4, and 6 cycles) with parameters $r=0.4$, 0.5, and 0.6 (a, b, and c, respectively) and the equivalent circular cones (relation 1, Fig. 2c).

The value of c_n^α for star-shaped bodies considerably exceeds that for the equivalent cones, which testifies to the excellent lift properties of the three-dimensional configurations in question. The increase in the derivative of the normal force, observed as the aspect ratio λ increases and the parameter r decreases, is due to the corresponding increase in the plan area of the star-shaped body $S_p \sim 2\lambda S_M/(rn \sin(\pi/n))$. The increase in c_n^α with increase in the number of cycles of the star-shaped body is attributable to the significant rise in pressure in the cycle on the windward side and the very weak dependence of the plan area S_p on n, which, the other parameters being equal, varies only by 15% on transition from $n=3$ to $n=6$ (see, for example, [2, 6]). These properties of the flow are especially clearly expressed for models of star-shaped bodies with an odd number of cycles ($n=3$, relations 2 in Fig. 2), when the orientation of the cycle with respect to the free stream is varied (the continuous curves represent the cycle on the leeward side, the broken curves the cycle on the windward side) for a constant plan area.

In Fig. 3 we present the experimental data for the drag coefficient c_τ of star-shaped bodies at an angle of attack $\alpha=0$ for $n=3$, 4, and 6 cycles (a, b, and c, respectively) and $r=0.6$, 0.5, and 0.4 (continuous curves 1, 2, and 3) and those for the equivalent cones (chain curve in Fig. 3c) as a function of the aspect ratio λ.

When $M=6$ all the star-shaped bodies on the range of variation of the design parameters λ, r, and n considered have a lower aerodynamic drag than the equivalent cone. Whereas for star-shaped bodies with $n=3$ and 4 cycles (Fig. 3, a, b) the minimum values of c_τ are not reached over almost the entire interval of aspect ratios λ as r varies from 0.6 to 0.4 (an exception is the interval of variation of λ in the neighborhood of 2.5 when $n=3$), for $n=6$ (Fig. 3c) the minimum value of the aerodynamic drag is reached when $r \approx 0.5$.

The results of the balance experiments on star-shaped bodies for $M=6$ and $\alpha=0$ also show that in order to minimize the aerodynamic drag for fixed parameters λ and r it is no longer possible to designate with certainty the necessary number of cycles of the star-shaped body, as was the case for smaller Mach numbers, when it is was sufficient to take $n=3$ or 4 [1, 3]. Thus, when $r=0.6$ and 0.4 bodies with $n=3$ and 4 cycles, respectively, have the minimum drag, but when $r=0.5$ the minimum drag corresponds to bodies with $n=6$ cycles. This indeterminacy was noted in [7] at hypersonic flow velocities.

When $r \in [0.4; 0.6]$ and $\lambda=1.3$ the aerodynamic drag of the optimum star-shaped body is approximately half the drag of

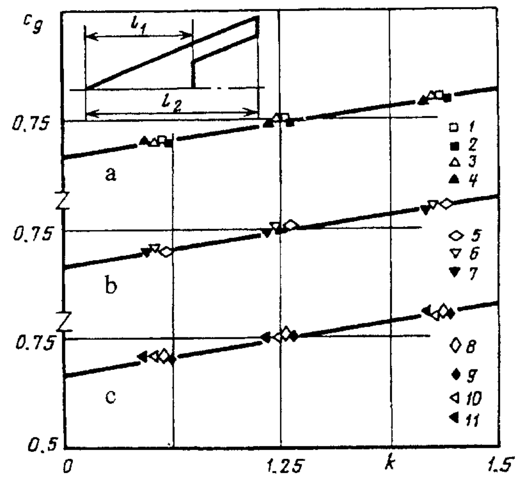

Fig 4

the equivalent cone and when $\lambda=2$ it is less than the drag of the equivalent cone by a factor of 1.3. These data, together with the results of other studies (see the bibliography in [1]), show that the minimum value of the ratio of the aerodynamic drag of star-shaped bodies to the c_τ of the equivalent cone is reached at Mach numbers $M>6$.

2. AERODYNAMIC CHARACTERISTICS OF STAR-SHAPED BODIES WITH OPTIMUM TRAILING EDGE SHAPE

One of the important problems in developing promising three-dimensional shapes for aircraft traveling at high supersonic velocities and possessing optimum or near-optimum aerodynamic characteristics is the problem of stabilizing them in flight. In [8] in the class of conical star-shaped bodies with plane faces the possibility of ensuring the maximum reserve of static stability by optimizing the shape of the trailing edge was studied under various isoperimetric conditions: either the area of the maximum cross section or the volume of the body was given. The diagram in Fig. 4 shows the projection of a lobe of the star-shaped body with optimum trailing edge shape on the plane of symmetry. At distances l_1 and l_2 from the nose there are plane base surfaces normal to the axis of the star-shaped body.

In [8] a variational problem for slender bodies was formulated and solved on the assumption that the pressure varies only weakly over the span of each of the panels of the three-dimensional body and that the "stabilizers" are not affected by the part of the trailing edge lying closer to the nose (Fig. 4). A similarity rule establishing the universality of the trailing edge shape and the value of the static stability reserve for constant values of the parameters $k=l_2/l_1$ (relative length of "stabilizers") and m was discovered. Here

$$m = \frac{(r/R)\cos(\pi/n)}{1-(r/R)\cos(\pi/n)} \equiv \frac{r^2(n/2\pi)\sin(2\pi/n)}{1-r^2(n/2\pi)\sin(2\pi/n)}$$

and r and R are the minimum and maximum radii of the transverse contour of the star-shaped body in some cross section.

In order to test the results of the theory [8] it is sufficient to measure the position of the center of pressure l_g of star-shaped bodies with the optimum trailing edge shape, determined, for example, for the case of a given maximum cross-sectional area, for various sets of governing parameters and to compare it with the theoretical values determined from the expression

$$c_g \equiv \frac{l_g}{l_1} = 1 - \tfrac{1}{3}z^2 + \tfrac{1}{2}(k-1)(1-z^2), \quad z^4 = [1 + \tfrac{4}{3}(1+m)(k-1)]^{-1}$$

In accordance with the above, we designed and built experimental models of star-shaped bodies with the optimum trailing edge shape, the geometry of each of which was determined by the values of the dimensionless parameters in the rows of Table 1 and by the value of the parameter $k=1.11$, 1.25, and 1.43.

It is easy to see that the models were so designed that it was possible to establish, on the one hand, the validity of the similarity rule (models with $n=3$ and 4 transverse cycles, constant m and the same λ) and, on the other hand, the validity of the theory for different values of the relative thickness of the body λ^{-1}.

The experimental investigation of the aerodynamics of the models described above and the processing of the results were carried out in accordance with the methods discussed in Sec. 1.

The dependence of the aerodynamic characteristics of models of star-shaped bodies with the optimum trailing edge on the

TABLE 1

m	n	r'	λ	Point
0,165	4	0,47	2,83	1
»	4	0,47	3,53	2
»	3	0,58	2,83	3
»	3	0,58	3,53	4
0,21	4	0,53	2,75	5
»	3	0,65	2,75	6
»	3	0,65	3,7	7
0,33	4	0,62	2,5	8
»	4	0,62	3,1	9
»	3	0,78	2,5	10
»	3	0,78	3,1	11

angle of attack is qualitatively the same as for pyramidal star-shaped bodies. In particular, over the entire range of angles of attack investigated ($-10° \leq \alpha \leq 10°$) on which the requirement with respect to the accuracy of the measurements of the quantities generating the characteristic in question (normal force, transverse moment) is satisfied, the experimental values of the position of the center of pressure l_g fit into a strip around a certain straight line l_g=const with a half-width that does not exceed the measuring error given in Sec. 1. This indicates that on the range of angles of attack investigated $l_g(\alpha)$ is either a constant or a very weakly varying function. Therefore, as l_g we took the mean of the values of $l_g(\alpha)$ obtained in the experiments.

This method of determining the position of the center of pressure l_g is reasonable in the case of axisymmetric star-shaped bodies (n=4 and 6). Considering the substantial difference in the values of $c_n(\alpha)$ for nonaxisymmetric bodies (n=3, relations 2 and 3 in Fig. 1) at angles of attack of different sign, when the plane of variation of α coincides with any of the planes of symmetry of the star-shaped body, the procedure for determining the position of the center of pressure requires special study. An investigation of the value of l_g for different models with n=3 cycles at varying angles of attack in planes coinciding with the planes of symmetry of the bodies and perpendicular to the latter showed that, within the limits of experimental error, l_g does not depend on the roll angle of the model.

In Figs. 4a, 4b, and 4c for m=0.165, 0.21, and 0.33, respectively, we have plotted the theoretical (continuous curves) and experimental values of the relative position of the center of pressure of star-shaped bodies with the optimum trailing edge against the aspect ratio of the "stabilizers" k (the experimental points are displaced relative to the corresponding coordinate lines k=const). The correspondence between the experimental points and the models is established in the right-hand column of Table 1. The difference between the experimental values of c_g and the theoretical curves is no greater than the error of the measurements.

Thus, the experimental investigation not only confirms the validity of the similarity rule established in [8] but also indicates that the theoretical relation can be used to determine the position of the center of pressure with a high degree of accuracy.

Apart from the position of the center of pressure, for star-shaped bodies with the optimum trailing edge an important characteristic for calculating the dynamic properties of these bodies in flight is the derivative of the normal force with respect to the angle of attack c_n^α. In Figs. 5a, 5b, and 5c for m=0.165, 0.21, and 0.33, respectively, using the same notation as in Fig. 4, we have plotted the experimental data for c_n^α as a function of the parameter k.

The bulk of the experimental data for models with an odd number of cycles (n=3, points 3, 4, 6, 7, 10, and 11) relate to flow regimes in which the plane of variation of the angle of attack is perpendicular to any of the planes of symmetry of the star-shaped body, i.e., $c_n(\alpha)$ is an odd function of the angle of attack. Points 1 in Fig. 5 (corresponding to points 4 and 7) for k=1.43 and points 2 (corresponding to points 10) for k=1.25 correspond to the flow regimes in which the plane of variation of the angle of attack coincides with one of the planes of symmetry of the body, while the cycle lies either on the leeward side (lower points of each pair in Figs. 5a, 5b, and 5c) or on the windward side (upper points). The mean value of c_n^α for these characteristic flow regimes coincides with the c_n^α for an odd function $c_n(\alpha)$ (points 4 and 7, k=1.43, and point 10, k=1.25).

An important experimentally determined fact (Fig. 5) is the virtual constancy of the derivative of the normal force in each series of models with the same values of the parameters m, r, and λ, but different k. Considering the substantial redistribution of the lifting surface on the optimum bodies in the neighborhood of the base from the inner ribs to the leading edges [8], this observation points, firstly, to a very small pressure variation in the transverse direction on each face of the star-shaped body and, secondly, to the weak (or nonexistent) effect of the part of the base section located closer to the nose of the body (see Fig. 4) on the flow over the "stabilizers."

The calculations showed that when M=6 every kind of flow regime may be realized around the experimental models: with the shock wave detached from the leading edges (n=3$-m$=0.21, λ=3.7; m=0.33; n=4$-m$=0.33; λ=3.1), with a smooth bow shock attached to the leading edges (n=3$-m$=0.165, m=0.21; λ=2.75; n=4$-m$=0.33, λ=2.5), and with a Mach shock

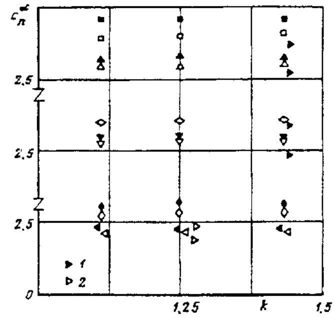

Fig 5

wave configuration in the cycles ($n=4-m=0.21$; $m=0.165$). For large aspect ratios of the star-shaped body and a small number of cycles ($n=3$, 4) the inner shocks in the Mach configuration are weak, and in the case of the flow regime with a single smooth shock wave in the cycle the pressure within the Mach cone differs little from the pressure behind the plane shock attached to the leading edge. These data confirm the validity of the simplifications made in [8] in formulating the variational problem.

The fact that the derivative of the normal force coefficient does not depend on the parameter k makes it possible to conclude that the value of c_n^α for star-shaped bodies optimal with respect to the static stability reserve coincides with that for star-shaped bodies with a plane base section (pyramidal bodies, $k=1$) having the same values of the design parameters λ, r, and n. In this connection it is of interest to compare the mean values of c_n^α with respect to k for star-shaped bodies with the optimum trailing edge shape with the c_n^α for star-shaped bodies with a plane base section (Sec. 1).

In Sec. 1 we investigated the dependence of c_n^α on the aspect ratio λ ($\lambda \leq 2.5$) for pyramidal star-shaped bodies when $r \in [0.4; 0.6]$. Among the models of star-shaped bodies with an optimum trailing edge (Table 1) the series 9 and 11 have a value of the parameter r lying on or close to the interval (0.4; 0.6) and an aspect ratio $\lambda \geq 2.5$. Therefore the experimental data on the values of c_n^α for pyramidal star-shaped bodies (Sec. 1) could be linearly extrapolated to $\lambda > 2.5$.

In Fig. 2 the broken straight lines ($\lambda > 2.5$) represent the extrapolated c_n^α relations for pyramidal bodies with the parameters $r=0.5$ (b), $n=4$ and $r=0.6$ (c), $n=4$, respectively, while the chain straight line in Fig. 2c represents the extrapolation of the mean values of the derivative of the normal force coefficient for models of star-shaped bodies with three cycles (relations 2). The deviation of the experimental points from the reference straight lines corresponds qualitatively to our notion of the variation of c_n^α as a function of the parameter r. For constant λ a decrease (increase) in the parameter r corresponds to an increase (decrease) in c_n^α, which is associated with the increase (decrease) in the lifting surface of the star-shaped body. The results are also in good quantitative agreement (the interpolation straight lines corresponding to models with an optimum trailing edge for $r \in (0,4; 0.6)$ are not shown).

Our experimental investigation of the aerodynamics of star-shaped bodies with an optimum trailing edge shape enables us to conclude that the recommendations of the theory [8] can be used at least for aspect ratios $\lambda \geq 2.5$.

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