EXCITATION OF UNSTABLE MODES IN A SUPERSONIC BOUNDARY LAYER BY ACOUSTIC WAVES

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The acoustic excitation of the first and second boundary layer modes in the neighborhood of the sharp leading edge of a plate in a supersonic gas flow is analyzed.

The results of experimental and theoretical investigations of the sensitivity of subsonic boundary layers to acoustic disturbances were described in [1—3]. The main conclusions of these studies were as follows. The sound waves excite instability waves at spatial inhomogeneities of the main flow. In subsonic flow the wavelength of the sound is much greater than the characteristic scale of the unstable mode; accordingly, intense excitation takes place at inhomogeneities whose scale is commensurable with the length of the instability wave. If a plate is free of irregularities and other sources of local nonuniformity, then the excitation is concentrated in the neighborhood of the leading edge on a scale of several instability wavelengths [4]. However, the oscillations that develop near the leading edge enter a zone of strong attenuation and arrive at the instability point with a small amplitude. Accordingly, the mechanism in question may have to compete with distributed generation due to the nonparallelism of the flow in the boundary layer [1]. This is much less efficient, but the buildup of the unstable waves takes place directly in the neighborhood of the lower branch of the neutral curve.

The transition to supersonic flow leads to a qualitative change. The oscillation damping rates decrease sharply in the region lying upstream from the neutral curve, and the screening of the leading edge weakens [5]. As the leading edge is approached, the boundary layer oscillations become synchronized with the sound waves propagating parallel to the surface of the plate [6]. Consequently, these waves may strongly excite the unstable modes. It is reasonable to assume that the neighborhood of the leading edge plays a dominant role in the excitation of instability of the supersonic boundary layer. This assumption is confirmed by experiment [7] and also finds support in the strong influence of leading edge bluntness on the transition point [8].

1. On the steady flow over a semi-infinite plate we superimpose a wave incident at zero angle to the wall (see Fig. 1)

$$P=(\gamma M^2)^{-1}+h\,\mathrm{Re}\{\exp[i\alpha_a(X-c_a{}^*\tau)]\}$$

Here, P is the pressure divided by $\rho_\infty U_\infty$, $M > 1$ is the free-stream Mach number, γ is the specific heat ratio, α_a is the wave number, $c_a^* = U_\infty(1 \pm 1/M)$ is the phase velocity, X is the longitudinal coordinate, and τ is time. It is assumed that the frequency parameter $F = \nu_\infty\omega^*/U_\infty^2 \ll 1$ ($\omega^* = \alpha_a c_a^*$ is the frequency) and the amplitude h is small enough for the linear theory to be applicable.

At a distance from the leading edge of the order of the wavelength, $L_1 \sim \alpha_a^{-1}$, Prandtl's hierarchical procedure can be employed: in the leading approximation the boundary layer does not distort the external acoustic wave, the boundary layer solution can be calculated for the given parameters of the latter, the corrections to the inviscid disturbance are then found (they are of the order of $\sqrt{F}$), and so on.

In a region with the characteristic scale $L_2 \gg \alpha_a^{-1}$ the flow can be represented in the form of a combination of a steady boundary layer varying slowly with respect to X and a rapidly oscillating perturbation. The nonparallelism of the main flow results in wave diffraction which distorts the acoustic field in a wall zone of thickness D. This scale can easily be estimated using the equation for sound waves in a uniform flow

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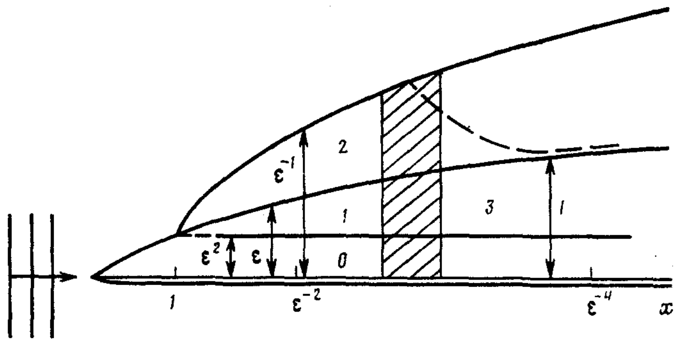

Fig. 1

$$p_{YY}=(M^2-1)p_{XX}+2M^2p_{X\tau}+M^2p_{\tau\tau}$$

Here, p are the pressure fluctuations. The distortion of the wave number of the monochromatic disturbance $\sim L_2^{-1}$, i.e., $p_{YY} = O(\alpha_a/L_2)p$, $D = \sqrt{L_2/\alpha_a}$. The changes in wave amplitude are determined by the behavior of the solution in the boundary layer, whose displacement thickness $\delta^* = \Delta\sqrt{\nu_\infty L_2/U_\infty}$. From the equation for the longitudinal and transverse momenta $p_Y \sim v_X$, $p_X \sim u_X$, and from the continuity equation $v_Y \sim u_X$; consequently, $p_Y = O(\delta^*\alpha_a^2)p$. The hierarchical scheme breaks down when the amplitude changes by its own order, i.e., $p_Y FD = O(p)$ or $L_2 = (\Delta\alpha_a\sqrt{F})^{-1}$. Thus, in the region $X = O(\Delta^{-1}\alpha_a^{-1}F^{-1/2})$, $Y = O(\Delta^{-1/2}\alpha_a^{-1}F^{-1/4})$ the nonparallelism of the boundary layer strongly influences the evolution of the disturbance.

When $X \sim L_2$ the viscous sublayer (Stokes layer) has a thickness $\delta_S = \sqrt{\nu_\infty/\omega^*}$ and lies at the bottom of the boundary layer, $\delta_S/\delta^* \sim F^{+1/4}\Delta^{-1/2}$; the bopundary layer lies at the bottom of the diffraction zone $\delta^*/D \sim F^{1/2}\Delta$. Moreover, $\alpha_a^{-1}/\delta^* \sim F^{-1/4}\Delta^{-1/2}$, $\alpha_a^{-1}/D \sim F^{1/4}\Delta^{1/2}$, i.e., the disturbance is shortwave in the metric of the diffraction zone and longwave in the metric of the boundary layer. These properties considerably facilitate the construction of the solution.

When the displacement thickness is of the order of the wavelength, the locally parallel approximation, which describes the acoustic field and the oscillations in the boundary layer, holds true. The latter include the first and second unstable modes. In this case the characteristic longitudinal scale $L_3 = \Delta^{-2}\alpha_a^{-1}F^{-1}$. Below it is shown that in the boundary layer as $X/L_2 \to \infty$ the asymptotic form of the disturbance can be divided into two parts, one of which can be matched with the asymptotic form of the first and second modes as $X/L_3 \to 0$. The dimensionless displacement thickness Δ has been included in the estimates in order to take into account the strong dependence of the boudnary layer thickness on the Mach number and the temperature factor.

We nondimensionalize the coordinates and time with respect to the length and period of the incident acoustic wave: $x = \alpha_a X$, $y = \alpha_a Y$, $t = \alpha_a U_\infty \tau$. We introduce the small parameter $\varepsilon = F^{1/4}\Delta^{1/2}$. When the above is taken into account, we have the following characteristic scales:

$$x=(x_1,\ \varepsilon^{-2}x_2,\ \varepsilon^{-4}x_3),\quad y=(\varepsilon^2 y_0,\ \varepsilon y_1,\ \varepsilon^{-1}y_2,\ y_3)$$

The characteristic regions are shown schematically in Fig. 1: 0 denotes the Stokes layer, 1 and 3 the boundary layer, and 2 the acoustic wave diffraction zone.

2. In region 1 with the variables $x_2 = O(1)$, $y_1 = O(1)$ we represent the solution in the form:

$$\begin{aligned}
U_s&=U(x_2,y_1)+O(\varepsilon^3)+h\,\mathrm{Re}\{u_1(x_2,y_1)\exp[i(x_1-c_at)]+O(\varepsilon)\}\\
V_s&=\varepsilon^3V(x_2,y_1)+O(\varepsilon^6)+h\,\mathrm{Re}\{\varepsilon v_1(x_2,y_1)\exp[i(x_1-c_at)]+O(\varepsilon^2)\}\\
P&=(\gamma M^2)^{-1}+O(\varepsilon^3)+h\,\mathrm{Re}\{p_1(x_2,y_1)\exp[i(x_1-c_at)]+O(\varepsilon)\}\\
T_s&=T(x_2,y_1)+O(\varepsilon^3)+h\,\mathrm{Re}\{\theta_1(x_2,y_1)\exp[i(x_1-c_at)]+O(\varepsilon)\}
\end{aligned}\tag{2.1}$$

Here and in what follows, $c_a = 1 \pm 1/M$ is the dimensionless phase velocity of the acoustic wave, U_S, V_S, and T_S are the x and y velocity components and the temperature divided by U_∞ and T_∞, respectively.

In the highest approximation with respect to ε we obtain the system

$$\left(\frac{v_1}{U-c_a}\right)'=i\left[\frac{T}{(U-c_a)^2}-\mathrm{M}^2\right]p_1,\qquad p_1'=0 \tag{2.2}$$

$$v_1(x_2,\,0)=0 \tag{2.3}$$

The prime denotes differentiation with respect to y_1, and the no-flow condition (2.3) is a consequence of the matching of the solution in the viscous sublayer 0 and the solution in the inviscid region 1. It follows from (2.2) that the pressure p_1 is constant across the boundary layer, and the vertical velocity component

$$v_1=ip_1(x_2)(U-c_a)\int\limits_0^{y_1}\left[\frac{T}{(U-c_a)^2}-\mathrm{M}^2\right]dy_1$$

For $c_a = 1 - 1/M$ the amplitude $v_1(y_1)$ has a singularity at the critical point y_{1c}: $U(y_{1c}) = c_a$. The singular point y_{1c} must be bypassed from below in the complex y_1 plane [9]. Considering that the undisturbed flow is self-similar, i.e., depends on the variable $\eta = y_1/\sqrt{x_2}$, we obtain the condition at the outer edge of the boundary layer

$$v_1=i(1-c_a)p(x_2)\sqrt{x_2}k,\quad y_1\to\infty \tag{2.4}$$

$$k=\int\limits_0^{\infty}\left\{\frac{T}{(U-c_a)^2}-\mathrm{M}^2\right\}d\eta \tag{2.5}$$

In the region 2 with variables $x_2 = O(1)$, $y_2 = O(1)$ we represent the solution in the form:

$$U_S=1+O(\varepsilon^3)+h\,\mathrm{Re}\{u_2(x_2,\,y_2)\exp[i(x_1-c_at)]+O(\varepsilon)\}$$

$$V_S=\varepsilon^3V(x_2,\,y_2)+O(\varepsilon^6)+h\,\mathrm{Re}\{\varepsilon v_2(x_2,\,y_2)\exp[i(x_1-c_at)]+O(\varepsilon)\}$$

$$P_S=(\gamma\mathrm{M}^2)^{-1}+O(\varepsilon^3)+h\,\mathrm{Re}\{p_2(x_2,\,y_2)\exp[i(x_1-c_at)]+O(\varepsilon)\}$$

$$T_S=1+O(\varepsilon^3)+h\,\mathrm{Re}\{\theta_2(x_2,\,y_2)\exp[i(x_1-c_at)]+O(\varepsilon)\}$$

In the highest approximation with respect to ε we obtain the problem for p_2

$$\frac{\partial^2p_2}{\partial y_2^2}=2i[\mathrm{M}^2(1-c_a)-1]\frac{\partial p_2}{\partial x_2} \tag{2.6}$$

$$p_2(x_2,\infty)=1,\qquad \frac{\partial p_2}{\partial y_2}(x_2,0)=-iv_2(x_2,0)(1-c_a) \tag{2.7}$$

$$p_2(0,\,y_2)=1 \tag{2.8}$$

The initial condition (2.8) is a consequence of the fact that when $x = O(1)$ Prandtl's hierarchical scheme, in accordance with which in the inviscid zone the acoustic wave has an amplitude $p = 1 + O(\varepsilon^2)$, is realized. From the continuity conditions $v_2(y_2 \to 0) = v_1(y_1 \to \infty)$, $p_2(y_2 \to 0) = p_1$ and relations (2.4) and (2.7) we find the boundary condition

$$\frac{\partial p_2}{\partial y_2}=(1-c_a)^2k\sqrt{x_2}\,p_2,\qquad y_2=0 \tag{2.9}$$

By direct substitution we find that the solution has the form:

$$p_2=-\int\limits_0^{x_2}\frac{\partial p_2}{\partial y_2}(\xi,0)\frac{\exp\{-2iy_2^2[\mathrm{M}^2(1-c_a)-1]/4(x_2-\xi)\}}{\sqrt{2\pi i(x_2-\xi)[\mathrm{M}^2(1-c_a)-1]}}d\xi+1$$

Here and in what follows $\arg(\sqrt{\pm i}) = \pm\pi/4$. Considering (2.9) and taking into account the fact that $p_2(x_2,\,0) = p_1$, we obtain the integral equation for the pressure in the boundary layer

$$p_1(x_2)=\lambda\int\limits_0^{x_2}\sqrt{\frac{\xi}{x_2-\xi}}\,p_1(\xi)d\xi+1 \tag{2.10}$$

$$\lambda=-(1-c_a)^2k\{2\pi i[M^2(1-c_a)-1]\}^{-1/3} \tag{2.11}$$

The solution of Eq. (2.10) is unique and can be represented in the series form:

$$p_1=\sum_{n=0}^{\infty} a_n(\lambda x_2)^n\pi^{n/2}, \quad a_0=1, \quad a_n=\prod_{j=1}^{n}\frac{\Gamma(j+1/2)}{\Gamma(j+1)} \tag{2.12}$$

It is easy to obtain the asymptotic form of the solution as $x_2 \to \infty$ [10]:

$$p_1=(8\pi)^{1/4}A(\pi\lambda^2x_2^2)^{1/4}\exp(\lambda^2x_2^2\pi/2)[1+O(x_2^{-2})]-(\pi\lambda x_2)^{-1}[1+O(x_2^{-1})],$$
$$A=\lim_{n\to\infty} n^{-1/4}a_n\sqrt{\Gamma(n+3/2)}=0.935\ldots, \quad \mathrm{Re}\,\lambda>0. \tag{2.13}$$

The second term corresponds to the acoustic field in the wall zone. As will be shown below, the first and second terms are the "seed" for the first boundary layer mode when $c_a = 1 - 1/M$ and $c_a = 1 + 1/M$, respectively.

3. In order to construct the solution in the region 3 with variables $x_3 = O(1)$, $y_3 = O(1)$ we employ the formalism developed in [1, 11]. Since the quantity α_a^{-1} has been taken as the characteristic length scale, the Reynolds number is determined from the expression $R = \alpha_a^{-1}U_\infty/\nu_\infty = c_a\Delta^2\varepsilon^{-4}$. We represent the characteristics of the main flow and the perturbation vector $\mathbf{z}$ in the form:

$$U=U(x_3, y_3), \quad V=\varepsilon^4V_0(x_3, y_3), \quad P=P(x_3), \quad T=T(x_3, y_3)$$
$$\mathbf{z}=[\mathbf{z}_0(x_3, y_3)+\varepsilon^4\mathbf{z}_1(x_3, y_3)+\ldots]\exp[i(S-\omega t)] \tag{3.1}$$
$$\mathbf{z}=\left(u, \frac{\partial u}{\partial y_3}, v, p, \theta, \frac{\partial\theta}{\partial y_3}\right)^T, \quad S=\varepsilon^{-4}\int_0^{x_3}\alpha\, dx_3$$

The perturbation amplitude z_0 and the eigenvalue α are determined from the boundary-value problem

$$\frac{\partial \mathbf{z}_0}{\partial y_3}=A_0\mathbf{z}_0, \quad z_{01}=z_{03}=z_{05}=0, \quad y_3=0; \quad |\mathbf{z}_0|\to 0, \quad y_3\to\infty \tag{3.2}$$

The nonzero elements of the 6 × 6 matrix A_0 are given in [11]; they correspond to the Lees–Lin system for two-dimensional disturbances. The eigenvalue problem is usually solved numerically, the eigensolution having the form:

$$\mathbf{z}_0=a(x_3)\zeta(x_3, y_3) \tag{3.3}$$

The amplitude function $a(x_3)$ is determined from the conditions of solvability of the following approximation:

$$\langle B\zeta, \xi\rangle\frac{da}{dx_3}+a\left[\left\langle B\frac{\partial\zeta}{\partial x_3}, \xi\right\rangle+\langle G\zeta, \xi\rangle\right]=0, \quad \langle \mathbf{f}, \mathbf{g}\rangle=\int_0^\infty\sum_{n=1}^{6} f_ng_n^*\,dy_3 \tag{3.4}$$

Here, ζ and ξ are eigenfunctions of the direct and adjoint problems; $B = -i\partial A_0/\partial\alpha$; the matrix G contains elements proportional to $\partial U/\partial x_3$, $\partial T/\partial x_3$, V_0, $\partial V_0/\partial y_3$; their explicit form is given in the Appendix to [11]. We normalize the eigenfunction with respect to the condition $\zeta_4(x_3, 0) = 1$; then the amplitude of the pressure perturbation at the wall is equal to $a(x_3)$. The function a is determined correct to a constant multiplier. In order to obtain a unique solution it is necessary to supplement the equation (3.4) with initial conditions, i.e., to find the relation between the amplitude of the external wave and the amplitude of the oscllations of the boundary layer. For this purpose we will investigate the behavior of this eigensolution as $x_3 \to 0$.

We first consider the asymptotic behavior of the eigenvalue α and the eigenfunction ζ. In the highest approximation with respect to ε the system (3.2) reduces to a boundary-value problem for inviscid disturbances which in the self-similar variables η has the form:

$$p''-\left(\frac{2U'}{U-c}-\frac{T'}{T}\right)p'+x_3\alpha^2\left[\frac{M^2}{T}(U-c)^2-1\right]p=0, \quad p'(x_3, 0)=0; \quad p(x_3, \eta)\to 0, \quad \eta\to\infty \tag{3.5}$$

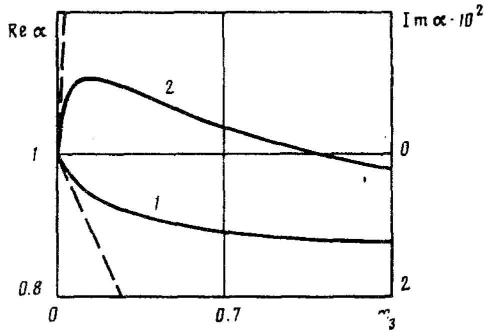

Fig. 2

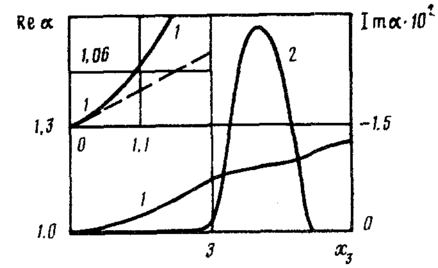

Fig. 3

Here, p is the amplitude of the pressure perturbation normalized with respect to the condition $p(x_3, 0) = 1$, and a prime denotes differentiation with respect to η. We represent the eigenvalue in the asymptotic form:

$$\alpha=1+bx_3+\ldots,\quad x_3\to 0 \tag{3.6}$$

The solution of the problem (3.5) has a two-layered structure $\eta = (\eta_1, x_3^{-1}\eta_2)$

$$\eta_1=O(1),\quad p=1+p_{11}x_3+\ldots,\quad p_{11}=\int_0^{\eta_1}\frac{(U-c_a)}{T}dz\int_0^z\left[\frac{T}{(U-c_a)^2}-\mathrm{M}^2\right]ds$$
$$\eta_2=O(1),\quad p=p_{20}+x_3p_{21}+\ldots \tag{3.7}$$

$$\frac{d^2p_{20}}{d\eta_2^2}+2[\mathrm{M}^2(1-c_a)-1]bp_{20}=0,\quad \frac{dp_{20}}{d\eta_2}(0)=\frac{dp_{11}}{d\eta_1}(\infty),\quad p_{20}\to 0,\quad \eta_2\to\infty \tag{3.8}$$

Solving problem (3.8), we find the eigenvalues for Re $k < 0$

$$\alpha_{1,2}=1-\frac{(1-c_a)^4k^2}{2[\mathrm{M}^2(1-c_a)-1]}x_3+\ldots,\quad c_a=1\mp\frac{1}{\mathrm{M}} \tag{3.9}$$

In Fig. 2 we have plotted the functions $\mathrm{Re}[\alpha_1(x_3)]$ and $\mathrm{Im}[\alpha_1(x_3)]$ (curves 1 and 2, respectively) obtained as a result of the numerical integration of Eq. (3.5) for a boundary layer with parameters: M = 4, surface temperature factor $T_f = 1$, $\gamma = 1.4$, Pr = 0.72, and viscosity coefficient calculated from the Sutherland formula at a stagnation temperature of 310°K. The broken lines represent the comparison with the asymptotic form (3.9). Similar data for the second mode are given in Fig. 3 and compared with the asymptotic form to an enlarged scale in the upper left-hand corner of the figure. There is good agreement between the exact and asymptotic solutions. Thus, as $x_3 \to 0$ the first and second modes are synchronized with the acoustic waves propagated parallel to the surface of the plate. According to the data of [12], this property exists over a broad range of values of the Mach number, temperature factor and frequency parameter. Calculations based on the complete system of linearized Navier–Stokes equations have shown that taking the viscosity into account does not disturb the synchronization effect.

We will find the asymptotic form of the amplitude function $a(x_3)$ by using the form of expansion (3.6) and analyzing the system (3.2). We represent the eigenfunctions of the direct and adjoint problems in the form of a linear combination of three vectors so that as $y_3 \to \infty$

$$\zeta\to\sum_{j=1}^{3}\mathbf{g}_j\exp(\lambda_jy_3),\quad \xi\to\sum_{j=1}^{3}\mathbf{f}_j\exp(\lambda_jy_3),\quad \mathrm{Re}\,\lambda_j<0$$

$$\lambda_1^2=i\alpha R(1-c_a)+O(1)=O(\Delta^2\varepsilon^{-4}),\quad \lambda_2^2=i\alpha\,\mathrm{Pr}\,R(1-c_a)+O(1)=O(\Delta^2\varepsilon^{-4})$$
$$\lambda_3^2=\alpha^2-\mathrm{M}^2(\alpha-\omega)^2+O(R^{-1})=2b[1-\mathrm{M}^2(1-c_a)]x_3+O(x_3^2)+O(\Delta^{-2}\varepsilon^4)$$

Here, λ_j are the roots of the characteristic equation. The first two vectors decrease rapidly outside the boundary layer, and the third forms the solution in the inviscid zone, where $\mathrm{Re}(\lambda_3) \to 0$ as $x_3 \to 0$. Therefore the main contribution to the scalar products entering into (3.4) is made by the asymptotic "tails" of the terms proportional

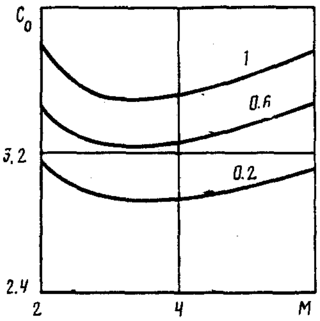

Fig. 4

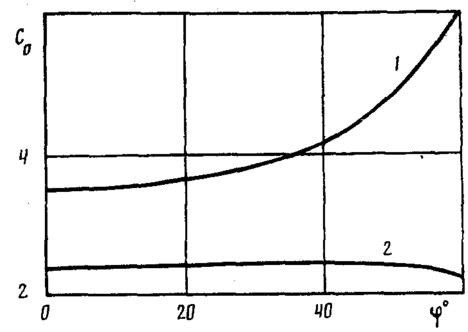

Fig. 5

to g_3 and f_3. Taking this property into account and using the self-similarity of the main flow, we easily obtain

$$\langle B\zeta, \xi\rangle = -\frac{(B_\infty g_3, f_3^*)}{2\mu\sqrt{x_3}}[1+O(x_3)], \quad \left\langle B\frac{\partial\zeta}{\partial x_3}, \xi\right\rangle = -\frac{(B_\infty g_3, f_3^*)}{8\mu x_3^{3/2}}[1+O(x_3)]$$

$$\langle G\zeta, \xi\rangle = O(x_3^{-1/2}), \quad \mu = 2b[1-M^2(1-c_a)] \qquad (3.10)$$

Here, B_∞ is the asymptotic form of the matrix B as $y_3 \to \infty$. Substituting (3.10) in (3.4) and integrating, in the leading approximation we find

$$a(x_3) = Cx_3^{1/4}, \quad C = \text{const} \qquad (3.11)$$

From relations (3.1), (3.3), (3.7), (3.9), and (3.11) we find the asymptotic form for the pressure perturbations in the boundary layer as $x_3 \to 0$

$$p = p_3(x_3)\exp[i(x_1 - c_a t)], \quad p_3 = Cx_3^{1/4}\exp[\Lambda^2\varepsilon^{-4}x_3^2], \quad \Lambda^2 = -i\frac{(1-c_a)^4 k^2}{4[M^2(1-c_a)-1]} \qquad (3.12)$$

The first term of (2.13) coincides with the asymptotic form (3.12) correct to a constant multiplier, i.e., the regions 1 and 3 overlap (shaded zone in Fig. 1). By matching we obtain

$$C = h\varepsilon^{-1/2}(8\pi)^{1/4}A(\pi\lambda^2)^{1/8} \qquad (3.13)$$

Thus, we have determined the relation between the eigensolution (3.3) describing the first and second boundary layer modes and the parameters of the external acoustic wave. The oscillations are excited over a much broader region than in the case of a subsonic boundary layer [4], which is attributable to the synchronization of the first and second modes and the sound waves. The quantity $C_0 = h^{-1}\varepsilon^{1/2}|C|$ depends only on the characteristics of the main flow through the integral parameter k. In Fig. 4 we have plotted the functions $C_0(M)$ for the first mode at three different values of T_f.

There exists a normalization of the eigenfunction ζ such that the initial condition for $a(x_3)$ depends only on the amplitude of the incident acoustic wave: $\zeta_4(x_3, 0) = [2i(S - S_a)]^{1/8}$, where $S_a = x_3\varepsilon^{-1}$ is the eikonal of the acoustic wave. With this normalization the initial value of the amplitude function

$$a(0) = hK_0, \quad K_0 = (8\pi)^{1/4}A = 2.093 \qquad (3.14)$$

The results can easily be extended to the case in which the acoustic wave impinges on the plate at a glancing angle φ, i.e.,

$$P = (\gamma M^2)^{-1} + h\,\mathrm{Re}\{\exp[i(\alpha_a x + \beta z - \omega t)]\}$$

Here, z is the transverse coordinate, $\varphi = \mathrm{arctg}(\beta/\alpha_a)$, and the phase velocity $c_a = 1 \pm 1/(M\cos\varphi)$. Calculations showed that the first and second modes are synchronized with the sound waves as $x_3 \to 0$ if $\varphi < \arccos(1/M)$, i.e., $c_{1,2} \to 1 \mp 1/(M\cos\varphi)$. In this case the initial amplitude of the modes is determined by relation (3.13) if (2.5) is replaced by the expression

$$k=\int\limits_0^\infty\left[\frac{T}{(U-c_a)^2\cos^2\varphi}-\mathrm{M}^2\right]d\eta$$

In Fig. 5 we have plotted the functions $C_0(M)$ for the first and second modes (curves 1 and 2, respectively) for M = 4 and $T_f = 1$. The expression (3.14) also holds for the renormalized eigenfunction.

We note that for the first mode the parameter λ, determined from relation (2.11), increases with increase in the Mach number and/or the glancing angle φ. It follows from Eq. (2.10) that the longitudinal scale of the diffraction zone 2 contracts $\sim|\lambda|^{-1}$. In this case the asymptotic theory in question is valid under the stronger constraint $\varepsilon \ll |\lambda|^{-1/2}$.

LITERATURE CITED

1. V. N. Zhigulev and A. M. Tumin, The Onset of Turbulence [in Russian], Nauka, Novosibirsk (1987).
2. V. V. Kozlov and O. S. Ryzhov, Boundary Layer Receptivity: Asymptotic Theory and Experiment [in Russian], VTs AN SSSR, Moscow (1988).
3. E. Kerschen, "Boundary layer receptivity," AIAA Pap., No. 1109 (1989).
4. M. E. Goldstein, "The evolution of Tollmien—Schlichting waves near a leading edge," J. Fluid Mech., 127, 59 (1983).
5. A. V. Fedorov, "Development of instability waves in a compressible-gas boundary layer," Chislenn. Metod. Mekh. Sploshnoi Sredy, 13, 144 (1982).
6. S. A. Gaponov, "On the development of disturbances in nonparallel supersonic flows," in: Laminar Turbulent Transition. (IUTAM Symp. Novosibirsk, 1984), Springer-Verlag, Berlin (1985), p. 581.
7. A. A. Maslov and N. B. Semenov, "Excitation of boundary layer oscillations by an external acoustic field," Izv. Akad. Nauk SSSR, Mekh. Zhidk. Gaza, No. 3, 74 (1986).
8. S. A. Gaponov and A. A. Maslov, Development of Disturbances in Compressible Flows [in Russian], Nauka, Novosibirsk (1980).
9. Chia Chiao Lin, The Theory of Hydrodynamic Stability, C.U.P (1955).
10. M. A. Evgrafov, Asymptotic Estimates and Integral Functions [in Russian], Nauka, Moscow (1979).
11. A. H. Nayfeh, "Stability of three-dimensional boundary layers," AIAA J., 18, 406 (1980).
12. L. M. Mack, "Boundary layer stability theory," Jet Propul. Lab. Doc. 900-277, Rev. A, Pasadena (1969).