

Averaged equations describing the turbulent diffusion of a chemically active admixture in coordinates tied to the instantaneous values of the concentration of another passive admixture are obtained. The results can be readily extended to the case of an arbitrary number of different chemically active admixtures. An advantage of this approach is the separation of the scales of the fluctuating and average motions, which makes the proposed average diffusion relations applicable even at times on the inertial interval.

The need to obtain the characteristics of multicomponent diffusion transport of various admixtures arises in many problems of physical kinetics in turbulent flows [1, 2]. In [1] certain particular solutions are considered and an approximate method of analyzing the fuel diffusion in the lean part of a flame is proposed.

1. Formulation of the Problem

The instantaneous equations describing the various kinetic processes in turbulent flow can be written in the following form:

$$\rho \frac{\partial c}{\partial t} + \rho(\mathbf{v}\nabla)c - \text{div}(D_c \rho \nabla c) = W\rho \quad (1.1)$$

$$\rho \frac{\partial z}{\partial t} + \rho(\mathbf{v}\nabla)z - \text{div}(D_z \rho \nabla z) = 0 \quad (1.2)$$

Equation (1.2) describes the turbulent transport of a passive admixture that does not take part in chemical reactions. The concentration z is so normalized that in the region in which the fluid is turbulent $0 < z < 1$, while in the regions of potential flow $z = 0$ and $z = 1$. The concentration of the active admixture c depends on the results of the reaction and satisfies Eq. (1.1) with the source term $W(z, c)$. In averaging Eq. (1.1) (especially when W depends strongly on z and c) it is necessary to find the combined characteristics of the z and c concentration fields. It is assumed that the active admixture does not have much effect on the dynamic parameters of the turbulence. Generally speaking, the system of kinetic equations (1.1), (1.2) may contain n equations for the concentrations of the various admixtures c_i with source terms $W_i(z, c_1, \dots, c_n)$ ($i = 1, 2, \dots, n$). Below we consider the relations for a single active admixture, but this does not impose any limitations on the system of kinetic equations, since all the arguments are applicable to each admixture concentration c_i .

In some special cases a simple steady-state relation may be obtained between the concentrations z and c . For example, if $D_z = D_c$ and $W = 0$, then the dependence $c(z) = a_1 z + a_2$ (a_1 and a_2 are constants) turns the system (1.1), (1.2) into an identity. We will call these equilibrium solutions. Another type of equilibrium solution was obtained in [1] for the turbulent combustion of nonpremixed gases in the presence of a source having a sharply expressed maximum in the neighborhood Δz of the stoichiometric value $z = z_s$. The solution can be represented by the following relations:

$$c(z) = 0, \quad 0 < z < z_s - \Delta z, \quad N \frac{d^2 c(z)}{dz^2} = -W(z, c(z)), \quad z_s - \Delta z < z < z_s + \Delta z \quad (1.3)$$

$$c(z) = \frac{c(1)}{1-z_s} (z-z_s), \quad z_s + \Delta z < z < 1$$

where N is the instantaneous or average value of the scalar dissipation $N = \nu(\nabla z)^2$. The dependence (1.3) can be used only if the quasistationarity condition is satisfied, for which it is necessary that $\Delta z \ll z_\sigma$ (z_σ is the characteristic macroscale of the fluctuations of the concentration z).

The existence of a steady-state relation between the instantaneous values of c and z is the exception rather than the rule. In condensation theory, in which z represents the frozen thermodynamic parameters and c the condensate concentration, a relation of type (1.3) cannot be constructed [2]. The same situation also arises in many problems of combustion theory, for example, in connection with fuel diffusion in the lean part of a flame, for which an approximate integral method is proposed in [1].

A detailed characteristic of the z concentration field is the admixture concentration probability distribution function $P(z)$ which satisfies the equation

$$\frac{\partial}{\partial t} (P(z)\rho) + \text{div}(\langle v \rangle_z P(z)\rho) + \frac{\partial^2}{\partial z^2} (\langle N \rangle_z P(z)\rho) = 0 \quad (1.4)$$

where $\rho = \rho(z)$ is the density of the medium.

Equation (1.4) was obtained in [1], where the coefficients $\langle V \rangle_z$ and $\langle N \rangle_z$ were also analyzed and where numerical calculations were made for $P(z)$ and analytic approximations constructed. An important experimentally confirmed result is the nondependence of the conditional mean scalar dissipation $\langle N \rangle_z$ on the value of z within the turbulent flow $0 < z < 1$, which is associated with the weak statistical correlation between the small-scale fluctuations (which determine the instantaneous value of N) and the large-scale fluctuations (which determine the instantaneous value of z) [1]. This property relates only to the passive admixture, for whose concentration the boundary conditions are assigned as described above and there is no source term.

2. Equation for the Conditional Mean Concentration $\langle c \rangle_z$

First of all, we will obtain a relation for the joint probability density of the passive and active admixtures — the function $P(z, c)$, which also depends on the parameters x and t . We start by assuming that the diffusion coefficients D_z and D_c are identical and equal to D . We will then extend the results to the case of different diffusion coefficients.

We introduce the function $\psi = \delta(z(x, t) - z^0) \delta(c(x, t) - c^0)$ of the physical coordinates x and t and the parameters c^0 and z^0 , which depends on the realizations of the fields $z(x, t)$ and $c(x, t)$. Applying the rules for the differentiation of Dirac delta-fractions [3], we obtain the following identities:

$$\frac{\partial \psi}{\partial t} = -\frac{\partial}{\partial c^0} \left(\frac{\partial c}{\partial t} \psi \right) - \frac{\partial}{\partial z^0} \left(\frac{\partial z}{\partial t} \psi \right) \quad (2.1)$$

$$\nabla \psi = -\frac{\partial}{\partial c^0} (\psi \nabla c) - \frac{\partial}{\partial z^0} (\psi \nabla z) \quad (2.2)$$

$$\begin{aligned} \text{div}(D\rho \nabla \psi) &= \frac{\partial^2}{(\partial c^0)^2} ((\nabla c)^2 D\rho \psi) + 2 \frac{\partial^2}{\partial c^0 \partial z^0} ((\nabla c \cdot \nabla z) D\rho \psi) + \\ & \frac{\partial^2}{(\partial z^0)^2} ((\nabla z)^2 D\rho \psi) - \frac{\partial}{\partial c^0} (\psi \text{div}(D\rho \nabla c)) - \frac{\partial}{\partial z^0} (\psi \text{div}(D\rho \nabla z)). \end{aligned} \quad (2.3)$$

We then substitute the relations (1.1), (1.2), (2.2), and (2.3) in the equation (2.1) and in order to write the convective terms in divergence form we add the equation obtained and the unaveraged continuity equation multiplied by the function ψ . We average the relation obtained taking into account the identities $\langle \psi \rangle = P(z^0, c^0)$, $\langle B\psi \rangle = \langle B \rangle_{z^0 c^0} P(z^0, c^0)$, where $\langle B \rangle_{z^0 c^0}$ denotes the averaging of the quantity B for $z = z^0$, $c = c^0$. Since the chemically active admixture does not affect the dynamic turbulence

characteristics, in the isobaric approximation the density depends only on the concentration z : $\rho = \rho(z)$. Finally, we will estimate the term on the left of Eq. (2.3) (setting, for simplicity, $\rho = \text{const}$ and $D = \text{const} \rightarrow 0$): $\langle D \text{div}(\rho \nabla \psi) \rangle = D \text{div}(\rho \nabla P(z^\circ, c^\circ)) \rightarrow 0$.

The result of these substitutions is the unclosed equation for the joint probability density:

$$\begin{aligned} & \frac{\partial}{\partial t} (P(z^\circ, c^\circ) \rho) + \text{div}(\langle v \rangle_{z^\circ c^\circ} P(z^\circ, c^\circ) \rho) + \\ & \frac{\partial^2}{(\partial z^\circ)^2} (\langle D(\nabla z)^2 \rangle_{z^\circ c^\circ} P(z^\circ, c^\circ) \rho) + 2 \frac{\partial^2}{\partial c^\circ \partial z^\circ} (\langle D(\nabla z \cdot \nabla c) \rangle_{z^\circ c^\circ} \\ & P(z^\circ, c^\circ) \rho) + \frac{\partial^2}{(\partial c^\circ)^2} (\langle D(\nabla c)^2 \rangle_{z^\circ c^\circ} P(z^\circ, c^\circ) \rho) = - \frac{\partial}{\partial c^\circ} (\langle W \rangle_{z^\circ c^\circ} P(z^\circ, c^\circ) \rho) \end{aligned} \quad (2.4)$$

The possibility of obtaining a relation for $P(z, c)$ was noted by Kuznetsov. In what follows, as in Eq. (1.4), the superscripts have been omitted from the variables z° and c° .

We now turn to the derivation of a relation for the conditional mean concentration $\langle c \rangle_z$. We introduce the notation: $c_z = \langle c \rangle_z$, $V_z = \langle V \rangle_z$, $W_z = \langle W \rangle_z$, $N_z = \langle N \rangle_z = \langle D(\nabla z)^2 \rangle_z$. We multiply Eq. (2.4) by c and integrate it with respect to c from $-\infty$ to $+\infty$ (from 0 to $+\infty$, if the concentration c takes only non-negative values). The last two terms on the left of Eq. (2.4) and the term on the right are integrated by parts:

$$\frac{\partial}{\partial t} (c_z P(z) \rho) + \text{div}(\langle vc \rangle_z P(z) \rho) = W_z P(z) \rho + \frac{\partial}{\partial z} \xi(z) \quad (2.5)$$

$$\xi(z) = 2 \langle D(\nabla z \cdot \nabla c) \rangle_z P(z) \rho - \frac{\partial}{\partial z} (\langle Nc \rangle_z P(z) \rho) \quad (2.6)$$

The physical significance of relation (2.5) becomes clear when it is integrated with respect to z from z_1 to z_2 ($0 < z_1 < z_2 < 1$). The terms on the left of (2.5) describe the convective transport of the active admixture averaged on the understanding that the instantaneous value of the concentration z lies on the interval (z_1, z_2) . On the right of (2.5) we have the conditional mean source term W and the mean flux ξ across the isoscalar surface $z = \text{const}$. We will now examine the suitability of Eqs. (2.4)–(2.6) for describing the combined diffusion of admixtures having different diffusion coefficients D_c and D_z . If we introduce an auxiliary admixture, whose concentration z^* satisfies the same initial and boundary conditions as the concentration z , but which has the diffusion coefficient D_c , then relations (2.4)–(2.6) can be used for determining the combined characteristics of the admixture concentrations c and z^* . In their turn, in accordance with the general notions of turbulence qualitatively confirmed by the experimental data of [4] the turbulent fields of the admixture concentrations z and z^* will differ only on scales of the order of the Kolmogoroff scale η_m determined from the diffusion coefficient $D_m = \max(D_c, D_z)$. Considering that $\eta_m \rightarrow 0$ as $D_m \rightarrow 0$ and that on these scales the admixture concentration c varies by an amount $c_\eta \rightarrow 0$, we may conclude that $P(z, c) \rightarrow P(z^*, c)$; $\langle c \rangle_z \rightarrow \langle c \rangle_{z^*}$. Thus, the relations obtained for identical diffusion coefficients are suitable for describing the combined dynamics of admixtures with different diffusion coefficients on scales that exceed the scale η_m .

In order to close Eq. (2.5) it is necessary to approximate the correlations entering into Eq. (2.6). We will proceed to construct an expression for the flux ξ after investigating the behavior of a selected particle in the passive admixture field.

3. Particle in Passive Admixture Field

Let us suppose that a certain admixture consists of a set of small noninteracting inertialess particles. We will call the instantaneous value of the passive admixture concentration in the neighborhood of a selected particle the coordinate of that particle in the space of instantaneous values of the passive admixture concentration (or in z -space). At the initial instant of time let the particle be located within the turbulent field in the neighborhood of the point x_0 in physical space and have the coordinate z_0 ($0 < z_0 < 1$) in z -space. We denote the deviation from the initial value in z -space by

$z_1(t)$ ($z_1(t_0) = 0$) and its derivative by $\dot{z}_1(t) = dz_1/dt$.

At times on the inertial interval the random process $\dot{z}_1(t)$ may be assumed to be stationary, since the particle is unable to leave the neighborhood of x_0 , where the turbulent field is homogeneous. We will consider the correlation $\langle \dot{z}_1(t_1)\dot{z}_1(t_2) \rangle_{z_0} = K(t_2 - t_1)$. The function $K(t)$ is proportional to the second derivative of the standard deviation $\langle (z_1(t))^2 \rangle_{z_0}$ (see, for example, [5, 6]). The subscript z_0 indicates that the average is conditional only with respect to those realizations of the turbulent field in which z takes the value z_0 in the neighborhood of x_0 at $t = t_0$.

We note that the time dependence of the standard deviation should relate two scales — the time scale and the scale of variation of the admixture concentration z . At times on the inertial interval of turbulence (see [1, 6]) this relation should be realized through the turbulent energy dissipation $\langle \varepsilon \rangle_{z_0}$ and the scalar dissipation $\langle N \rangle_{z_0}$, parameters of the inertial interval that do not depend on the Reynolds number Re . From dimensional considerations it follows that this relation is unique

$$\langle (z_1(t))^2 \rangle_{z_0} = A \langle N \rangle_{z_0} (t - t_0), \quad A = \text{const} \quad (3.1)$$

consequently

$$K(t) = \frac{1}{2} \frac{d^2}{dt^2} \langle (z_1(t))^2 \rangle_{z_0} = 0 \quad (3.2)$$

Equation (3.2) indicates that at times on the inertial interval the motion of the particle in z -space may be regarded as a process with uncorrelated increments. In order to simplify the calculations as far as possible, we will assume that in the first approximation the velocities of the particle and the medium coincide:

$$\dot{z}_1 = D(\nabla^2 z) \quad \text{when} \quad D = \text{const}, \quad \rho = \text{const}.$$

The order of magnitude of $K(0) \approx \langle P(\nabla^2 z)^2 \rangle_{z_0}$ is determined by the corresponding Kolmogoroff scales $K(0) \approx (z_\eta/t_\eta)^2 \approx \langle N \rangle / t_\eta \sim \sqrt{Re} \gg 1$. If it is assumed that the correlation does not decrease sufficiently rapidly, then the particle should "escape" from the turbulent fluid ($0 < z < 1$). The integral of the function $K(t)$ with respect to time, which gives the diffusion coefficient in z -space, cannot increase without bound as the Reynolds number Re increases; from this it follows that the characteristic time corresponding to the fall in the correlation is t_η . The qualitative behavior of the function $K(t)$ is represented in Fig. 1, where the quantities that do not depend on the Reynolds number Re are assumed to be quantities of the order of unity.

For fairly high Reynolds numbers the turbulent field has a "spongy" structure consisting of zones with high scalar dissipation (where $\nabla^2 z$ is large) and zones with weak dissipation having a characteristic dimension of the order of the Kolmogoroff scale (where $\nabla^2 z$ is small) [6, 7]. The coordinate $z_1(t)$ changes in jumps of the order of the corresponding Kolmogoroff scale z_η . Strengthening the property (3.2) somewhat, we will assume these jumps to be not only uncorrelated but also statistically independent. Then, in accordance with the central limit theorem of the theory of probability the probability density of the sum z_1 of a large number of jumps z_η will be distributed in accordance with a Gaussian law, and the motion of the particle in z -space will be an analog of Brownian motion.

4. Closure of the Equations for the Conditional Mean Concentration c_z

We now turn to the construction of the relations for the flux ξ entering into Eq. (2.5). We will formally consider the active admixture to consist of small inertialess noninteracting particles. This assumption will facilitate the discussion and does not impose any limitations on the possibility of particles appearing or disappearing (i.e., on participation in chemical reactions). If the motion of each of these particles in z -space is similar to Brownian motion, then the mean flux ξ must be determined by the following first-order diffusion relation:

$$\xi = A_1 \frac{\partial c_z}{\partial z} + A_2 c_z \quad (4.1)$$

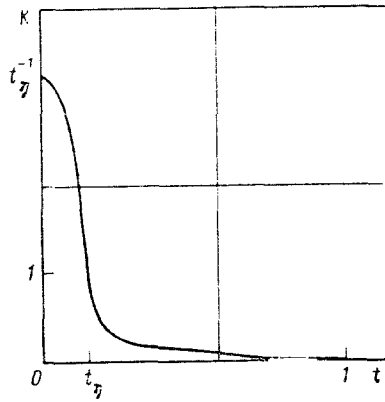


Fig. 1

The coefficients A_1 and A_2 are not known in advance and depend on the coordinates, time, and various turbulence parameters (and their derivatives), but do not depend on the c concentration field.

We will determine the coefficients A_1 and A_2 . We substitute in Eqs. (2.5) and (4.1) the equilibrium solution $c(z) = a_2$ (when $W = 0$), for which $\partial c(z)/\partial t = 0$, $\nabla c(z) = 0$, $\langle v'c \rangle_z = v_z c(z)$. Taking Eq. (1.4) into account, we have $A_2 = -\partial(N_2 P(z)\rho)/\partial z + \mu$, where μ is a constant of integration that does not depend on z . Nonzero values of μ are physically meaningless, since as z varies and the probability density $P(z)$ rapidly falls the mean flux $\xi(z)$ of active admixture across the isoscalar surface, on which the concentration $z = \text{const}$, remains constant. Proceeding in the same way with respect to the equilibrium solution $c(z) = a_1 z$, we obtain an expression for the other coefficient which gives

$$\xi = P(z)\rho N_z \frac{\partial c_z}{\partial z} - c_z \frac{\partial P(z)\rho N_z}{\partial z} \quad (4.2)$$

For a certain field B we will determine the fluctuating component by means of the relation $B'(x, t) = B(x, t) - \langle B \rangle_{z=z(x, t)}$, where $\langle B' \rangle_z = 0$. This quantity characterizes the instantaneous deviation of the value of B at a certain point in space from the conditional mean for a fixed joint realization of the fields $B(x, t)$ and $z(x, t)$. As a result of Eqs. (1.4), (2.5), and (4.2) we have two equivalent ways of writing the equation for the conditional mean concentration:

$$\frac{\partial}{\partial t} (c_z P(z)\rho) + \text{div}(\langle v'c \rangle_z P(z)\rho) = W_z P(z)\rho + P(z)\rho N_z \frac{\partial^2 c_z}{\partial z^2} - c_z \frac{\partial^2 P(z)\rho N_z}{\partial z^2} \quad (4.3)$$

$$\frac{\partial c_z}{\partial t} + (v\nabla)c_z + \frac{\text{div}(\langle v'c' \rangle_z P(z)\rho)}{P(z)\rho} = W_z + N_z \frac{\partial^2 c_z}{\partial z^2} \quad (4.4)$$

It is possible to run a simple check on the equations obtained, since averaging them with respect to the z coordinate should yield the well-known completely averaged equations of turbulent admixture transport. Since Eq. (4.3) was derived in order to describe the diffusion processes within the turbulent fluid $0 < z < 1$, we will consider the case in which there is no active admixture in the potential flow: $c = 0$ when $z = 0$ and $z = 1$. Then the conditional averaging of the characteristics of the active admixture within the turbulent field will coincide with the complete averaging of these characteristics. Since fluid can only flow into the eddying turbulent zone, the admixture flux $\xi(z)$ across the boundaries of the turbulent flow $z = +0$ and $z = 1 - 0$ will be equal to zero. Hence, taking into account the relation $P(z) \rightarrow 0$ as $z \rightarrow 0$, $z \rightarrow 1$ obtained in [1] and expression (4.2), we have $c(z) = 0$ when $z = +0$, $z = 1 - 0$. This equation is a boundary condition for the solution of Eqs. (4.3) and (4.4). It signifies that on transition from the potential to the turbulent zone the concentration c does not suffer discontinuities. Integrating Eq. (4.3) with respect to the variable z , we obtain the well-known average transport equations for a chemically active admixture. The last two terms on the right can be integrated by parts.

In deriving Eqs. (4.3) and (4.4) with the aid of relation (4.4) [sic] we used the fact that the linear equilibrium dependence $c(z) = a_1 z + a_2$ should turn these equations

identities. We will check whether Eqs. (4.3) and (4.4) are satisfied by the equilibrium dependence (1.3), for which $\partial c(z)/\partial t=0$, $\nabla c(z)=0$, $\langle v c(z) \rangle_z = v_z c(z)$, $c' = 0$, $W_z = W(z, c(z))$. Substituting these relations in Eq. (4.4), we obtain

$$N_z \frac{d^2 c(z)}{dz^2} = -W(z, c(z)) \quad (4.5)$$

The solution (1.3) satisfies Eq. (4.5) when in (1.3) the conditional mean value of the scalar dissipation $\langle N \rangle_{z_0}$ is used. The reason for the differences between approaches (1.3) and (4.5) consists in the following: the need to take into account the fluctuations of N arises in considering reaction zones with a characteristic scale of the order of the Kolmogoroff scale, and on such small scales the diffusion approximation (4.1) is not applicable.

Another method of deriving a relation for the flux ξ is based on the use of the assumption that the dependence $c(z)$ is close to the equilibrium linear dependence, i.e., that the small-scale structures of the c and z concentration fields differ only slightly. Locally (with respect to small fluctuations) this dependence may be assumed to be equilibrium $c = a_1(z - z_0) + a_2$ ($a_2 = c(z_0)$, $a_1 = \partial c(z)/\partial c$ when $z = z_0$). We will use this relation and the hypothesis of statistical independence of the large-scale and small-scale fluctuations in order to decouple the correlations entering into expression (2.6):

$$\langle N c \rangle_z = N_z c_z \quad (4.6)$$

$$\langle D(\nabla c \nabla z) \rangle_z = \left\langle D(\nabla z)^2 \frac{\partial c(z)}{\partial z} \right\rangle_z = N_z \frac{\partial c_z}{\partial z} \quad (4.7)$$

It can be seen that these relations lead to the previously obtained equation (4.2). This conclusion may seem unexpected: in Sec. 3, on the basis of which the closure (4.2) was constructed, we considered essentially strongly nonequilibrium distributions — the particles of active admixture are concentrated in the neighborhood of $z = z_0$. On the other hand, the condition of applicability of Eqs. (4.6) and (4.7) is that the admixture concentration c be close to the equilibrium value. The explanation of the universality of relation (4.2) may be that local equilibrium (local similarity of the admixture concentrations c and z) also exists for c fields that are nonequilibrium as a whole. This assumption is consistent with the approach of [7], in which it is assumed that in the region of large gradients the admixtures are the result of the convergence of the trajectories of different fluid particles (carrying different values of c and z) in a certain direction. Of course, in this case ∇z and ∇c both have the same direction.

Approximations of the various terms entering into (4.3) and (4.4) — v_z , N_z , $P(z)$ — were proposed in [1]. For example, we will use $N_z = N_t$ ($0 < z < 1$), $N_z = 0$ ($z = 0$, $z = 1$). In order to close relations (4.3) and (4.4) completely it is necessary to find the conditional mean turbulent transfer term for the active admixture $\langle v'c' \rangle_z$ and to average the source term $W_z = \langle W \rangle_z$. We note that the $\langle c \rangle_z$ dependence gives much more information for averaging the source than that for $\langle c \rangle$. Equations (4.3) and (4.4) contain the additional independent variable z , which makes them much more difficult to use in practice. Below we examine some specific turbulent flows in which these difficulties can be overcome.

5. Equation for the Conditional Mean Concentration c_z

in the Homogeneous Case in the Absence of a Source Term

It is easy to see that in the homogeneous case when $W = 0$ and $\rho = \text{const}$ Eqs. (4.4) and (1.4) yield the relations

$$\frac{\partial c_z}{\partial t} = N_t \frac{\partial^2 c_z}{\partial z^2} \quad (5.1)$$

$$\frac{\partial P(z)}{\partial t} = -N_t \frac{\partial^2 P(z)}{\partial z^2} \quad (5.2)$$

Equation (5.2) was obtained in [1]. It is also possible to arrive at Eq. (5.1) for the integral of c_z over the physical volume in the case of a homogeneous passive

admixture field and a localized active admixture field. For the purpose of analyzing relations (5.1) and (5.2) we also introduce the following notation:

$$m = \int_0^{1-0} P(z)c_z dz, \quad \tau = - \int_t^{t_1} N_t(t) dt \quad (5.3)$$

The fact that Eqs. (5.1) and (5.2) have a similar structure is not accidental. In fact, we specify as boundary conditions the absence of active admixture particles in the potential flow when $z = 0, z = 1$; consequently, $c = 0$ when $z = +0, z = 1 - 0$. Multiplying Eq. (5.1) by $P(z)$ and Eq. (5.2) by c_z and integrating their sum with respect to z , we see that the average number of active admixture particles in the turbulent fluid remains unchanged: $\partial m / \partial t = 0$. In general, however, the admixture concentration c decreases with time. This is associated with the increase in the volume occupied by the turbulent fluid, i.e., with the presence of intermittency.

Going over in relations (5.1) and (5.2) from the variable t to the variable τ , we obtain an equation with constant coefficients. In many turbulent flows as $t \rightarrow \infty$ the integral of N_t remains finite, which makes it possible to formulate initial conditions for the inverse parabolic equation (5.2): $P(z) = \delta(z - z_\infty)$ when $t = \infty$ [1].

For convenience we will assume that $t_1 = \infty$. If $N_t < \infty$ at the initial instant of time t_0 , then Eq. (5.1) will have a finite diffusion time $\tau(\tau(t_0) < \tau < 0)$. Therefore as $t \rightarrow \infty$ the solution of Eq. (5.1) $c_z(t, z)$ will tend to a certain, generally speaking nonequilibrium distribution. However, Eq. (5.1) has real physical significance only when $P(z)$ is considerably greater than zero (despite the fact that Eq. (5.1) can be solved independently of (5.2)). The probability of deviations from equilibrium $c_z = a_1(z - z_\infty) + a_2$ (where $a_1 = \partial c_z / \partial z, a_2 = c_z$ when $t = \infty, z = z_\infty$) tends to zero as $t \rightarrow \infty$.

6. Multicomponent Diffusion of Admixtures in Turbulent

Jets and a Turbulent Wake

A characteristic feature of these flows is the presence of a direction x_2 (x_2 and x_3 in the three-dimensional case) in which the turbulence characteristics vary much more rapidly than in the direction x_1 . The probability density $P(z)$ also varies rapidly with increase in the x_2 coordinate. We will assume that the dependence $P(z)$ is a stronger function of the transverse coordinate than the conditional mean concentration c_z . We note that this assumption does not imply a weak dependence of all the characteristics of the c field on x_2 . For example, the fully averaged concentration $\langle c \rangle$ may depend significantly on x_2 . As a rule, the source term in the physical kinetics equations is a function of the concentrations c and z and does not depend on the physical coordinates: $W = W(z, c(z, x_1)) = W(z, x_1)$.

The rapid variation of the average characteristics of the turbulent field on the periphery of the jet or wake is primarily associated with the change in intermittency. On the other hand, the turbulence characteristics averaged on condition that the observation point lies within the turbulent field vary only weakly [1]. The assumption made above is also consistent with this approach.

Depending on the dimensionality of the problem for the arbitrary quantity B we introduce the following notation:

$$\{B\} = \int_{-\infty}^{+\infty} B dx_2, \quad \{B\} = \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} B dx_2 dx_3$$

We integrate Eqs. (4.3) and (1.4) with respect to the coordinate x_2 (x_2 and x_3 in the three-dimensional case), assuming that $0 < z < 1$. In integrating we take into account the fact that $P(z) \rightarrow 0$ as $x_2 \rightarrow \pm\infty, z \neq 0, z \neq 1$. As a result of the weak dependence of c_z on x_2 for the arbitrary quantity B we assume that $\{c_z B P(z)\} = c_z \{B P(z)\}, \{W_z B P(z)\} = W_z \{B P(z)\}, \{\rho B P(z)\} = \rho \{B P(z)\}$. Here, the c_z removed from the braces is the effective (independent of x_2) value of the conditional mean concentration. We also neglect the fluctuations of the longitudinal velocity component: $\langle v_1 \rangle_2 = \langle v_1 \rangle$. We assume the flow to be steady. After integration we obtain

$$\frac{\partial}{\partial x_1} (c_z \langle \{v_1\} P(z) \rangle \rho) = W_z \{P(z)\} \rho + \{N_1 P(z)\} \rho \frac{\partial^2 c_z}{\partial z^2} - c_z \frac{\partial^2}{\partial z^2} (\{N_1 P(z)\} \rho) \quad (6.1)$$

$$\frac{\partial}{\partial x_1} (\langle \{v_1\} P(z) \rangle \rho) = - \frac{\partial^2}{\partial z^2} (\{N_1 P(z)\} \rho) \quad (6.2)$$

$$\langle \{v_1\} P(z) \rangle \frac{\partial c_z}{\partial x_1} - \{N_1 P(z)\} \frac{\partial^2 c_z}{\partial z^2} = \{P(z)\} W_z, \quad 0 < z < 1 \quad (6.3)$$

Equation (6.3) was obtained by deducting from (6.1) the equation (6.2) multiplied by c_z .

For the source W_z the following expression is proposed: $W_z = W(z, c_z(z, x_1))$. We note that this equation takes into account the fluctuations of the passive admixture, since for the total mean $\langle W \rangle$ we have

$$\langle W \rangle = \int_0^1 W(z, c_z(z, x_1)) P(z | x_1, x_2) dz \quad (6.4)$$

Relation (6.4) is an expression for averaging the source term in calculations in physical space that includes the dependence on x_2 . A similar relation can be used for calculating $\langle c \rangle$.

As a result of the sharp variation of the characteristics of the z field with the x_2 coordinate the surface $z = \text{const}$ is, on average, "similar" to the surface $x_2 = \text{const}$. Essentially, Eq. (6.3) represents the transition in the averaged transport equations to the new moving coordinates x_1, z . Information about the large, nonuniversal fluctuations will be unnecessary, since the isoscalar surfaces $z = \text{const}$ move together with these fluctuations. In accordance with the analysis in Sec. 3 Eq. (6.3) is the diffusion relation even at times on the inertial interval. The equations in z -space have the further advantage that the averaging of the source term (6.4) is more correct.

Equation (6.3) is the equation for the unknown dependence $c_z(z, x_1)$; the procedure for determining all the coefficients in the braces is known [1]. The difficulties associated with taking into account the reaction of the kinetic processes on the turbulence dynamics can be overcome by using various combined methods of calculation in both z -space and physical space.

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