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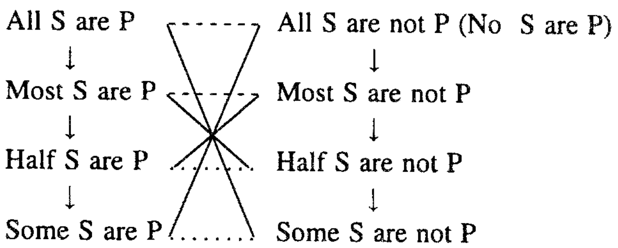
DISTRIBUTION AND PROPORTION

My aim is to defend the Aristotelian concept of distribution by expanding Robert Carnes' application of it (cf. Carnes & Peterson 1991, Peterson & Carnes ms. and 1983). In Section I, I apply distribution to k-quantity "fractional" syllogistic systems. In Section II, I consider unrestricted "proportional" syllogisms and sorites. Proportional systems with an infinite number of quantities are described in Section III.[1] Finally, in Section IV, I show how the new understanding of distribution survives Geach's criticisms.

I

If we add "Most" and "Half" statements to the traditional categoricals, we obtain eight basic forms:

(1)



where dashes connect contraries, dots connect sub-contraries, straight lines connect contradictories, arrows indicate sub-alternation, and each form is interpreted as possessing a tacit "or more" rider attached to the quantifier-phrase.

With eight such forms, it is a 4-quantity rather than a 2-quantity syllogistic system – one with 2048 syllogistic forms and 72 valid forms (cf. p. 354, Peterson 1985). This 4-quantity syllogistic system is NOT

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simply the 5-quantity system of Peterson and Carnes (ms. and 1983, and Carnes and Peterson 1991), with one of the quantities removed. Rather, the system based on (1) results from removing *two* of the quantities from the 5-quantity system ("almost-all", "few" and "many") and then adding the new quantity ("half").

The syllogistic system of "fractional" quantities (cf. Peterson 1985) is most easily introduced as a generalization of this 4-quantity system:

(2)

All S are P	All S are not P
↓	↓
More than $\frac{n-m}{n}$ S are P	More than $\frac{n-m}{n}$ S are not P
↓	↓
m/n S are P	m/n S are not P
↓	↓
Some S are P	Some S are not P

where $1/2 \geqslant m/n > 0$.

The number of quantities of a particular fractional syllogistic system will depend on how many different (non-reducible) fractions m/n (all equal to or less than $1/2$) are introduced: $q = 2f + 2$, for "q" quantities and "f" different fractions in reduced forms.[2]

The basic idea in Carnes' application of the traditional rules to the 5-quantity syllogism can be developed for the fractional syllogistic. First, the categorical forms are understood as follows:

(3)

English	*Schematized*	*Algebraic Interp.*
All S are P	$\geqslant n/n$ S are P	SP/S $\geqslant n/n$
More than $(n-m)/n$ S are P	$>(n-m)/n$ S are P	SP/S $>(n-m)/n$
m/n S are P	$\geqslant m/n$ S are P	SP/S $\geqslant m/n$
Some S are P	$>0/n$ S are P	SP/S $>0/n$

where (as above) $1/2 \geqslant m/n > 0$, "not P"/"$\overline{\text{P}}$" replaces "P" for negatives, and it cannot be that $S = 0$ since then "SP/S" is *un*defined.

In the algebraic formulae, read "SP/S" as "the ratio of the quantity of Ss that are Ps (or S that are P) to the quantity of Ss (or S)". The traditional form "All S is P" is given an algebraic interpretation wherein it is true if and only if the ratio of Ss that are Ps to the Ss is equal to or greater than 1. Since *no* ratio will be considered that is greater than 1 (for you cannot have 8 out of 7 Ss for example), SP/S = 1. However, stating the ratio for the truth of "All S is P" as SP/S $\geqslant$1 is required by the demand that its denial be equivalent to "Some S is not P" – viz., $S\overline{P}/S > 0/n$. The truth-condition SP/S = n/n is proved equivalent to SP = 0 via use of S = $S\overline{P}$ + SP. Universal forms entail corresponding particulars, since existential import is assumed.

Here are my extensions of Carnes' application of the traditional rules,

(5) Validity Rules for Fractional Syllogisms

Distribution

R1: The sum of the DIs (distribution indices) of the middle terms exceeds 1 (*is* >1).

R2: No term bears a larger DI in the conclusion than it bears in the premises.

Quality

R3: At least one premise must be affirmative.

R4: The conclusion is negative if and only if one of the premises is.

Quantity (dispensable)

R5: At least one premise must have a quantity of majority ("most") or higher (i.e., subject-term DI must be ">1/2" or higher).

R6: If any premise is non-universal, then the conclusion must have a quantity (DI) that is less than or equal to that premise.

WHERE the Distribution Indices (DIs) are as follows:

$\geqslant x/x$ for subjects of universals and predicates of negatives, where "x" is a non-zero whole numeral (i.e., DI $\geqslant$1)

Qx/y for subjects of fractional forms, where "Q" = ">" or "⩾" and "x" and "y" are non-zero whole numerals;

$>0/x$ for subjects of particulars and predicates of affirmatives, where "x" is any non-zero whole numeral (i.e., DI >0).

AND WHERE DIs are "summed" as follows

DI(Qx/y) + DI(Ru/v) *is* $(Q + R)(x/y + u/v)$
where "Q" and "R" are each one of ">" and "⩾",
and $> + > = >$, $> + \geqslant = >$, and $\geqslant + \geqslant = \geqslant$.

AND WHERE Qx/y *exceeds* Ru/v if and only if

either (1) $x/y > u/v$
or (2) "Q" = ">" and "R" = "⩾" when $x/y = u/v$.

Every valid syllogism is such that the algebraic formulae for the premises together with the formula for the denial of the conclusion are inconsistent. When a syllogism is invalid a counter-example can be formed (premises true and conclusion false). Failed attempts to demonstrate validity supply reduced formulae for hypothesizing counter-examples. The following assumptions, definitions, and axioms are adopted for the algebraic manipulations:

(6) (a) Existential Import: $X > 0$, for every *term* X (e.g., S, P, M) in a categorical

(b) Ratios: $(x)(y) \sim (x/y > 1)$; e.g. "7/8 S" is interpreted as 7 out of the 8 Ss (or 7/8 of the amount of S). "⩾9/8 S" has NO interpretation (since "9 out of the 8 Ss" is incoherent), although "9/9 S" means 9 out of the 9 (i.e., *all*) of the Ss.

(c) "$x \neq 0$"=df. "$\sim (x = 0)$"=df. "$x > 0$"

(d) Axioms: $(x) \sim (0 > x) \ldots$ non-negativity
$(x) \sim (x > x) \ldots$ anti-reflexivity.

The following are some k-quantity fractional syllogisms (for finite k) illustrating (a) how R1–R6 distinguish valid from invalid syllogistic forms[3] and (b) how algebraic proofs confirm valid cases and provide counter-examples (via failed proofs) for invalid cases. Any proof of the soundness of R1-R6 must show that every form they deem valid possesses an algebraic proof and any proof of the completeness rules must show that every form which possesses an algebraic proof is deemed valid by R1–R6.

(7)

All M are P	(a form analogous to AKI-1, p. 350,
$\geqslant$1/8 S are M	Peterson 1985)
Some S are P	Satisfies R1–R6. DIs of middle term are "summed" to obtain: $\geqslant 1 + > 0$ *is* >1. DI of each of S and P in the conclusion do not exceed their occurrences in the premises (DI(P3)>0 and DI(P1)>0, where DI(S3)>0 which does not exceed DI(S2)$\geqslant$1/8.

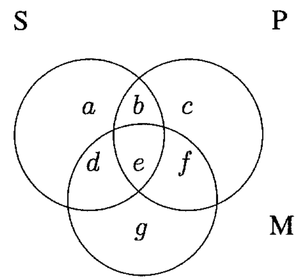
Proof (via RAA)

1. $d + g = 0$ .... premise All M are P
2. $(d + e)/(a + b + d + e) \geqslant 1/8$ .... premise $\geqslant$1/8 S are M
3. $b + e = 0$ .... denial of (Some S are P)
4. $8(d + e) \geqslant (a + b) + (d + e)$ .... from 2
5. $7(d + e) \geqslant (a + b)$ ..... from 4
6. $7e \geqslant a + b$ .... from 1 and 5
7. $(a + b + d + e) > 0$ ... axiom: $S > 0$
8. $a > 0$ .... from 1, 3, and 7
9. $0 \geqslant a$ .... from 3 and 6
10. $a = 0$ .... from axiom and 9

(8 and 10: a contradiction (QED).)

To read the proof (and others like it below) the following Venn Diagram is used.

(8)



(9) No M are P (a form analogous to EPB-1, *ibid.*)

$>$3/4 S are M

$>$3/4 S are not P

Satisfies R1–R6. DIs of middle terms are summed: $\geqslant 1 + >0$ *is* >1. DI(P3) is same as DI(P1), viz., $\geqslant$1; and DI(S3) is the same as DI(S2), viz., $>$3/4.

Proof by *reductio ad absurdum* is analogous to (7).

If we modify (9) by replacing the minor premise with "$\geqslant$3/4 S are M", then R2 would be violated (confirmed invalid via counter-example $d = 3$, $b = 1$, and $a = 0$). Such a modification departs from the fractional syllogisms defined in (2), but will be exploited below.

(10) All M are P (a form analogous to APP-1, *ibid.*)

$>$3/4 S are M

$>$5/6 S are P

This form is invalid, since R2 is violated. Middle term DIs do sum to $>$1, but DI(S) in conclusion is larger than the DI(S) in minor premise.

By attempting an algebraic proof, the following formulae can be derived:

$$5a \geqslant e \qquad e > 3(a+b)$$

To fit these formulae, the following values can be hypothesized:

$$a = 3 \qquad b = 1 \qquad e = 13$$

which together with $d = g = 0$ make premises true and conclusion false.

(11)

$>1/2$ M are P
$\geqslant 1/2$ M are S
———
Some S are P

(analogous to TTI-3, *ibid.*)

This form is valid, satisfying R1–R6. Proof (by *reductio*) is analogous to (7).

(12)

$\geqslant 1/2$ M are P
$\geqslant 1/2$ M are S
———
Some S are P

(a slight modification of (11))

This form is invalid, because R1 is violated; DIs of middle term sum as: $\geqslant 1/2 + \geqslant 1/2 = \geqslant 1$. But $\geqslant 1$ is not large enough to satisfy R1.

A counter-example is easily derived by examining the proof in (11). It would appear the same for (12) except that all occurrences of "$>$" would be replaced with "$\geqslant$".

(13)

All P are M
No M are S
———
$\geqslant 1/8$ S are not P

(analogous to AEG-4, p. 354, *ibid.*)

This form is valid since R1–R6 are satisfied. Middle terms' DIs sum to > 1, and DIs of conclusion terms do not exceed occurrences in premises (e.g., ($\geqslant 1/8$) does not exceed $\geqslant 1$).

Proof need *not* be by *reductio* to $S\overline{P}/S \geqslant 1/8$ via $8(a + d) > (a + d) + (b + e)$.

(14)

All P are M
$\geqslant 1/8$ M are not S
———
No S are P

(modification of (13))

This form is INvalid, of course. The sum of the middle term DIs is NOT >1 (but $>1/8$), so R1 is violated (though R2 is not since the DIs for the terms in the conclusion are $\geqslant 1$, the same DIs they have in the premises).

Counter-example: For 24 Ms, distributed as $d = 1$, $e = 2$, $g = 10$, and $f = 11$ (to make the minor premise true) and $b + c = 0$ (to make the major premise true), the conclusion (requiring $b + e = 0$) is false since $e = 2$.

(15)

>1/8 M are not P	(analogous to GPO-3, *ibid.* and
⩾7/8 M are S	cf. (11) and (12) above)
Some S is not P	This form is valid since R1–R6 are satisfied. Middle term DIs sum to >1, and DIs of conclusion terms do not exceed occurrences in premises.

Proof by *reductio*, analogous to (7).

(16)

All M are P	(almost similar to AKK-1, *ibid.*)
⩾1/3 S are M	
>1/3 S are P	This form is INvalid since R2 is violated; i.e., DI(S3) is >1/3, which exceeds DI(S2), which is only ⩾1/3. (Sum middle DIs is >1.)

Counter-example: Let there be 3 Ss distributed as $a = 2$, $e = 1$, and $b = 0$, where $d = 0$ due to the major premise. Then though the minor premise is true (1/3 or more S are M because exactly 1/3 are), the conclusion is false since it is not the case that more than 1/3 the S are P.

II

Since we can expand the 5-quantity syllogism to fractional quantifiers, we ought to consider whether the fractional syllogistic can be expanded to encompass, for example, (i) more quantifiers, (ii) arguments with more than two premises and three terms, and (iii) relations. If *all* of these extensions were successful, then a new approach to the foundations of mathematics may arise in a quantifier-theory of numerically-relevant intermediate quantifiers (wherein a particular number is not identified with a set, but rather with the *meaning* of one of the quantitative adjectives). I merely *begin* the envisioned expansion in this paper, not reaching relations.

First, it is extremely simple to include the inverse of "$>$" and "$\geqslant$". The following square is NOT proposed for new *categoricals*, even though all its relations are *true*:

(17) $< m/n$ S are P – – – $< m/n$ S are not P

$\leqslant (n-m)/n$ S are P .... $\leqslant (n-m)/n$ S are not P.

Rather, introduce all propositions expressing "$<$" and "$\leqslant$" by *definitions*. The UNdefined elements in the definitions are the basic categoricals with "$>$" and "$\geqslant$" so that the defined forms are NOT categoricals (and remain just defined forms, or "non-basic categoricals" if you like):

(18) "Qy/x S are P" =df. "Q' $(x-y)/x$ S are neg-P"

where if "Q" is "$<$", then Q' is "$>$"; if "Q" is "$\leqslant$", then Q' is "$\geqslant$";

if "are P" contains no negative element, then "neg-P" is "not P"; but

if "are P" contains "not" (is "are not B"), then "neg-P" is simply "B".

Also, an affirmatizing of traditional universal-negatives is required:

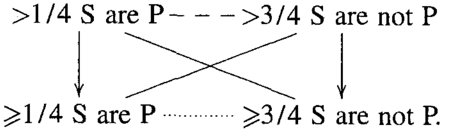
(19) "No S are P" (i.e., $\leqslant 0/n$ S are P) =df.
"All S are neg-P" (i.e., $\geqslant n/n$ S are neg-P),

where "neg-P" is defined as in (18). (See note 13 below for a defense of (19).) Finally, the customary inter-relations will be frequently used in discussion and proofs below (and have already been used above):

(20) "$x > y$" = "$\sim (x \leqslant y)$" = "$\sim (y \geqslant x)$" = "$y < x$", and
"$\sim (x > y)$" = "$(x \leqslant y)$" = "$(y \geqslant x)$"
= "$\sim (y < x)$".

Another way to grasp the point of (18) is via "rotated" squares, such as (5) and (11) of Peterson 1991. Here is a particular rotated square (matrix of embedded ratios "rotated" 90° from previous patterns in fractional squares):

(21)



(21) supports the equivalence of "–(>1/4 S are P)" with "⩾3/4 S are not P". However, if we looked at the denial of ">1/4 S are P" as a denial of the ">" relation, then we might be inclined to say that "⩽1/4 S are P" is equivalent to "–(>1/4 S are P)". But *then* "⩽1/4 S are P" is equivalent to "⩾3/4 S are not P" via (18).

To add more quantifiers to the restricted set of fractional systems, define k-quantity *proportional* systems as follows:[4]

(22) A k-quantity *proportional* syllogistic system is a system wherein each well-formed syllogistic form contains three categorical propositions, three terms (one, the middle term), two premises, and a number of quantities k (two of which are the traditional universal and particular) such that the "intermediate" quantities (i.e., non-universal and non-particular) are distinct ratios (between zero and one) modified with "more than" or "or more".

This definition makes any k-quantity proportional system (for finite k) distinct from a similar k-quantity *fractional* system by permitting quantifiers with ratios x/y, where $1/2 > x/y$, to be prefixed with ">" AND those such that $x/y > 1/2$ to be prefixed with "⩾". However, it does not permit the quantities expressed by "(less than) x/y (or less)"; i.e., all propositions are to be reduced (no "less") and "affirmatized" via (18) and (19). There will be the following KINDS of categorical propositions in any k-quantity proportional system ($1 > x/y > 0$):

(23)

English	*Schematic form*
All S are (not) P	$\geqslant x/x$ S are (not) P
More than x/y S are (not) P	$>x/y$ S are (not) P
x/y or more S are (not) P	$\geqslant x/y$ S are (not) P
Some S are (not) P	$>0/x$ S are (not) P.

The number of well-formed syllogistic forms, s, in any k-quantity *proportional* system will be discovered with the same formula used for fractional systems – i.e., $s = 32k^3$, for k the number of quantities. Also, for the number of ratios, r, contained in the intermediate quantifiers: $s = 256\ (r + 1)^3$, since $k = 2r + 2$. Fractional systems produce patterns of valid forms that are direct expansions of those for the 5-quantity system (e.g., in (6) of Peterson 1985). Consequently, the number of valid forms, v, for any k-quantity fractional system can be calculated: $v = 3k^2 + 6k$. So far the best that can be said for k-quantity proportional systems is that the number of valid forms, v', for any k-quantity proportional system won't be less than the number of valid forms for a similar fractional system (value of k the same). So, $v' \geqslant 3k^2 + 6k$. But *will* it be more? Comparing a proportional system to a fractional one it is generated from may raise doubts. For example, take the 6-quantity fractional system with the quantifiers ALL, >3/4, >1/2, ⩾1/2, ⩾1/4, and SOME will have 6912 syllogistic argument forms of which 144 will be valid. If we add to this system the quantifiers ⩾3/4 and >1/4, then we have an 8-quantity system with 16,384 argument forms. So, what if these two small additions produced exactly the same proportion [*sic*] of valid forms as the 6-quantity system – and no more? Would that be such a low number – viz., 240 (vs. the previous 144)? Maybe the very same formula holds: $v = 3k^2 + 6k$ ($= 12r^2 + 36r + 24$, for r = number of ratios, here 3).

Now, how do we modify the rules of (5) for determining validity of k-quantity fractional syllogisms so they can be used for *proportional* syllogisms? The answer, happily, is that no modification is required. THE RULES FOR DETERMINING VALIDITY OF PROPORTIONAL SYLLOGISMS ARE IDENTICAL TO THOSE FOR DETERMINING VALIDITY OF FRACTIONAL SYLLOGISMS – viz., those in (5). I won't prove this herein, but only illustrate it.

Illustrations are easily produced by simply inspecting examples used above. In (7), if the minor premise quantifier is changed to ">1/8", then the form is still valid for the same reasons. In the proof substitute ">" for "⩾". Further, if ANY intermediate quantifier of the form Qx/y (for "Q" = ">" or "⩾" and any ratio "$x > y$", $1 > x/y > 0$) is substituted for the quantifier in the minor premise of (7), the form is still valid. Similarly, for (9). Replace both occurrences of ">" with "⩾". Then the rules are satisfied identically, and the algebraic proof would appear the same except for interchange of ">" and "⩾".

No combination of replacements of "$\geqslant$" for "$>$" in (10) will make it valid. On the other hand, if we switch the minor premise and the conclusion (so "P" is middle term), the form is valid no matter what combination of "$>$" and "$\geqslant$" are substituted in the two intermediate quantifiers.

If we change the quantifier in the conclusion of (13) above to "$>$ 1/8 S are not P", then the form is still valid (via the rules of (5)) and the algebraic proof will be the same – except one line shorter. Similarly, (14) is still invalid if a similar change of intermediate quantifier is made – replacing "$\geqslant$1/8" with "$>$1/8". To prove it, use the same counter-example as for (14) but change $d = 1$ to $d = 2$.

Notice that (15) is NOT a well-formed fractional syllogism! For since the ratio in the quantifier of the major premise is 1/8, its degree should be $\geqslant$ (not $>$, as it is in (15)). Similarly, since the ratio in the quantifier of the minor premise is 7/8, its degree should be "$>$" (not "$\geqslant$"). So, (15) should NOT have been listed as an illustration of a fractional syllogism (say, one of a 9-quantity fractional system). But now we can see exactly what it is, a valid *proportional* syllogism. Finally, notice that the invalid (16) (also NOT a well-formed fractional syllogism) would BE a valid proportional if the quantifiers of the minor premise and the conclusion were interchanged.

The following reduction test is proposed to extend the methods of the proportional syllogistic to arguments with *more* than two premises (and one middle term).

(24) A sorites (or syllogistic-like form with more than two premises and one middle term) is valid IF either

(1) the conclusion follows validly from two of its premises OR

(2) from two of the premises a proposition follows validly which when added to the other premises permit step (1) to be satisfied, OR

(i) from two of the premises a proposition follows validly which when added to the previous premises permit step ($i - 1$) to be satisfied – for $i = 3, 4, 5$, etc. as required to produce a series of valid forms (polysyllogism) wherein the conclusion of the last is the conclusion of the original argument form.

The standard example is a Barbara-sort of sorites such as "All A are B, All B are C, All C are D, All D are E, All E are F / thus, All A are F" – wherein the intermediaries required to show that a valid polysyllogism can be generated are "All A are C, All A are D, and All A are E". This procedure, of course, gives us only a sufficient (not a necessary) condition for validity of more-than-two-premises "syllogisms".

By using (18) additional kinds of premises and conclusions can be included. Also, premises and conclusions can be allowed which begin with negatives – such as "Not all S are P", "Not more than 1/2 S are not P", "Not 3/22 or less S are P", etc. However, just as each proposition with a "less" quantifier in it must be replaced with one without it (i.e., reduced via (18)), so each externally negated proposition must be replaced with an equivalent proposition which is a categorical. Simply use the relevant squares of oppositions in (1), (2), (17), and (21).

Before considering propositions which have proportional quantifiers in the *predicate* (Finch 1957), consider the quantifier type "$=x/y$", in English "*exactly* x/y". Why can't we also have "=1/8" ("exactly one-eighth") and "=3/4" ("exactly three-quarters") as quantifiers? If forms containing "=" quantifiers are simply added to the basic categoricals of (23) above, then their occurrences *totally disrupts* the use of the rules in (5) (though NOT the use of the algebraic semantics of the proofs). Some argument forms like those discussed above remain valid when "=" quantifiers replace the proportional quantifiers "$>x/y$" and "$\geqslant x/y$". For example, replacing "⩾1/8" in (7) above with "=1/8" preserves validity. And replacing "⩾" in (11) with "=" preserves validity and "=" for both "⩾" in (12) preserves *in*validity. On the other hand, the valid (9) becomes invalid if one or the other or both of the ">" in it are replaced with "=". The only way (apparently) to obtain a valid modification of (9) with "=" is to replace ">" in the minor premise with "=" and ">" in the conclusion with "⩾". But in all these modifications of (9), the rules of (5) would deem the forms to be valid (with appropriate emendations in (5) so that DIs about phrases with "=" in them can be calculated).

The reason that "=" quantifiers added to proportional categoricals raise havoc is that they are complex forms *vis à vis* the basic categoricals; e.g., "=3/4 S are P" is equivalent to "⩾3/4 S are P *AND* ⩽3/4

S are P" – or better, to "$\geqslant 3/4$ S are P *AND* $\geqslant 1/4$ S are not P". So, when the "=" quantifier appears in a conclusion, we have *two* conclusions and to be valid two forms must be valid. And when a proportional statement containing "=" in its quantifier occurs as a premise, two separate premises are really involved. In general:

(25) " $=x/y$ S are P" = df.
"($\geqslant x/y$ S are P) & ($\geqslant (y-x)/y$ S are not P)".

To analyze any apparently well-formed proportional syllogism containing an "=", first replace the "=" quantified proposition with its equivalent form via (25). If the replacement occurred in the conclusion, the form is valid if and only if the two argument forms (the same premises in each and one conjunct for the conclusion of the first and the other for the conclusion of the second) are both valid. Analogously, an argument form is valid if it contains an "=" quantified proposition in the premises if and only if it is replaced by two separate propositions – each of which is one conjunct obtained by applying (24) – and the resulting sorites is valid.

Finally, consider quantifiers in the predicate. Finch (1957) analyzes categorical-like propositions containing two quantifiers, such as "At most 3/4 of the P are at least 1/2 of the M"[5] – which I call "Finch propositions". Finch propositions are reducible to two (or a conjunction of) singly quantified categorical-like propositions – e.g.,

(26) At most 3/4 of the P are at least 1/2 of the M $\equiv$ (At most 3/4 of the P are M) AND (At least 1/2 of the M are P).

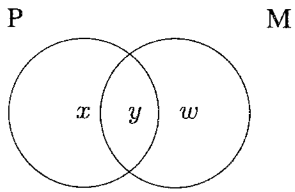
Finch does not consider the same kind of examples with negated copulas, such as "More than 3/4 of the S *are not* 9/10 or more of the Ps". The correct reduction makes both conjuncts negative categoricals (perhaps a surprise); e.g.,

(27) More than 3/4 of the S are not 9/10 or more of the P $\equiv$ (More than 3/4 of the S are non-P) AND (9/10 or more of the Ps are non-S).

That (26) and (27) are correct rests, at bottom, on *empirical* evidence about the meanings and uses of the component English sentences in (26) and (27).

Consider, for example, the first proposition in (26), "At most 3/4 of the P are at least 1/2 of the M", with respect to a Venn Diagram. Let "x" represent the one or more individuals in the $P\overline{M}$ region (i.e., that are P but not M), let "y" represent the individuals (one or more) in the PM region (that are P and M), and "w" represents the individuals in the $\overline{P}M$ region.

(28)



The statement in (26) refers, to begin with, to at most 3/4 of the Ps. Are those Ps, the x ones or the y ones? It seems to me that the meaning of the whole sentence is such that they have to be the y ones in (27), for the rest of the sentence identifies these Ps (the at most 3/4) with a certain quantity of Ms. So they have to be in the PM overlap in (28) AND the relation between the x individuals and the y individuals is that 3 times the number of x equals or exceeds the number of y – i.e., $3\#x \geqslant \#y$. Now these Ps, the y, are said by the statement to be identical to a certain quantity of Ms – to *be* the very same objects (not to be merely equal in number). So, the "is" of identity which seems to be involved in the first statements in (26) and (27) is not used to identify two quantities (as arises with "exactly", "precisely", and "=" inside the proportional quantifier phrases). So they must be in the overlap with the M region in (28). But the statement says more – viz., that these particular Ps are not just some Ms or other, but that they are identical to at least half the Ms. So, where "w" are the Ms which are not also Ps, the relationship between the y individuals (those Ms) and the w ones must be that $\#y > \#w$. In sum, we have *two* facts resulting from Venn Diagram analysis of (26) – that $3\#x \geqslant \#y$, and $\#y > \#x$. But the former fact is expressible in English by "At most 3/4 of the P are M" and the latter by "At least 1/2 of the M are P".[6]

(27) is analyzed similarly, but is more surprising – due to the negatives. Examination of (27) via a Venn Diagram helps again. The first statement of (27) says that more 3/4 of the Ss are *not identical to* a certain quantity of the Ps. So, those Ss have to be the x individuals in the diagram in (28) (no Ps are among them). Further 9/10 or more of the Ps are *not identical to* those Ss just mentioned. So, two facts result: $\#x > 3\#y$ (since the ratio of x to y is more than 3 to one); and $\#w \geqslant 9\#y$ (since the ratio of w to y is 9 to one or more). But these two facts are the same ones expressed in English by the two conjuncts of the right hand side of (27). Again, this is an empirical proposal, made with the help of a Venn Diagram representation of what the sentences mean. The least I would claim is that the two statements (one a conjunction) are logically equivalent.

It seems clear to me that these kinds of examples can be generalized:

(29) For any Finch proposition "Qx/y S are Ru/v P" or "Qx/y S are not Ru/v P" with quantifiers "Qx/y" and "Ru/v" ranging over basic and derived proportional quantifiers,
Qx/y S are Ru/v P = (Qx/y S are P & Ru/v P are S)
Qx/y S are not Ru/v P = (Qx/y S are $\overline{\text{P}}$ & Ru/v P are $\overline{\text{S}}$)

Here is one of Finch's "syllogism-analogues":

(30)
At most 4/7 P are at most 2/3 not-M
At least 3/5 S are precisely 7/8 M
so, At most / 3 S are P.

To analyze (with a k-quantity proportional system) any syllogism-analogue or sorites-analogue in which a Finch proposition occurs requires *replacing* the Finch proposition as follows:

(31) (i) If the Finch proposition occurs in the premises, replace it with the two propositions which are the conjuncts of the conjunction equivalent to it (so the overall argument form is "reduced" by increasing its premises); and

(ii) If the Finch proposition occurs in the conclusion, replace the whole argument with two forms each with the same premises but one with a conclusion which is one conjunct of the equivalent proportional conjunction and the other with a conclusion that is the other conjunct.

THEN an argument-form containing a Finch-proposition which is otherwise a well-formed syllogism or sorites of the k-quantity proportional syllogistic is valid if and only if (i) the "reduced" argument (Finch proposition was in the premises) is valid and/or (ii) the two new arguments are each valid (Finch proposition in the conclusion).

To evaluate (30), then, replace it with[7]

(30)′

At most 4/7 P are not-M
At most 2/3 not-M are P
At least 3/5 S are M
Precisely 7/8 M are S
so, At most 1/3 S are P

Schematically, (30)′ is

(32)

	Derived Qs		*Basic Qs*
1.	$\leqslant 4/7$ P are not M	1.	$\geqslant 3/7$ P are M
2.	$\leqslant 2/3$ not M are P	2.	$\geqslant 1/3$ not-M are not P
3.	$\geqslant 3/5$ S are M	3.	$\geqslant 3/5$ S are M
4.	$= 7/8$ M are S	4.	$\geqslant 7/8$ M are S
		5.	$\geqslant 1/8$ M are not S
6.	$\leqslant 1/3$ S are P	6.	$\geqslant 2/3$ S are not P.

Here is a counter-example, making the premises and denial of conclusion of (32) all true:

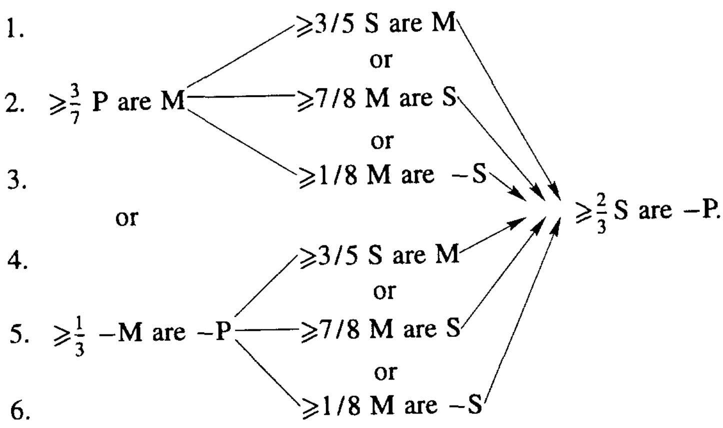
(33)

$$\left.\begin{array}{l} 1.\ 4(e+f) \geqslant 3(b+c) \\ 2.\ 2(a+h) \geqslant b+c \\ 3.\ 2(d+e) \geqslant 3(a+b) \\ 4.\ d+e \geqslant 7(g+f) \\ 5.\ 7(g+f) \geqslant d+e \end{array}\right\} \text{premises}$$

6. $2(b+e) > a+d \ldots$ denial of conclusion
$a = c = d = f = 0$; $b = g = h = 1$; $e = 7$ (where $h = \overline{\mathrm{SPM}}$ is added to (8) above).

So, (30) is invalid. Which of rules R1–R6 does (30) violate, however?

To answer this question, we must first determine what the rules R1–R6 of (5) are supposed to apply to. For they do not apply directly to a sorites, but rather to each syllogism of the series of syllogisms which need to be evaluated in order to decide whether the sorites they are generated from is valid. To obtain a series of candidate syllogism from the right hand side of (32), or from lines 1–5 of (33), one must first select a pair of propositions which will validly entail an intermediary that might, by repeating the process, lead to the desired conclusion. However, these premises are not typical of what sorites are classically envisioned to be, where the number of premises is the same as the number of middle terms. For some pairs selected from these five premises are not coherent or sensible candidates to start with (e.g., 1 and 2, or any pair of 3–5). Rather, the first premise must be paired with one of 3–5, then the second paired with each of 3–5, to see if any one path through 1–5 can produce 6, the conclusion. Here is a useful diagram of the alternatives:

(34)


NONE of these inferences, paths 1–6 on (34), should be valid according to rules R1–R6.

Alternate 1 of (34) violates R1. Similarly, alternatives 2 and 3. Alternatives 5 violates R2, since DI(S3) exceeds DI(S2). Alternatives

4–6 may well seem very problematic. For the first premise contains a negated subject term – something NOT permitted in the classical syllogism, and for which no allowances in the intermediate systems have been provided. In alternative 5 the issue can be avoided since the rule violated, R2, does not concern the negated subject of the major premise. So, we are left with only alternatives 4 and 6 as problematic. If we ignore the negativity of the subject term in the major premise, we will be led to think that R1 is NOT satisfied in 6. However, R5 is also violated by the same reasoning. I do NOT favor this approach; i.e., we can't just ignore the negativity. The best way to dispose of 6 is to say that it violates R4. I favor the speculation that essentially negated (with ineliminable negation) subject terms with intermediate quantifiers be taken to be fully distributed exactly like negated terms in the classical predicate position. Thus, the DI for the subject term of the major premise of alternatives 4–6 in (34) *is* $\geqslant 1$. That means R1 *is* satisfied for alternatives 4–6 (since the sum of the middle term DIs is >1). So, we are only left with alternative 4 as problematic.

Concerning alternative 4, we know it *is* invalid since the counter-example in (33) confirms its invalidity. Re-inspection of 4 suggests that R2 is violated, for DI(S3) exceeds DI(S2). This does resolve the problems with (34) and show the applicability of the rules to (30) and (32). But it is not a good solution. For we can imagine another inference just like alternative 4 but with the ratio in the conclusion changed from 2/3 to 1/3. Then the new inference would not violate R2. Nor would it seem to violate any other of R1–R6. Yet, it ought to, since it's invalid – as shown by the following counter-example (exactly the same one used in (33), by the way):

(35)	$\geqslant 1/3$ –M are –P	.... $2(a+h) \geqslant b+c$
	$\geqslant 3/5$ S are M	.... $2(d+e) \geqslant 3(a+b)$
	so, $\geqslant 1/3$ S are –P	.... $2(a+d) \geqslant b+e$

where premises true and conclusion false, if
$a=c=d=0,\ h=b=1$, and $e=7$.

BUT in (35) R2 is NOT violated. DI(S3) does not exceed DI(S2). So, what *is* violated by (35) of rules R1–R6?

If we are to permit negated subject terms in the proportional syllogistic, then we have a definite problem with R1–R6. We might regard this as a "syntactic" problem. Semantically, we know that (35) is invalid – i.e., via the algebraic semantics. But the "syntactic" rules R1–R6 say it is valid. So, R1–R6 are "unsound". My conclusion is that rules R1–R6 simply cannot handle negated subject terms (just as the traditional rules cannot).[8] So, rather than enter a revision of them, I propose that a qualification be added to their statement in (5): they do not apply to syllogisms with negated subject terms.

III

Throughout above, I have spoken of many *separate* fractional and proportional syllogistic systems. Each one has some finite number of quantities, k, which result from what the intermediate quantities are for the system – e.g., a 4-quantity system derived from (1) in which only two fractional quantities are added or an 8-quantity system with the proportional quantities based on the ratios 3/4, 1/2, 1/4 (each combining with ">" and "⩾"). The latter system would not be identical to another 8-quantity system – viz., one which is a *fractional* system containing ">6/7", ">5/7", ">4/7", "⩾3/7", "⩾2/7", "⩾1/7". NOW, however, we can envision combining such systems to obtain higher-quantity systems. Combining the two just mentioned would produce a 14-quantity system containing varieties of "fourths" and "sevenths". So, let's combine them all for all values of embedded ratios "x/y", where $1 > x/y > 0$. Since there are an infinite number (denumerably) of them, the combination will have an infinite number of quantities, well-formed arguments, and valid ones. Call this ultimate *proportional* syllogistic system the infinite-quantity – or "i-quantity" – syllogistic system. All the same rules, R1–R6, apply as does the algebraic semantics!

Isn't the i-quantity proportional syllogistic what we were thinking of all along – in short, utilizing all the finite, genuine ratios? (Irrationals are *not* included. Lines 2–4, p. 289, Peterson 1991 are a joke.) There can, then, exist an infinite number of quantities added to the classic syllogistic which sets the stage for a quantifier-theoretic approach to mathematics. But *this* quantifier-theoretic approach need *not* be reduced to some species of relational-quantifiers or restricted quantifiers in the predicate calculus. It is an ARISTOTELEAN theory of quantifiers based on propositional quantity.

The i-quantity system appears to be equivalent to Johnson's (1994) syllogistic with "fractional quantifiers". (Also, see Johnson 1991.) In the i-quantity system "<", "$\leqslant$", and "=" are introduced by definitions (18) and (25), even though in Johnson's system they are basic. And sorites in the i-quantity system are evaluated by reduction to polysyllogisms via (24), rather than directly as in Johnson's system. Still, Johnson's necessary and sufficient conditions for validity ought to determine the same syllogisms and sorites to be valid that the rules R1–R6 do for the i-quantity system. I think of Johnson's theorems as material for proving the soundness and completeness of R1–R6 i-quantity-wise. These rules are, of course, (a) sound if each form they deem valid is such that its premises together with the denial of its conclusion is inconsistent and (b) complete if each form which is such that its premises are inconsistent with the denial of its conclusion is deemed valid by them. *Proving* soundness and completeness may be easier with Johnson's Theorems 1 and 4. Briefly, R1–R6 for i-quantity syllogistic are sound, if every valid inference form (valid according to R1–R6) has a T-form – where each of the five T-forms (listed in Johnson's Theorem 1) is a different set of inconsistent proposition-types (each such generable from premises and denials-of-conclusions of valid forms). Similarly, using Theorem 4, R1–R6 for i-quantity syllogistic is *complete* if every I-quantity inference form that does have a T-form is deemed valid via R1–R6. Justification for this use of Johnson's theorems rests in the obviously semantic nature of his characterization of T-forms.[9]

It is not surprising to hold that an i-quantity syllogism or sorites should be considered valid if and only if it's premises are inconsistent with the denial of its conclusion – the idea behind Johnson's Theorem 1 and which was central above. However, it IS surprising to learn that no other types of inconsistencies will (or ought to) be generated from valid i-quantity syllogisms and sorites than just the five types Johnson listed. So, does every valid i-quantity syllogism or sorites have a Johnson T-form and is every inference-form that does have a T-form valid (according to R1–R6)? Yes – though I leave the proof as an exercise for the reader.[10]

IV

Geach's objections to distribution ought to apply to this Aristotelian account of rational proportions. For Geach, there are two concepts (or two aspects) of distribution – the formal (perhaps "syntactic"), and the semantic. Formally, distribution is simply a property (call it the "D-factor") which subjects of universals and predicates of negatives have. Although Geach's best known, and most discussed, criticisms concern distribution semantically speaking, he does criticize the formal concept (1960). He says that "by the doctrine of distribution the inversion will be valid *only* if ... the predicate term of q is not distributed unless that of p is" (1972, p. 63), where "an inversion is an inference from a categorical p to a categorical q such that (i) p and q have the same predicate term, but the subject terms of p and q are contradictory; [and] (ii) the quality of p is opposite to the quality of q." (*ibid.*) I agree with Geach that this rule does not work and that adjustments from some others (Keynes and Lukasiewicz) are not convincing. MY solution to this criticism is to find the culprit elsewhere. Geach blames distribution. I blame negative subjects. We have seen above at the end of Section II that negative subjects lead to troubles with R1–R6. The same kind of troubles occur in allowing negative subjects in the traditional syllogism. So, prohibit negative subjects in genuine traditional categoricals, where only genuine categoricals occur in well-formed syllogisms (256) and in the valid ones (24). The traditional rules do not necessarily work for validating inferences outside the syllogism.[11]

The *semantic* concept of distribution is that a term "T" with respect to its occurrence in a proposition "B" is distributed if and only if "B" says something (or implies something) about ALL of the Ts. The weaker way of expressing this is the easiest to defend – viz., "T" is distributed in "B" if and only if "B" *entails* a proposition about all of the Ts. I understand this to mean that the entailed proposition has the form "All (each, every) T are(is) ...", where "..." is filled in with some appropriate continuation. Then, in "All S are P", "S" is distributed because "All S are P" is about all the Ss (entails itself); in "No S are P", each term is distributed because "All S are not P" and "All P are not S" are both entailed; and in "Some S are not P", "P" is distributed because "All P are-not (are distinct from) some S" is entailed by "Some S are not P".

Geach objects very plausibly (1956). For if the truth of "All P is not some S" (entailed from the O-form) shows that "P" is distributed semantically, then a closely analogous thing can be done for *both* terms of the I-form. "Each S is either a P or is distinct from some P" AND "Each P is either an S or is distinct from some S" can both be validly inferred from "Some S is P". So, the semantic test fails. For neither term of the I-form categorical CAN be distributed traditionally.

I conclude that Geach *has* shown that a very typical statement of the semantic concept of distribution is faulty. In particular, it is too broad, permitting terms to be distributed which can't be (by the formal criterion). In light of the advancements herein, I offer a new defense of a semantic concept of distribution. I propose that the *kind* of semantic explication Geach has formulated and criticized is the wrong kind.

First of all, R1 and R2 *replace* the two traditional rules. R1–R2 concern *relative* quantities. A term's being distributed or not is replaced in R1–R2 with a term's being more, or less, distributed than some other term in the same context. Distribution is NOT, then, about all the members of the class designated, BUT is about *how much or many* of something you have got. In "More than 3/4 of the S are P", the term "S" has a distribution property which is a function of its *quantity* as expressed by its non-traditional quantifier "more-than-3/4". At first, it seems that the distribution value just concerns the proportion of the term in question. But this is not true, for the value "more-than-3/4" for "More than 3/4 the S are P", for example, concerns the relation of the ratio "3/4" to the ratio of S-*that-are-P* to all the Ss; i.e., the *semantic* character of the distributedness of "S" is expressed in the *whole* algebraic formula. So, "P" *is* involved. Yet, the quantity of Ps that there are, or are referred to or designated via the proposition, might seem *not* to be involved. "More than 3/4 of the S are P" is true if 9 out of 10 of the S are P – whether there are 9 P or 90 P (or an infinite number, a topic Johnson attends to but I ignore). But for 9 out of 10 S to *be* P, there must be at least 9 Ps! So, the quantity of Ps there are *is* involved.

Crucial to using the rules R1–R2 are decisions concerned with summing distribution indices and judging relative size. Distribution indices, DIs, might have been taken to just be formal notations. But now we can consider what they really are – they are quantities of concrete things. "More-than-3/4 of the musicians (M)" refers to (semantically!) a certain *proportion* of the musicians. So, distribution is certainly semantically characterizable. Distribution concerns what the distribution indices ARE (viz., proportions) and what *in* a statement refers to these indices. So, distribution IS proportion!!

What happens, then, to the traditional notion of distributed vs. UNdistributed? Well, it is not now a question of whether a term is distributed or not, but of what the distribution index IS – which is some ratio between 0 and 1, *with* various modifiers. Every distribution level is a genuine level of distributedness. (Is the old "*un*distributed" now the minimal level of distribution? Or, is it the non-maximal? Does it matter?)

This still leaves something hanging – viz., the distribution indices assigned to terms which do NOT have an explicit modifier expressing relative quantity. This is also the problem of distribution of predicate terms in basic categoricals, for *every* subject term in the i-quantity syllogistic has an explicit distribution-index-referring component as its explicit quantity-expressing modifier.[12] BUT predicates of negatives are stipulated as referring to $\geqslant n/n$, and predicates of affirmatives are stipulated as referring to $>0/n$ (similar to stipulations in the traditional syllogistic).

First, the predicate of the E-form categorical is legitimately assumed to have the distribution index "$\geqslant n/n$" because the same term when it appears in the logically equivalent *converted* form acquires an explicit expression for the maximal distribution level ($\geqslant n/n$). For example, in "No singers are musicians" the term "singers" has the maximal distribution index because the compact quantifier "No" in the quantifier-phrase "No singer" DOES express universality in English.[13] Alternatively, in "All singers are *not* musicians", the term "singers" has maximal distributedness because "all" very plainly refers to as large a proportion of the singers as there could be. In either way of looking at the E-form, it is validly convertible. Converting either form, the previous predicate term acquires an explicit quantity indicator. I conclude this is a good reason to hold that in its original position the predicate *tacitly* referred to $\geqslant n/n$. The same kind of argument holds for predicates of particular affirmatives, but there it is minimal rather than maximal distribution level that is in question.[14]

But there is *no* similar account for the O-form. And *why* are the predicates of all *intermediate* negative categoricals said to have the DI of "$\geqslant n/n$" and the predicates of *intermediate* affirmatives said to have DI of "$>0/n$"? The reasons these claims are not pure stipulations might be difficult to see prior to considering Finch's extensions of categoricals.

I defined "Finch proposition" above in Section II, and gave analyses of affirmative and negative forms in (29). Now I introduce the idea of "hamiltoning" a propositional form. A basic or non-basic categorical proposition (extended to include intermediate categoricals with all types of proportionals) has only one quantifier phrase in it, in subject position. A Finch proposition is similar to a categorical, but the predicate in it has an explicit quantifier. I shall call the process of adding an explicit quantifier to the predicate term in a categorical proposition a "hamiltoning" of it, where ONLY the explicit quantifier is added which produces a proposition logically equivalent to its pre-hamiltoned form AND exactly the same terms in the same positions remain (a term does *not* become negated in the process).[15] In the cases which are negative categoricals, the hamiltoned proposition retains the negated copula. (Also, the E-form that appears as "No S are P" is replaced *before* hamiltoning with "All S are *not* P"; cf. note 13.) Here are some pairs of unhamiltoned and hamiltoned propositions:

(36)	Most S are P;	Most S are some of the P
	More-than-2/3 S are P;	More-than-2/3 S are some of the P
	1/8-or-more S are not P;	1/8-or-more S are not all of the P.

Here is the traditional square of opposition with each hamiltoned form appearing below the standard representation:[16]

(37)	A: Every S is P Every S is some P	E: Each S is *not* P Each S is not each P.
	I: Some S is P Some S is some P	O: Some S is not P Some S is not each P.

Now I will *use* hamiltoned forms of the traditional categoricals in the basic test for distribution-level of predicates, as follows:

(38) (i) First, "hamilton" the categorical so that a logically equivalent categorical is obtained, one with exactly the same number of terms as the original form;

(ii) Second, take the explicit quantifier of the predicate in the hamiltoned form to refer to the DIs used above – i.e., "each", "all", or "every" refers to the DI $\geqslant n/n$ and "some" or "at least one" refers to the DI $> 0/n$.

Applying this to the traditional negatives, the distribution index for the predicate terms – "P" in the right-hand column of (37) – will be $\geqslant n/n$. In sum, the distribution-levels of the predicates of each categorical are determined by the DIs of each predicate in the hamiltoned transformation of the form (and see last three sentences of note 17).

This method also applies to all of the forms containing proportional quantity. But before discussing this, consider the following objection. Reducing a Finch proposition via substituting the right-hand sides of (instances of) the equivalences in (29) for the left-hand sides might be thought to be something which should apply to a hamiltoned proposition (since a hamiltoned one *looks* like a Finch proposition, both having two quantifiers and two terms). But this is a mistake. A hamiltoned proposition is simply not a Finch proposition. Thus, deFinching via (29) does not apply to the hamiltoned transforms of the basic four categoricals. (DeFinching reveals a compound nature for Finch propositions, not something genuine hamiltoneds possess.)

I have only argued that the predicates in negative categoricals have maximal DIs and the predicates in affirmative categoricals have minimal DIs for the four traditional categoricals. *Should* this carry over to all the basic proportional categoricals? Yes, for two good reasons. First, any such basic categorical is "in between", logically speaking, two traditional categoricals in such a way that the corresponding universal entails it and it entails the corresponding particular. So, shouldn't the predicate of the "in between" proposition have the same distribution level that it has in the entailing and entailed propositions? For example:

(39) All S are P $\rightarrow$ > 3/4 S are P $\rightarrow$ Some S are P
No S are P $\rightarrow$ Half S are not P $\rightarrow$ Some S are not P.

Since "P" in the first *proportional* form in (39) occurs "between" (entailment-wise) two forms in which it is minimally distributed (first row), I conclude "P" is minimally distributed in ">3/4 S are P". Similarly, for the same term being maximally distributed in "Half the S are not P".

Second, the reasoning for "P" being maximally distributed in "Some S is not P" (justification via hamiltoned forms) applies to *every* form "Qm/n S are not P" ("Q" is ">" or "$\geqslant$" and $1 > m/n > 0$). For example,

(40) (>3/4 S are not P) $\equiv$ (>3/4 S are not each P),

where the right-hand side is the hamiltoned version of the left. On the right side, "P" has the distribution indicator "each" which designates maximal distribution, $\geqslant n/n$. This applies repeatedly for every basic negative categorical with a proportional quantifier. Similarly, for all affirmative proportionals. Each of their hamiltoned forms will reveal that their predicates have minimal distribution, $>0/n$.[17]

But now we can ask: What if a proposition occurs in an inference to be evaluated by the i-quantity rules, R1–R6, which is one of the hamiltoned forms in (39)? Obviously, wherever such a hamiltoned form occurs it must be replaced for the sake of evaluating the inference by an acceptable categorical, the result of applying the *inverse* of the hamilton function.

One problem remains – *mixed* forms, with the "is" of identity but with one *traditional* quantifier and one *proportional* quantifier. It is no surprise that their analysis is mixed as well. The affirmative variations all succumb to treatment as genuine Finch propositions (e.g., (41)), but the negatives do not. For neither of (42) are true.

(41) (>2/3 S are all P) $\equiv$ (>2/3 S are P) & (all P are S)
(All P are $\geqslant$2/3 S) $\equiv$ (All P are S) & ($\geqslant$2/3 S are P)
(>2/3 S are some P) $\equiv$ (>2/3 S are P) & (some P are S)

(Some P are >2/3 S) ≡ (Some P are S) & (>2/3 S are P).

(42) (>2/3 S are not some P) ≡ (>2/3 S are non-P & Some P are non-S)
(>2/3 S are not each P) ≡ (>2/3 S are non-P & Each P is non-S).

So, perhaps the negative mixed cases need *un*hamiltoning rather than *de*Finching. The first thing to notice, on this hypothesis, is that mixed cases with the traditional quantifier in the subject cannot plausibly be unhamiltoned (since proportions would disappear). So, to *try* unhamiltoning these, invert the forms where necessary. Secondly, there is no alternative to dropping the traditional quantifier from the grammatical predicate. For the left-hand sides of (42), for example, unhamiltoning produces:

(43) >2/3 S are not P.

This *is* reasonable when the quantifier dropped is universal (paralleling negative categoricals which had been hamiltoned). An example reveals the problem with negative mixed cases with *particular* quantity. Consider:

(44) More than 2/3 of the singers (S) are not all the pianists (P).

(45) More than 2/3 of the singers are not some of the pianists.

(46) More than 2/3 of the S are not some particular P (at least one).

Even though (44) can be equivalently expressed by (43), this is plausible for only some uses of (45). Reading "some" in (45) as "a" or "any", it is synonymous with (44). But (45) can be read as (46). (44) and (43) each entail the existence of Ss that are non-P. (46) *permits* the non-existence of S that are non-P. For example, if all and only S are P, but they are sub-divided into two groups of 21 and 9 (say, 9 Ms and 21 non-Ms), then (46) is true. For more than 2/3 (21 of 30) of the Ss are *distinct* from some particular P (viz., any one of the 9 Ms). But this reading is incompatible with the (43) reading of (45).

So, the i-quantity syllogistic can be extended to cover the occurrences of propositions that are mixed cases by unhamiltoning them if the traditional quantifier is universal and if the quantifier is particular on one reading of the sentence. On the other reading, (46), it might be hoped that deFinching applies. It doesn't. To try it, wide-scope of the particular quantifier must be recognized (cf. note 17) so the product of deFinching is:

(47) Some P are not S & >2/3 S are not P.

But this can't be right, since each conjunct is incompatible with the truth conditions for (46) just given. So, there *is* one reading of negative mixed forms with particular quantity which simply cannot be transformed into forms the i-quantity syllogistic can accept.[18]

The fact that negative particular mixed forms do not always permit equivalent unhamiltoning (nor deFinching) does NOT disprove anything I have said about the i-quantity syllogistic. It only shows that empirical facts of English syntax and meaning are in control. The limits of logical analysis of English or any other natural language can be reached. However, these limits have been pushed back considerably with regard to analyzing sentences containing quantifiers expressing rational proportions in English – whether you take Johnson's formalistic (metatheoretical) approach or my own (with Carnes') Aristotelian approach. And if you are particularly interested in the *empirical* semantics of natural language quantifiers, my advice is that you adopt the Aristotelian approach.[19]

NOTES

[1] Johnson (1994) motivated me to try to give an Aristotelian alternative to his excellent extensions, which I return to in Section III.

[2] The "middle-quantifiers", Peterson 1985, p. 354, should not be included in a purely fractional syllogistic. I use "fractional" in a narrower sense than Johnson.

[3] The k-quantity fractional syllogistic was introduced in Peterson and Carnes (ms.), Section 5.2, and later appeared in Peterson 1985. What is new here is the formulation and application of R1–R6.

[4] I use the term "proportional" to acknowledge Finch (1957), about which cf. Peterson 1993.

[5] He does not, however, consider doubly quantified propositions containing relations – such as "Half the students failed 3/4 of the tests". I *hope* the systems herein can eventually cover relations.

[6] I have not *proved* this, since the English meanings are crucial. I hold that the reason the two sides of (26) are logically equivalent is that their meanings are identical. However, I don't need to *defend* this synonymy for anything herein.
[7] This analysis is drawn from (15)–(18) of Peterson 1993.
[8] Many syllogistic forms with negative subjects can be shown valid via appropriate replacements. For example, in "All non-M are P, All M are non-S, so All non-P are non-S", replace the first premise with its contrapositive and then replace "non-P" with "S" and "non-S" with "P′" and a case of Barbara is obtained. But using such replacements does not amount to extending R1–R6 to negative subjects, but rather shows how to obtain substitute forms to which R1–R6 can apply. As far as I can tell, no such replacements for intermediate/proportional categoricals with negative subjects exist. For one thing, the traditional immediate inference rules of obversion, conversion, and contraposition do NOT apply to intermediate/proportional categoricals (even though some immediate inferences for intermediates are valid – e.g., inferences read off various squares and reductions via (18)). I admire Williamson's (1971) treatment of negative terms. Although his techniques can save R1–R6 for traditional forms with negated subjects, I do not see how to apply them to intermediates/proportionals.
[9] I speak of inference forms having T-forms. This is briefer than Johnson puts it. He says, for an inference-form "X, so y" ("X" premises and "y" conclusion), that $\{X, y^*\}$ is the set of premises and denial of conclusion. When this set is ordered in a certain way (as a "chain"), it may satisfy one of the five T-forms of Johnson's Theorem 1. Only five patterns of inconsistent chains turn out to be needed! So, Johnson says that "X, so y" is valid only if $\langle X, y^*\rangle$ satisfies one of the five T-forms. I say "X, so y" itself satisfies a T-form when its associated chain, $\langle X, y^*\rangle$, does.
[10] Such a reader should note the following. What if there *is* some kind of evidence or argument for (a) a form being valid according to R1–R6, (b) the form not having a T-form, and (c) the form still having premises and denial-of-conclusion inconsistent? If such an example turns up, it might be as much evidence against Johnson's proofs as against the soundness of R1–R6 i-quantity-wise. Similarly, with respect to completeness, though in the reverse. Rather than finding too easy a *dis*proof of soundness, we might find too easy a proof of completeness. AND, a further question: Do Johnson's natural deduction rules of inference (expansions of Smiley 1973) generate all and only the same deductions (valid inference forms) that R1–R6 do in the i-quantity system? The answer should be "yes", and understanding why it is ought to be revealing for R1–R6. (And, are Finch forms assimilable to Johnson's fractional syllogistic? Or could an example like (32) be a counter-example to Johnson's Theorem 1, as it appears to be to R1–R6?)
[11] Williamson (1971) finds the culprit in distribution applied to universal-to-particular immediate inferences. Sommers (1975, 1982) has a quite different concept of distribution, which Englebretsen (1979, p. 117) uses on this example via treating it as an enthymeme (omitted premise "Some non-P are non-P").
[12] "More than 3/4 of the musicians" is a quantifier *phrase*. "More than 3/4" is its quantity-expressing component, a *modifier* (logico-grammatically) of its component *term*, "musician". "More than 3/4" refers to the distribution index >3/4. And "all" and the tacit universal-quantity component of "no" refer to the distribution index $\geqslant n/n$.

[13] This is the usual account. I prefer another one – that "No S are P" is a direct expression of denial of the I-form, a contracted form of "Not any S are P", where the latter is the grammatical expression in English for explicit denial of the I-form. "Not some S are P" is not grammatical in English, even though "Not all S are P" is. But you SAY the former grammatically by replacing "some" with "any". ("Any" after "not" in English often *means* what "some" ordinarily does.) So, "No S is P" is an explicit denial of the categorical "Some S is P", just the way "Not all S are P" is an explicit denial of "All S are P". So, "No S are P" (contrary to the entire tradition in the history of logic *and* to current practice!) is not a good candidate for expressing the E-form categorical. It is just as bad as saying "Not all S are P" *is* the O-form categorical. So, what *is* the E-form categorical? Simply "All S are *not* P". Emphasis is required in English so that the E-form won't be confused with the similar form pronounced with Shakesperean intonation, "*All* S *are* not P" (an alternative expression for the denial of the A-form). For more discussion, see Peterson 1988.

[14] There may be a Geach-like objection that distribution must be used to justify the validity of these inferences (about converted forms), so this test for distribution is circular. I reply that conversion is otherwise valid for E- and I-forms, via the algebraic semantics.

[15] Finch's idea of quantifiers in the predicate is not new. Sir William Hamilton made the most of such quantifiers, leading to the controversy between him and De Morgan. P. Heath sums it up: "Bentham was the first writer in English to quantify the predicate, Hamilton the first to make any extensive use of it, and De Morgan the first to grasp what it was all about. Since the result of [De Morgan's] investigation was to expose the hollowness of the claims that had been made for it, the dispute over priority has ceased, in any case, to be a matter of serious concern." (De Morgan 1966, p. xxiv). I have *not* herein been influenced by Hamilton or De Morgan (or Bentham either). Rather, I wonder whether looking at distribution and the syllogism as I propose might be used to cast new light on that debate. (Here's a start: only four of Hamilton & De Morgan's eight basic propositions are hamiltoned. The other four are what I would call "compounds", inequivalent to any traditional categorical. And all Finch propositions are similar to the compound "some S are not some P" which De Morgan (mistakenly, as I see it) said has "no ordinary use" and "no denial".)

[16] I alternate "every", "each", and singular copula with "all" and plural copula to increase plausibility. Also, in negative Finch and hamiltoneds, it can be useful to replace "is/are not" with "is/are distinct from".

[17] It is instructive to imagine the right side of (40) formalized as "$(>3/4\mathrm{S}x)[(\forall \mathrm{P}y)[x \neq y]]$" just as "$(\exists \mathrm{S}x)[(\forall \mathrm{P}y)[x \neq y]]$" is a restricted-quantifier notation for "Some S is not P" *after* hamiltoning. With this speculation, I can clear up a possible misunderstanding. Even though the "=" of predicate logic notation is appropriate for formalizing the "is" of identity mentioned above, still that does NOT mean an exchange of quantifier-positions in the notation corresponds to inversion in the English forms. (I refer to ordinary mathematical inversion, of functions and relations, NOT what Geach mentioned, as quoted above in Section IV.) This is important for the hamiltoned O-form. In English, it is "Some S is not each P". Schematizing the English yields "$>0/n$ S is-not $\geqslant n/n$ P". Both the English and the schematization are invertible, yielding "Each P is not some S" and "$\geqslant n/n$ P is not $>0/n$ S". (An ambiguity arises with the inverted English. The side of it I am pursuing is where it is synonymous with "Each P is not some one (or more) particular S", NOT with "Each

P is not any S" which is equivalent to "Each P is not S".) But the two forms of English and the two schematizations both have *exactly one* representation in predicate logic notation – e.g., "$(\exists Sx)[(\forall Py)[x \neq y]]$". So, quantifier scopes are not necessarily, nor usually, reversed via inversion in the English and schematizations. This may help to explain why I should *not* be understood to be advancing entailment-distribution vs. Geach; e.g., that the O-form entails, but is not entailed by, "Each P is such that some S is distinct from it", formalized "$(\forall Py)[(\exists Sx)[x \neq y]]$". I am *not* considering any variant of the argument wherein an entailed statement is found with predicate term transferred to grammatical subject. The relevant predicates "P" in negative categoricals, on *my* view, have the maximal distribution level *simply* because their explicitly present quantifier in *hamiltoned* forms designate that level.

[18] Actually, the merely apparent hamiltoneds which I call 'compounds' in Note 15 are also not covered. Something akin to deFinching them seems needed – but is only necessary, not sufficient, for affirmatives and is mistaken for negatives.

[19] I am grateful to Fred Johnson as I explained in note 1 and elsewhere above. As always on this topic, I have been importantly aided by Robert Carnes. Also, I have profited from Joel I. Friedman (via unpublished remarks) and from the anonymous reviewer for this journal (for several important corrections and additions).

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