Plastic Zones and Characteristics-line Families for Openings in Elasto-plastic Rock Mass

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Summary

Numerical solutions of the plastic zone and deformation distribution along tunnel wall due to excavation are presented in this paper. These solutions are obtained from elasto-plastic finite-element analysis, which assumes the Hoek-Brown or Mohr-Coulomb criteria, and the analyzation is performed to simulate a tunnel section far away from the excavation face, where the three-dimensional effect can be neglected. Factors, including the effects of anisotropy in the initial stress state and the tunnel shape on the plastic zone around an excavated tunnel have been examined. Furthermore, a concept called "the characteristic-line family of rock mass" is proposed to describe the varying relations between normalized stress and normalized deformation at various points on the tunnel wall in gradual unloading process due to excavation. The characteristic-line family of rock mass appears to depend on the rock mass property and the tunnel's characteristics.

1. Introduction

Tunnel-supports are often adopted in underground excavation to stabilize the rock mass around a tunnel. The design of tunnel-support usually requires the knowledge of the fracture zone as well as the stress change in the rock mass around a tunnel. The in-situ stress of rock mass remains in equilibrium before excavation of the tunnel; as excavation takes place, the rock mass is subjected to unloading, which results in stress re-distribution. If the new equilibrium stress state exceeds the yield limit or the strength of rock mass, the rock mass may develop cracks and reach a plastic state. This region of the rock mass in the plastic state is called the "plastic zone", which may propagate in the course of tunnel excavation.

Understanding the plastic zone is essential for tunnel-support design. An evenly distributed plastic zone (of a uniform thickness of the plastic zone) only exists for a circular tunnel under isotropic initial in-situ stress and axisymmetrical conditions. The distribution of the plastic zone around a tunnel may depend on a number of factors, such as the anisotropy in initial stress state, the tunnel's shape, the rock-mass properties and so on.

In the rock-support interaction analysis based on the "convergenceconfinement method" (G e s t a et al., 1980 ; H o e k and B r o w n, 1980), both the required support line for rock mass and the available support line have to be evaluated. The required support line for rock mass portrays the pressure change of the rock mass against tunnel deformation. The available support line, on the other hand, stands for the pressure that the applied support provides against tunnel deformation.

The fundamental concept of the convergence-confinement method asserts that the intersection of the required support line for rock mass and the available support line represents the pressure and tunnel deformation at the equilibrium state. This concept has commonly been accepted for designing the supports for deep tunnels (H o e k and B $r o$ w n, 1980; B $r a d v$ and B r o w n, 1985). Clearly, the determination of the required support line for rock mass plays an tremendously important role in designing tunnelsupport.

The gradual unloading (due to excavation) of isotropic and homogeneous rock mass, with hydrostatic initial stress surrounding a circular tunnel can be treated as an axisymmetrical cavity problem in infinite space. Conceptually, this cavity problem is equivalent to a one-dimensional model (D u d d e c k, 1980); and a one-to-one relationship exists between the cavity pressure and the cavity deformation. In the circular tunnel problem, this relationship corresponds to the required support line for the tunnel, which is a unique curve representing the characteristic of the rock mass surrounding the tunnel, hence is also termed "the characteristic line of rock mass". This concept indeed delivers a basic method for analyzing rock-support interaction through determination of the equilibrium position for rock mass and support (Fenner, 1938; Pacher, 1964). Several analytical solutions for the characteristic line of rock mass under specified assumptions are available (Ladanyi, 1974; Kennedy and Lindberg, 1978; Florence and Schwer, 1978; Hoekand Brown, 1980; Brown et al., 1983; O gawa and Lo, 1987). All the existing analytical solutions are subjected to a variety of assumptions, such as, isotropic initial in-situ stress, continuous, homogeneous and isotropic rock mass, and circular tunnel shape. These assumptions result in radial symmetrical in geometry and stress state of the considered problem.

The above-mentioned assumptions largely simplify the boundaryvalue problem into an axisymmetrical load-deformation problem (which is equivalent to a one-degree-of-freedom model), but are not necessarily satisfied by general in-situ conditions. Hence, analytical solutions can likely be inadequate for designing the general rock-tunnel supports, since in-situ conditions may violate the assumptions from which those analytical solutions are derived (Duddeck, 1980).

Due to the axisymmetrical nature of the existing analytical solutions, the plastic zone and deformation distribution around the tunnel wall remain uniform and axisymmetrical. The real plastic zone and deformation distribution, however contradictory to analytical solutions, often reveal non-uniform and non-axisymmetrical conditions resulting from anisotropic initial in-situ stress condition, a non-circular tunnel, or material heterogeneity and anisotropy (Lombardi, 1980).

In most cases, the in-situ stress condition in rock mass, prior to the excavation of a tunnel, is anisotropic. In other words, the initial horizontal stress differs from the initial vertical stress. Furthermore, non-circular shapes of tunnel cross section (such as rectangular, elliptical, and horseshoe shapes) are rather common. Consequently, the pressure and deformation of the rock mass vary significantly along different locations on the tunnel wall, and are far different from the ideal one-dimensional model. Indeed, a unique characteristic line of rock mass does not exist at all (Lombardi, 1980).

In this paper, numerical solutions of the plastic zone and deformation distribution along the tunnel wall (due to pressure release) are presented. These numerical solutions are obtained from elasto-plastic finite-element analysis, which assumes the yield criterion of rock mass to be Hoek-Brown or Mohr-Coulomb criteria. Numerical analyses are performed to simulate a tunnel section far away from the excavation face, where the three-dimensional effect can be neglected and the problem can be idealized as a planestrain problem. The numerical and analytical solutions of an example problem are first compared to verify the applied elasto-plastic finite-element analysis. Factors, including the effects of anisotropy in initial stress state and the tunnel's shape on the plastic zone around a hypothesized tunnel wall (after excavation), are then examined by the numerical analyses. Furthermore, a concept of the "characteristic-line family of rock mass" is proposed to describe the relationship and the distribution of pressure and deformation around the tunnel wall, in order to reflect the multi-degree-offreedom nature of this boundary value problem.

2. Yield Criteria and Plastic Potential

In the plasticity theory, elastic response is assumed before the stress state of a material reaches the yield surface, $F = F(\{\sigma\})$, which defines the boundary between the elastic and the plastic stress states. When a stress state reaches the yield surface, plastic flow is assumed to occur along the normal direction on the plastic potential surface: this condition is so-called the "normality flow rule". If a further assumption is made such that the plastic potential surface coincides with the yield surface, the flow rule is 'associated flow".

A variety of mathematical functional forms of yield criteria have been applied for rock material (Brady and Brown, 1985; Go odman, 1989). Among them, two of the most widely used, are the Mohr-Coulomb (e. g., Ogawa and Lo, 1987) and the Hoek-Brown criteria (e.g., Hoek and Brown, 1980; Hoek and Brown, 1983).

The Mohr-Coulomb criterion is a linear function in the following form,

$$
F = (\sigma_1 - \sigma_3) - (\sigma_1 + \sigma_3) \sin \phi - 2c(\cos \phi), \tag{1}
$$

in which ϕ and c are the internal frictional angle and the cohesion of rock, respectively.

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The Hoek-Brown criterion is a non-linear function in the following form.

$$
F = (\sigma_1 - \sigma_3) - \sqrt{(m \sigma_c \sigma_3 + s \sigma_c^2)},
$$
 (2)

in which σ_c is the uniaxial compression strength of the rock mass, while m and s are material parameters.

The yield criterion of a material can be regarded as the yield surface of this material, and the plastic potential surface is assumed to be in a similar form as the yield surface from time to time. In this study, yield surface in the forms of the Mohr-Coulomb criterion as well as the Hoek-Brown criterion are considered. Furthermore, the associated normality flow rule and perfectly plastic material behavior are assumed.

3. Verification of Numerical Analysis

An analysis of axisymmetrical circular opening in plane-strain condition within an infinitive rock-mass domain is performed, and the result is compared with analytical solution (Kennedy and Lindberg, 1978) to verify the applied numerical approach. The rock mass is assumed to be homogeneous and isotropic, and behaves as a perfectly plastic material when it yields. The yield criterion is assumed to follow the Mohr-Coulomb function. The hypothetical initial stress state and material parameters are listed as follows:

initial isotropic stress, $p_o = 20.68 \text{ MPa}$, uniaxial compressive strength, $\sigma_c = 8.96$ MPa, frictional angle, $\phi = 30^\circ$, Young's modulus, $E = 1930$ MPa, Poisson's ratio, $v = 0.25$, initial radius of opening, $A = 1.0$ m.

Derivation of the analytical solution for the considered problem can be referred to several references (e. g., Kennedy and Lindberg, 1978). The resulting analytical solutions for this problem are listed in order.

When $A \le r \le R$: plastic state.

$$
\sigma_r = \frac{\sigma_c}{1 - N_\phi} + \left(\frac{\sigma_c}{N_\phi - 1} + p_i\right) \left(\frac{r}{A}\right)^{N_\phi - 1},\tag{3}
$$

$$
\sigma_{\theta} = \frac{\sigma_c}{1 - N_{\phi}} + N_{\phi} \left(\frac{\sigma_c}{N_{\phi} - 1} + p_i \right) \left(\frac{r}{A} \right)^{N_{\phi} - 1} \tag{4}
$$

and

$$
u_i = \frac{r}{E'} \left\{ \frac{1 - v'}{1 - N_\phi} \sigma_c - \left(\frac{N_\phi^2 - 1}{2 N_\phi} - N_\phi + v' \right) \left(\frac{\sigma_c}{N_\phi - 1} + p_i \right) \left(\frac{r}{A} \right)^{N_\phi - 1} + \left(\frac{N_{\phi}^2 - 1}{2 N_\phi} \right) \left(\frac{\sigma_c}{N_\phi - 1} + p_i \right) \left(\frac{R_e}{A} \right)^{N_\phi - 1} \left(\frac{R_e}{r} \right)^{N_\phi + 1} \right\}.
$$
 (5)

When $R_e \le r \le \infty$: elastic state.

$$
\sigma_r = p_o - \frac{\sigma_c + p_o (N_\phi - 1)}{N_\phi + 1} \left(\frac{R_e}{r}\right)^2, \tag{6}
$$

$$
\sigma_{\theta} = p_o + \frac{\sigma_c + p_o \left(N_{\phi} - 1\right)}{N_{\phi} + 1} \left(\frac{R_e}{r}\right)^2 \tag{7}
$$

and

$$
u_i = \frac{r}{E'} \left\{ (1 - v') p_o + (1 + v') \left(\frac{\sigma_c + p_o (N_\phi - 1)}{N_\phi + 1} \right) \left(\frac{R_e}{r} \right)^2 \right\}, \quad (8)
$$

in which σ_r is the radial stress, σ_θ is the tangential stress, and u_i is the radial deformation on the tunnel wall. In the above equations, r is the radial distance from the center of opening, and p_i represents the pressure acting on the tunnel wall as it deforms; in addition,

$$
R_e = A \left\{ \frac{2}{N_\phi + 1} \left(\frac{\sigma_c + p_o (N_\phi - 1)}{\sigma_c + p_i (N_\phi - 1)} \right) \right\}^{\frac{1}{N_\phi - 1}},
$$
(9)

$$
N_{\phi} = \frac{1 + \sin \phi}{1 - \sin \phi},\tag{10}
$$

$$
E' = \frac{E}{1 - v^2} \tag{11}
$$

and

$$
v' = \frac{v}{1 - v}.\tag{12}
$$

Figure 1 shows the finite-element mesh of the discretized model for the considered problem; in this figure, r is the radial distance from the axisymmetric axis (which is the center of the opening) and z is the longitudinal direction of the opening. In the finite-element strip, an axisymmetrical condition with respect to z direction is considered. The deformation in z direction is also restrained in order to model the plane-strain condition in an effective and economic manner. The (tunnel) wall of the assumed opening is first subjected to an initial isotropic stress of 20.68 MPa. Then it is subsequently unloaded from 20.68 MPa to 0 MPa (stress free) step by step. Stress and deformation fields are calculated for each unloading stage. Figure 2 a presents the normalized deformation versus the inner normalized stress on

Fig. 1. Discretized model of verified problem of a circular opening in infinite space

the opening wall for this problem. The normalized stress (p_i/p_o) denotes the ratio of the stress on the tunnel wall at a certain stage to the initial stress. The normalized deformation is defined by $(u_i E)/(A p_o)$; where u_i is the deformation of the tunnel wall, E is the Young's modulus of rock mass, A is the initial half-width (i. e., the radius) of tunnel, and p_o is the initial vertical stress. Solid line in Figure 2 a represents the analytical solution, and the marked labels are the numerical solutions at various unloading stages. The numerical solution agrees with the analytical solution remarkably. Figure 2b displays the stress-state distribution along the radial direction of opening when the opening wall becomes stress-free. Solid lines in this figure are the analytical solution, while the marked labels are the numerical solutions. The numerical solutions of circumferential stress and radial stress both show good agreement with those of analytical solutions. From this example, the validity of the numerical analysis is clearly demonstrated.

Subsequently, parametric study based on numerical analysis is performed to investigate the effects of the initial stress ratio (of the horizontal stress to the vertical stress), K, and the shape of the tunnel's cross section

Fig. 2. Analytical (--) and numerical ($\circ \circ \circ$) solutions of verified problem: (A) ground **response curve, and (B) distribution of stresses**

on: (a) the plastic zone around a tunnel resulted from excavation, and (b) the ground response curve. In the parametric study, Mohr-Coulomb yield criterion and Hoek-Brown yield criterion are considered respectively, assuming associated flow rule and perfect plasticity when yielding of the rock mass occurs.

4. Parametric Study on Plastic Zone Resulted from Excavation

The distribution of the plastic zone around a tunnel depends on a number of factors. The effect of tunnel shape and anisotropy in the initial stress state on the developed plastic zone around an excavated tunnel will be examined by numerical analysis in this section.

Various shapes of tunnel cross-sections including circular, elliptical, rectangular, and horse-shoe shapes are considered. Figure 3 a-d represents the finite-element models (meshes) for the hypothetical tunnels of circular,

Fig. 3. Discretized models of (A) a hypothetical circular tunnel, (B) a hypothetical elliptical tunnel, (C) a hypothetical rectangular tunnel, and (D) a hypothetical horse-shoe shaped tunnel

elliptical, rectangular and horse-shoe shapes, respectively. In every case, assumed material parameters and the initial stress state are as follows.

Fig. 5. Plastic zones around a hypothetical elliptical tunnel when initial stress ratio equal to (A) 0.75, (B) 1.0, and (C) 1.5

4.1 Effect of Initial Stress Anisotropy on Plastic Zone

The effect of initial stress anisotropy on the plastic zone around a hypothetical tunnel is examined in this section. Various ratios of the horizontal initial stress to the vertical initial stress are considered for tunnel cross-sections, including circular and elliptical shapes.

Plastic Zone Around a Circular Tunnel

Figure 4a-c shows the plastic zones around a hypothetical circular tunnel when K (the initial lateral pressure ratio) is equal to 1.0, 1.5, and 2.0, respectively, while Mohr-Coulomb yield criterion and the associated flow rule are assumed.

Based on these results, it is noted that, for the circular tunnel, an axisymmetrical plastic zone, which looks like a circular ring, can exist only when the stress ratio, K equal to 1. As the stress ratio K departs from one, the plastic zone becomes more and more like an elliptical ring, and the region of the plastic zone expands in the direction of the small initial stress component.

Plastic Zone Around an Elliptical Tunnel

Figure $5a - c$ shows the plastic zones around a hypothetical elliptical tunnel when K (the initial lateral earth pressure ratio) is equal to 0.75 , 1.0, and 1.5, respectively, while the Mohr-Coulomb yield criterion is assumed.

The above results reveal that, for elliptical tunnels, when the direction of initial minor principal stress coincides with the shorter axis of ellipse, the plastic zone seems to be more evenly distributed. On the other hand, when the initial minor principal stress is in the direction of the longer axis of ellipse, an obvious unevenly distributed plastic zone appears, and expands in the direction of the initial minor principal stress.

4.2 Effect of Tunnel-shape on Plastic Zone

The effect of tunnel shape on the plastic zone around a hypothetical tunnel is investigated subsequently. Various shapes of tunnel cross-sections including circular, elliptical, rectangular, and horse-shoe shapes are considered. For all the cases considered here, identical stress states of isotropic initial stress of 20.68 MPa (stress ratio $K = 1$) are assumed.

Figure 6 a-d illustrates the plastic zone around circular, elliptical, rectangular, and horse-shoe shaped tunnels, respectively, when the Mohr-Coulomb yield criterion is considered. It is observed that an axisymmetrical plastic zone can exist only in the condition of a circular tunnel with initial hydrostatic stress state. For tunnels with corners, such as rectangular and horse-shoe shapes, relative limited thin plastic zone may appear near the corners, as those can be seen in Figure 6 c and d: this phenomenon was also reported by D a e m e n (1977). The limited plastic zone near a corner may be explained by the fact that the confining pressure near a corner is

relatively higher so that the yield strength of the rock mass near the corner is higher as well; consequently, the plastic zone near a corner is limited. In the case of a horse-shoe shaped tunnel, it can also be observed that the plastic zone in the tunnel roof is relatively thinner than that in the tunnel sidewall and in the tunnel floor due to the arch effect.

Fig. 6. Plastic zones around various shapes of hypothetical tunnels $(K = 1)$: (A) circular tun**nel, (B) elliptical tunnel, (C) rectangular tunnel, and (D) horse-shoe shaped tunnel**

The results based on the Hoek-Brown yield criterion, although not shown (in order to reduce repetition), are similar to those based on the Mohr-Coulomb yield criterion.

5. Characteristic-line Family of Rock Mass

A unique characteristic-line of rock mass only exists in such extreme conditions as those which have been discussed in the foregoing part of this paper. However, in most real and non-ideal in-situ conditions, a unique relationship between the pressure and deformation on the tunnel wall does not exist. As a matter of fact, the pressure and deformation of rock mass vary significantly along different locations on the tunnel wall, and are far different from the conditions considered by the ideal single-degree-of-freedom model (which is the basic assumption of an analytical solution).

To account for the multi-degree-of-freedom nature of the varying deformation distribution along tunnel wall during unloading process, a concept called "the characteristic-line family of rock mass" is used to describe the varying relations between normalized stress, (p_i/p_a) , and normalized deformation, $(u_i E)/(A p_0)$, at various points on the boundary of a tunnel in the course of gradual unloading due to excavation. The stresses at various points on the tunnel wall are first evaluated and are then gradually reduced proportionally to allow for the growth of deformations. The considered half-width of tunnel, A , in this study is defined as (a) the radius of a circular tunnel, or (b) the horizontal radius of an elliptical tunnel, or (c) half of the actual width of a rectangular tunnel, or (d) half of the largest horizontal distance between the two side-walls of a horse-shoe shaped tunnel.

The characteristic-line family of rock mass depends on the rock mass property and the tunnel's characteristics. For demonstrative purpose, a set of characteristic-line families of rock mass, with the same properties as those described in the last section for a variety of tunnelshapes, will appear in order.

Figure 7 a-c shows the characteristic-line families of rock mass for a hypothetical circular tunnel, assuming the Mohr-Coulomb yield criterion and $K = 1.0$, 1.5, and 2.0, respectively. Only when $K = 1.0$ will a unique characteristic-line family of rock mass exist. This figure clearly reveals that the characteristic-line family of rock mass largely depends on the initial lateral stress ratio, K, even for a circular tunnel. The deformations that occur at the roof and the sidewall of the tunnel can differ by more than 100 percent.

Figure 8 a-c shows the characteristic-line families of rock mass for a hypothetical elliptical tunnel, assuming Mohr-Coulomb yield criterion and $K = 0.75$, 1.0, and 1.5, respectively. It can be noted that large variation of deformation takes place along the tunnel wall. This figure also reveals that the characteristic-line family of rock mass largely depends on the initial lateral stress ratio, K , for an elliptical tunnel.

Fig. 7. Characteristic-line families of rock mass for a hypothetical circular tunnel when initial stress ratio equal to (A) 1.0, (B) 1.5, and (C) 2.0

Fig. 8. Characteristic-line families of rock mass for a hypothetical elliptical tunnel when initial stress ratio equal to (A) 0.75, (B) 1.0, and (C) $1.\overline{5}$

Figure 9 a and b shows the characteristic-line families of rock mass for a hypothetical rectangular tunnel and a hypothetical horse-shoe shaped tunnel, respectively, assuming the Mohr-Coulomb yield criterion and the stress ratio, $K = 1$. This figure also reveals significant variation of deformation along tunnel wall.

Fig: 9. Characteristic-line families of rock mass for (A) a hypothetical rectangular elliptical tunnel and (B) a hypothetical horse-shoe shaped tunnel

Again, the results based on the Hoek-Brown yield criterion (not shown to avoid repetition) are very similar to those based on the Mohr-Coulomb yield criterion.

6. Summary and Conclusions

Due to the unloading and stress redistribution, a plastic zone may develop and propagate in the process of tunnel excavation. An evenly distributed plastic zone rarely exists except for a circular tunnel in an isotropic initial stress condition. The distribution of plastic zone around a tunnel depends on various factors. Effects of the anisotropy in initial stress state and the shape of tunnel's cross section on the plastic zone around an excavated tunnel have been examined by numerical analysis. These numerical solutions are obtained from elasto-plastic finite-element analysis which assumes the yield criterion of rock mass to be the Hoek-Brown criterion of Mohr-Coulomb criteria. For a circular tunnel, an axisymmetrical plastic zone can exist only when the stress ratio, K , equals to 1. As the initial stress state differs from hydrostatic, the plastic zone expands in the direction along which the initial stress component is smaller. For an elliptical shape tunnel, when the initial minor principal stress is in the direction of the longer axis of ellipse, apparent uneven distribution of plastic zone appears and expands in the direction of the initial minor principal stress.

The determination of the required support line for rock mass plays an tremendously important role in designing tunnel-support based on convergence-confinement method. A unique characteristic-line of rock mass or required support line for rock mass only exists for extremely ideal conditions. In most real cases, large variation of deformation may take place along the tunnel wall. To account for the multi-degree-of-freedom nature of deformation distribution along the tunnel wall varying along with the unloading process, a concept called "the characteristic-line family of rock mass" is proposed to describe the varying relations between normalized stress and normalized deformation at various points on the boundary of a tunnel in the course of gradual unloading, due to excavation. The characteristic-line family of an opening in rock mass has been shown to depend on the property of rock mass and the characteristics of a tunnel.

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