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**Rock Mechanics and Rock Engineering**


# Technical Note

# New Description of the Shear Strength for Rock Joints

By

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## 1. Introduction

The failure criterion of a rock discontinuity is usually nonlinear. The angle of shearing resistance decreases as the level of normal effective stresses increases. At low normal stresses, motion takes place by climbing up the ridges and asperities which compose the rough face of the discontinuity. The result is a high apparent frictional resistance due to this effect of dilatancy. At higher normal stresses failure occurs in a very complex manner by ploughing, by shearing through the ridges and fracturing of the rock material adjacent to the contact. The contribution of dilatancy to the total shearing strength gradually decreases by the rise of stress level. If the normal effective stress is large enough, all dilatancy would be suppressed and the rock discontinuity would shear at nearly a constant volume.

Empirical strength criteria in the form of power or logarithmic relationship have been reported in literature. The main shortcoming of these proposals is the validity in the limited stress range and the lack of physical meaning. It is the author's opinion (Maksimović, 1979; 1988; 1989 a, b, c), that the function of hyperbolic type offers excellent possibilities for simple description of the nonlinear failure envelope within the widest possible range of stresses from zero to practical infinity for most non-cemented soils. In this note it will be shown that the same or similar failure law can be derived for rock discontinuities from a mechanical analog model of rock joint with non-rigid asperities.

## 2. Mechanical Model for the Nonlinear Failure Envelope

The expression, originally proposed for granular materials by Newland and Allely (1957), and for rock discontinuities by Patton (1966) and Goldstein et al. (1966), for relating the angle of shearing resistance, dilatancy and friction at the discontinuity between two planes with surface irregularities may be written as

$$\phi = \phi_B + \psi \tag{1}$$

where $\phi_B$ is a physical friction or "the basic angle of friction" and $\psi$ the angle of dilatancy, or the angle between displacement vector and the shearing plane.

It is assumed that the rock discontinuity of certain small scale can be described by the mechanical model with rigid zones sliding upwards at $\psi$ over the slipping zones, separated by series of simple "pneumatic" mechanisms. These simulate the non-rigid behavior of asperities, implying the positive dilatancy only, as shown in Fig. 1. The mechanism in the zone of the discontinuity consists of the element unit, of the unit size, the simulated surface of the asperity linked to the dash pot governed by the Boyle's law.

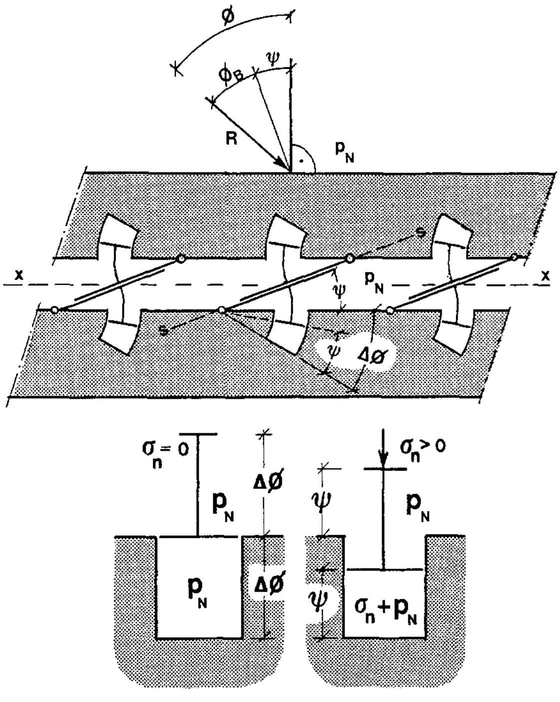



**Fig. 1.** The mechanical analog model of dilatancy

The mechanical analog model is such that, in an unstressed state, the angle between the planes $x-x$ and $s-s$ is equal to a value $\Delta\phi$ at the model ambient pressure $p_N$, which also exists in a curved chamber closed by a rotating piston. When the normal stress increases to a value of $\sigma_n$, the distance between two blocks decreases causing the rotation of planes $s-s$

with respect to $x-x$, which, connected to a piston, increases the value of the pressure by the same value of the stress in such a manner that for some angle $\psi$ between planes $x-x$ and $s-s$ the pressure increases to a value $p_N + \sigma_n$.

In terms of the model variables shown in Fig. 1, Boyle's law applied to a volume in a chamber segment can be written as:

$$p_N \Delta\phi = (p_N + \sigma_n)\, \psi. \tag{2}$$

Solving (2) for $\psi$:

$$\psi = p_N \Delta\phi/(p_N + \sigma_n) = \Delta\phi/(1 + \sigma_n/p_N). \tag{3}$$

Substituting (3) into (1) a very simple form of the model equation for the angle of the shearing resistance is obtained as:

$$\phi = \phi_B + \Delta\phi/(1 + \sigma_n/p_N) \tag{4}$$

and the shearing strength of the rock discontinuity is

$$\tau_f = \sigma_n \tan\left[\phi_B + \Delta\phi/(1 + \sigma_n/p_N)\right]. \tag{5}$$

The meaning of each parameter is shown in Fig. 2.

$\phi_B$ is the "basic angle of friction", the angle of the shearing ressistance mobilized at high normal stress levels at which all dilatancy effects are suppressed, as all the asperities are sheared off forming the smooth shearing plane. This angle could be approximately equal to the angle of the physical friction between mineral grains.

$\Delta\phi$ the "joint roughness angle", reflects the surface roughness of the discontinuity. The associated dilatancy effects at zero stress level and it can be described as the "angle of maximum dilatancy" which occurs on undamaged rugged surface.

$p_N$ "the median angle pressure" is equal to the value of the normal stress at which the contribution of dilation is equal to one half of the angle of dilatancy for the zero normal stress. It mainly reflects the deformability and the resistance of the asperities against crushing.

The function of the stress level, Eq. (3), which describes the rate of decrease of the second term in Eq. (4) as the stress level increases, has a property that decreases asymptotically towards zero with increasing normal stress. The total angle of shearing resistance tends towards the lower constant limiting value defined as a "basic angle of friction" $\phi_B$. The initial angle $\phi_0$, a tangent to the failure envelope in the origin, is simply the sum of the basic angle and the inclination of surface asperities ($\phi_B + \Delta\phi$) as shown in Fig. 2.

Expressions (4) and (5) are dimensionally consistent. Values of the angles $\phi_B$ and $\Delta\phi$ can be taken in degrees or radians, and the description of stresses is in the same units as the "median angle pressure" $p_N$.

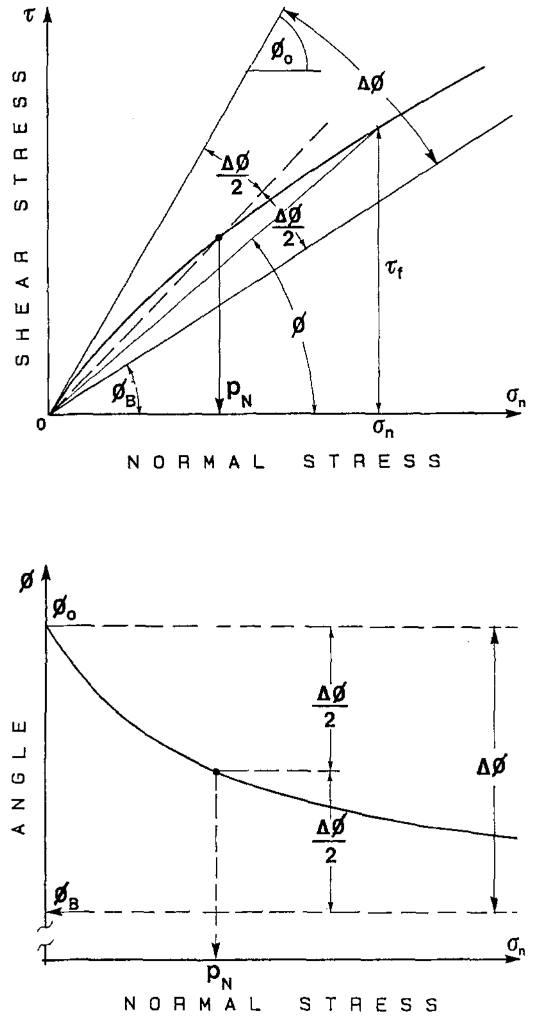



**Fig. 2.** Definition of parameters of the nonlinear failure envelope

## 3. Examples and Justification

*Example 1. Data by Barla et al. (1985)*

The results of the direct shear tests on sandstone discontinuity are evaluated. Six data points are processed, using the least square fit for the Eq. (4) with the method described in the Appendix, and a very good approximation with the proposed failure law obtained, as shown in Fig. 3.

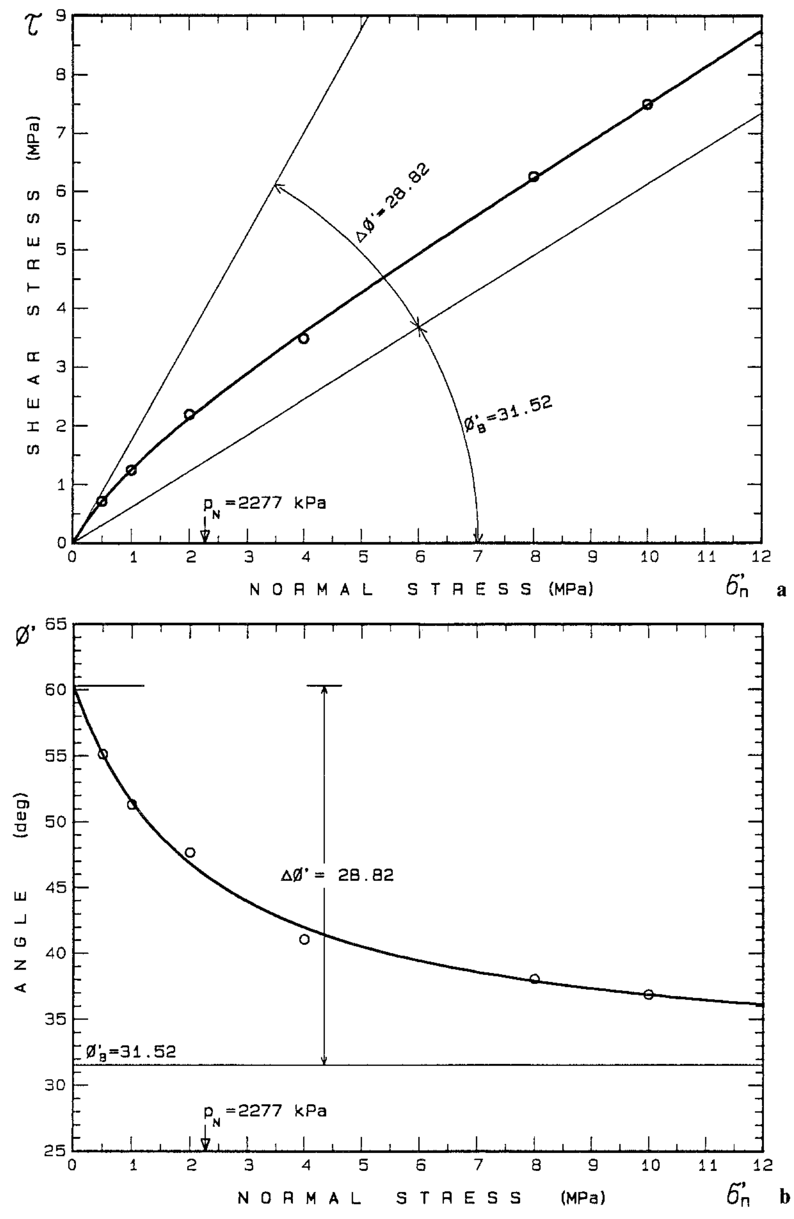
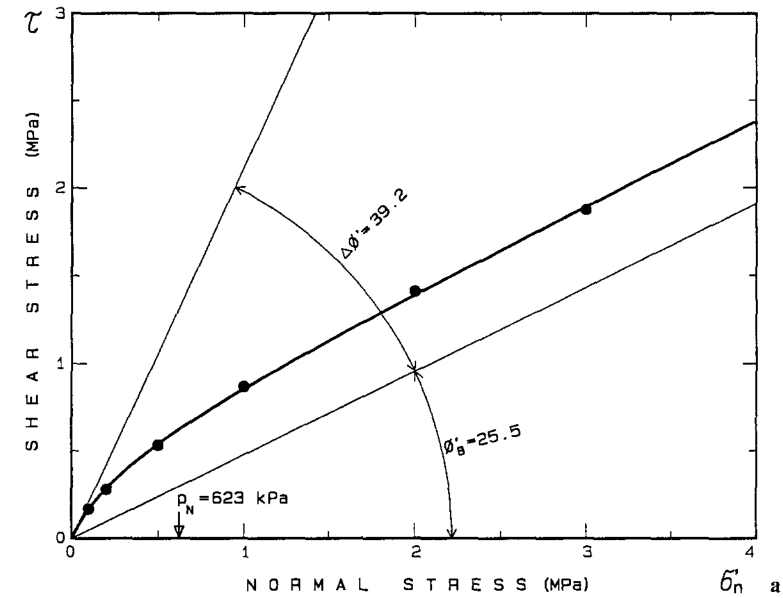



**Fig. 3.** Sandstone discontinuity, data by Barla et al. (1985).
**a** failure envelope, **b** secant angle of the shearing resistance versus normal stress

*Example 2. Data by Nilsen (1985)*

To compare the proposed failure criteria with the power law, an example of interpretation of test results obtained on foliation joints in micaschist, with a portable shear machine will be presented. The failure law was described by power law $\tau_f = 0.87\ \sigma^{0.70}_n$. Applying the regression analysis for



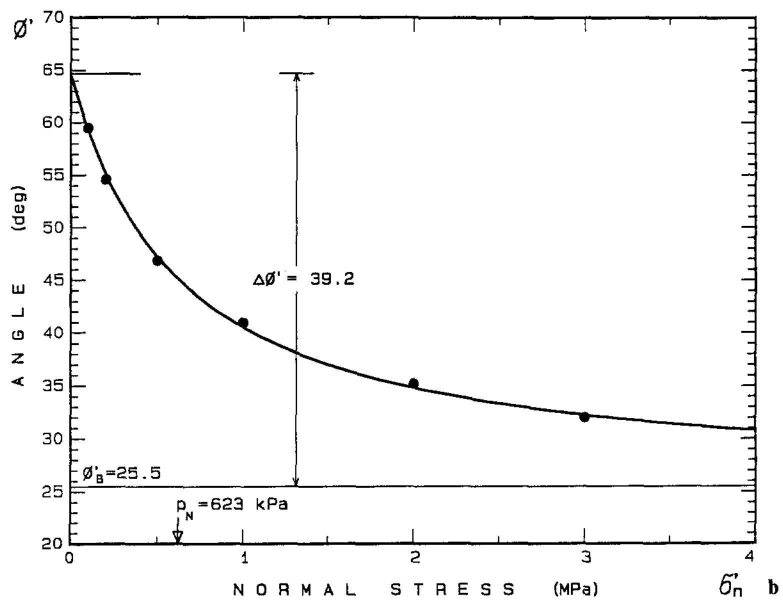




**Fig. 4.** Foliation joints in micaschist, data by Nilson (1985)
**a** failure envelope, **b** secant angle of the shearing resistance versus normal stress

a proposed hyperbolic variation of the angle of shearing resistance using six points computed from the above given expression, taking $\phi = \arctan(\tau_f/\sigma_n)$, failure envelope and parameters are obtained and presented in Fig. 4. As can be seen, the agreement is remarkable.

## 4. Concluding Remarks

In a semi-logarithmic plot (Fig. 5), it can be noted that very good approximation could be also obtained by using the linear relationship in the rather wide interval. Extrapolation of the straight line to small stress range would overestimate and extrapolation in the very high stress range would underestimate the angle of the shearing resistance.

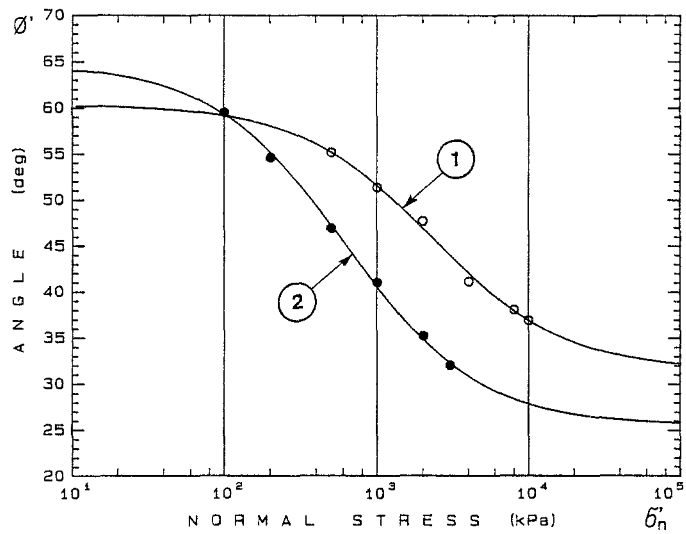



**Fig. 5.** Semi-log plot, secant angle versus normal stress.
*1* Sandstone discontinuity, data by Barla et al. (1985), *2* foliation joints in micaschist, data by Nilsen (1985)

The power law in the form $\tau_f = A\,\sigma^B$ described the nonlinear failure envelope with parameters that depend on units and have no physical meaning. This law defines a vertical tangent to the failure envelope in the origin, suggests the angle $\phi_0 = 90°$, which makes it unusable in dealing with dilatancy at zero stress level. It lacks a reasonable asymptote and is therefore unable to define the basic angle of friction.

Both examples shown in figures, which could be described by a logarithmic and power law, show excellent conformity with the simple form of the variation of the shearing resistance angle derived from the analogical model. The proposed model has significant advantages; parameters have physical meaning, law is valid from zero to infinity, and it is simpler from the mathematical point of view, as it has only a few divisions and additions.

## Appendix

### *Determination of Parameters from Experimental Data*

In order to cover the whole range of stresses with best accuracy, it is desirable to select normal stress levels for testing so that the angles of the shearing resistance span the range from $\phi_B$ to $\phi_0$. Spacing of data points should be such that the whole range of $\Delta\phi$ is filled with data in nearly equal intervals. The suggested normal stress levels, which will provide proper results both in the range of the high curvature, as well as in the range where the envelope is almost linear are expressed in terms of the median angle normal pressure, $p_N$ as shown in Table 1. Still, as the value of the median angle pressure $p_N$ is not known in advance, the initial estimate might be taken as $\sigma_C/35$ as a preliminary guess, where $\sigma_C$ is the compressive strength. For the weathered joints, $p_N$ value can be significantly smaller.

**Table 1.** Suggested stress levels for testing

| Number of stress levels | Good stress levels for testing $\sigma_n/p_N$ Stress level number | | | | | | |
|---|---|---|---|---|---|---|---|
| | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
| 5 | $\leq 1/5$ | 1/2 | 1 | 2 | $\geq 5$ | | |
| 6 | $\leq 1/6$ | 2/5 | 3/4 | 4/3 | 5/2 | $\geq 6$ | |
| 7 | $\leq 1/7$ | 1/3 | 3/5 | 1 | 5/3 | 3 | $\geq 7$ |

Two procedures for derivation of parameters will be outlined here under separate headings. The choice of the procedure will depend on the number of data points.

Procedure No. 1. The regression analysis:
More than three points

$$\sigma_{n,i}, \phi_i \qquad i = 1, N, \ N > 3$$

For the derivation of the proposed parameter from the results of the direct shear tests, in a form of $N$ pairs $\sigma_{n,i}$ and $\phi_i$, the Eq. (4) can be rewritten in the form

$$a + b\,\sigma_n = \sigma_n/(\phi_0 - \phi) \tag{1-1}$$

where

$$b = 1/\Delta\phi, \ a = p_N/\Delta\phi, \text{ and } \phi_0 = \phi_B + \Delta\phi. \tag{1-2}$$

Coefficients $a$ and $b$ in a straight line Eq. (1-1) are obtained from solution of the normal regression equations for the set of $N$ data points:

$$b = \frac{N\Sigma\sigma_{n,i}(\phi_0 - \phi_i) - (\Sigma\sigma_{n,i})(\Sigma(\phi_0 - \phi_i))}{N\Sigma\sigma_{n,i}^2 - (\Sigma\sigma_{n,i})^2} \tag{1-3}$$

$$a = \frac{\Sigma(\phi_0 - \phi_i)}{N} - b\frac{\Sigma\sigma_{n,i}}{N}. \tag{1-4}$$

The solution to the problem is the value of $\phi_0$ that gives the smallest norm of error in the regression analysis.

After calculating $a$ and $b$, parameters of the nonlinear envelope can be derived by using relationships (1-2).

Procedure No. 2. The perfect fit:
Three points only,

$$\sigma_{n,i}, \phi_i, \qquad i = 1, 2, 3.$$

If results for only three stress levels are considered, (this being the least number of results required for the derivation of parameters), the system of three equations can be solved for unknown parameters. The solution of three simultaneous equations for unknown parameters of the nonlinear envelope is:

$$\phi_B = \frac{Q_{1,2} - Q_{2,3}}{P_{1,2} - P_{2,3}} \tag{2-1}$$

where

$$Q_{1,2} = (\phi_1\sigma_{n,1} - \phi_2\sigma_{n,2})(\phi_3 - \phi_2) \tag{2-1-a}$$

$$Q_{2,3} = (\phi_2\sigma_{n,2} - \phi_3\sigma_{n,3})(\phi_2 - \phi_1) \tag{2-1-b}$$

$$P_{1,2} = (\sigma_{n,1} - \sigma_{n,2})(\phi_2 - \phi_3) \tag{2-1-c}$$

$$\mathrm{P}_{2,3} = (\sigma_{n,2} - \sigma_{n,3})(\phi_2 - \phi_1), \tag{2-1-d}$$

and

$$p_N = \frac{R_{1,2} - R_{2,3}}{P_{1,2} - P_{2,3}} \tag{2-2}$$

where

$$R_{1,2} = (\phi_2\sigma_{n,2} - \phi_3\sigma_{n,3})(\sigma_{n,1} - \sigma_{n,2}) \tag{2-2-a}$$

$$R_{2,3} = (\phi_1\sigma_{n,1} - \phi_2\sigma_{n,2})(\sigma_{n,2} - \sigma_{n,3}), \tag{2-2-b}$$

and the third unknown can be computed, for example, from

$$\Delta\phi = (\phi_1 - \phi_B)(1 + \sigma_{n,1}/p_N). \tag{2-3}$$

This method is called "the perfect fit method" as the error is equal to zero for given three data points. To achieve the highest accuracy in the wide range of stress, the data points should be widely spaced. One stress level should be in the vicinity of $p_N$ value, the second should be preferably less then $p_N/3$, and the third larger then $3\,p_N$.

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