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AMPLITUDE AND POLARIZATION EFFECTS IN SELF-FOCUSING OF LASER RADIATION IN MEDIA WITH SPATIAL DISPERSION OF NONLINEARITY

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The phenomenological theory of the propagation of beams with Gaussian transverse intensity profiles in isotropic media with spatial dispersion of nonlinearity is developed in the aberrationless approximation, using the method of moments. The self-focusing threshold is determined and different propagation regimes are identified in the case of elliptically polarized incident radiation. The latter are illustrated by analytically and numerically obtained dependences of intensity, polarization, degree of ellipticity, and rotation angle of the polarization ellipse of the output light along the beam axis as functions of propagation coordinate.

1. More than 25 years have passed since the possibility of self-focusing of laser radiation was first demonstrated [1]. Since this time a large number of theoretical and experimental papers have been published, dedicated to the study of various aspects of this phenomenon. The results of these investigations have been reflected in review articles [2-6]. However, practically all of these works considered only the cases of linear and circular polarization of the incident radiation, which was assumed to be constant during the process of propagation, and only a few contain results of experiments or numerical investigation of self-focusing of elliptically polarized light [7-9].

Recently, the relatively recently [13] predicted phenomenon of nonlinear optical activity, which is, in essence, the analog (in the case of strong light fields) of the classical effect of natural optical activity, has been experimentally detected [10] and theoretically investigated in detail [11, 12]. It is associated, in particular, with the existence of spatial dispersion of nonlinearity – the nonlocality of the optical response of the medium to an external influence, and takes place in many crystals, isotropic liquids, and liquid crystals. The phenomenological theory of this phenomenon which has been developed at present [11-13] does not take into account the spatial boundedness of the laser radiation. The rotation angle of the polarization ellipse in this case grows linearly with increasing intensity of the incident wave and the length of the sample. This permits us to assume that self-focusing of laser radiation in media with spatial dispersion mus be accompanied by strong variation of polarization of the propagating wave, i.e., that growth of intensity along the beam axis, associated with variation of beamwidth, will lead to a sudden rotation of the polarization ellipse.

In the present paper a phenomenological theory of the propagation of laser radiation in nonabsorbing, isotropic media with spatial dispersion is developed, which takes into account the finite width of the light beam. For arbitrary values of the parameters of a non-

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linear, gyrotropic medium and the linearly polarized radiation incident on the sample, analytic dependences of the nondimensional widths of the partial beams on the propagation coordinate are obtained, which are easily subject to analysis and permit the intensity and degree of ellipticity of the transmitted light to be determined. The rotation angle of the polarization ellipse is also found. All of the characteristics in the case of elliptical polarization of the light upon entrance to the medium were determined by numerical methods. The results obtained are valid if the duration of the incident pulse is many times greater than the relaxation time of the nonlinearity and demonstrate the influence of self-focusing effects on nonlinear optical activity.

2. Amplitude and polarization self-action of light beams in a isotropic, nonabsorbing medium with weak spatial dispersion of cubic nonlinearity is described by a system of parabolic equations for slowly varying amplitudes of circularly polarized waves $A_\pm = A_x \pm iA_y$

$$\frac{dA_\pm}{dz_1} + \frac{i}{2k}\Delta_\perp A_\pm = i\left\{\pm \rho_0 - \left(\frac{\sigma_1}{2} \mp \rho_1\right)|A_\pm|^2 - \left(\frac{\sigma_1}{2} + \sigma_2\right)|A_\mp|^2\right\}A_\pm , \tag{1}$$

which differs from that considered in [12] by the presence of the transverse Laplacian $\Delta_\perp = \partial^2/\partial x^2 + \partial^2/\partial y^2$. Here $k = \omega\sqrt{\varepsilon}/c$ is the modulus of the wave vector parallel to the axis z_1, $\sigma_{1,2} = 2\pi\omega^2\chi_{1,2}/kc^2$, $\chi_{1,2}$ are associated with the nonzero components of the cubic nonlinearity tensor (in the absence of frequency dispersion, $\chi_1 = 2\chi_2$), $\rho_{0,1} = 2\pi\omega^2\gamma_{0,1}/c^2$, and $\gamma_{0,1}$ are the pseudoscalar constants of linear and nonlinear gyration. In ordinary optically active media [12] $|\rho_1/\sigma_1| \ll 1$.

The propagating radiation is completely characterized by intensity

$$\bar{I}(x, y, z_1) = (|A_+|^2 + |A_-|^2)/2 , \tag{2}$$

degree of ellipticity

$$\bar{M}(x, y, z_1) = (|A_+|^2 - |A_-|^2)/(|A_+|^2 + |A_-|^2) \tag{3}$$

and the rotation angle of the polarization ellipse

$$\bar{\Psi}(x, y, z_1) = \operatorname{Arg}(A_+A_-^*)/2 .$$

Assuming that a beam of Gaussian shape

$$\bar{I}(r, 0) = (E_0^2/2)\exp(-2r^2/r_0^2), \quad r^2 = x^2 + y^2, \quad E_0^2 = E_+^2 + E_-^2 \tag{4}$$

with ellipticity $M_0 = \bar{M}(r, 0) = (E_+^2 - E_-^2)E_0^2$, where $E_\pm = |A_\pm(0, 0, 0)|$, is incident at the boundary of the medium $z_1 = 0$, we will seek a similar solution of Eq. (1) in the following form:

$$A_\pm = \frac{E_\pm}{f_\pm}\exp\left[-\frac{r^2}{r_0^2 f_\pm^2} - \frac{ikr^2}{2f_\pm}\frac{df_\pm}{dz_1} + i\varphi_\pm(z_1) \pm i\rho_0 z_1\right]. \tag{5}$$

The nondimensinal widths of the partial beams $f_\pm(z_1)$ satisfy the boundary conditions $f_\pm(0) = 1$, $f_\pm'(0) = 0$. The lack of collimation of the incident beam can be easily taken into account in analogy with [14]. We find the equation for the nonlinear rotation angle of the polarization ellipse in the center of the beam $\Psi(z_1) = \bar{\Psi}(0, 0, z_1) - \rho_0 z_1$ by substituting Eq. (5) into Eq. (1) and equating the terms in the expansion of the latter in powers of r:

$$\frac{d\Psi}{dz} = \frac{1}{2}\left(\frac{1}{f_+^2} - \frac{1}{f_-^2}\right) + \frac{\rho_1 L}{2}\left(\frac{E_+^2}{f_+^2} + \frac{E_-^2}{f_-^2}\right) + \frac{\sigma_2 L}{2}\left(\frac{E_+^2}{f_+^2} - \frac{E_-^2}{f_-^2}\right). \tag{6}$$

Here $L = kr_0^2/2$ and $z = z_1/L$. Intensity (2) and degree of ellipticity (3) along the beam axis, taking Eq. (5) into account, have the forms

$$I(z) = \bar{I}(0, 0, z) = E_+^2/2f_+^2 + E_-^2/2f_-^2 ; \tag{7}$$

$$M(z) = \bar{M}(0, 0, z) = (E_+^2/f_+^2 - E_-^2/f_-^2)/2I(z) \tag{8}$$

respectively.

For analysis of the dependences of Ψ, I, and M on the parameters of the radiation and the medium, it remains to obtain equations for $f_\pm$. We multiply Eq. (1) respectively by $\partial A_\pm^*/\partial z_1$ and take the sum of these expressions with their complex conjugates. Substituting Eq. (5) into the thusly found equalities and integrating the latter over the transverse coordinates (i.e., using the methods of moments [15]), we obtain the following system of equations for $f_\pm$:

$$\frac{d^2 f_\pm}{dz^2} = \frac{m_\pm}{f_\pm^3} - \frac{n(1 \mp M_0) f_\pm}{2(f_+^2 + f_-^2)^2}, \tag{9}$$

where $m_\pm = 1 - (\sigma_1 \mp 2\rho_1)LE_\pm^2/4$ and $n = (\sigma_1 + 2\sigma_2)LE_0^2$. Note that Eq. (9) possesses symmetry properties. Its solution does not change when ρ_1, $f_\pm$, and M_0 are exchanged, respectively, for $-\rho_1$, $f_\mp$, and $-M_0$. This allows us to restrict our consideration to the case $\rho_1 > 0$.

It is evident from Eq. (9) that $f_+(z) \neq f_-(z)$ for arbitrary values of the parameters of the radiation and the medium. As a result, already in the approximation of the similar solution for $A_\pm$ the transverse distribution of the total intensity loses its Gaussian shape in the process of propagation. Use of the integral definition of beamwidth

$$r_0^2 f_e^2(z) = \int \bar{I}(r,z) r^3 dr / \int \bar{I}(r,z) r dr \tag{10}$$

in this case can lead to invalid results. Thus, for exmaple, if $f_+ \gg f_-$, but $|M_0| \ll 1$, then $f_i \sim f_+$. But close to the beam axis, where, especially, our treatment also has meaning, A_+ gives a practically constant background, and the narrower beam actually determines the width. It is advantageous to use a second definition for the effective beamwidth $f_e^2 = E_0^2/2I(z)$. In accord with this, f_e is the dimension to which the light beam incident on the sample should be focused (or defocused), while still preserving its Gaussian shape, in order to provide the intensity along the axis which the partial beams give when taken together. Such an approach gives reasonable results for $f_\pm \gg f_\mp$ as well as for $f_+ \sim f_-$. In accord with the definition of f_e, we will understand by self-focusing unbounded growth of $I(z)$.

Spatial dispersion of nonlinearity, and also ellipticity of the incident radiation, substantially influence the amplitude and polarization characteristics of the propagating light and the effective beamwidth f_e. Thus, for example, ρ_1 gives a quite complicated contribution to all three terms of Eq. (6). For comparison we recall [12] that in the case of propagation of plane waves in the media under consideration $\bar{M}$ and I remain constant, and $\Psi = (\rho_1 + \sigma_2 M_0)E^2 z_1/2$.

3. If the radiation incident on the sample is circularly polarized ($E_- = 0$ or $E_+ = 0$), then its polarization, as can be seen from Eqs. (1) and (9), is conserved during propagation. The nondimensional width of the laser beam both in this case and the other case varies according to the law $f_\pm^2 = 1 + m_\pm z^2$. For $\sigma_1 > 0$ the beam is focused if its nondimensional power $E_\pm^2\sigma_1 L/2$ exceeds the threshold value $2\sigma_1/(\sigma_1 \mp 2\rho_1)$, and for $\sigma_1 < 0$ defocusing takes place for arbitrary parameters of the incident light. Thus, for $|M_0| = 1$, spatial dispersion does not have any effect on the qualitative picture of self-action; however, it varies the threshold conditions differently for oppositely oriented circular polarizations.

In the general case $|M_0| \neq 1$, system (9) has two integrals

$$(1 + M_0) f_+^2 + (1 - M_0) f_-^2 = Cz^2 + 2 = \Phi(z, C); \tag{11}$$

$$(1 + M_0)(df_+/dz)^2 + (1 - M_0)(df_-/dz)^2 + m_+(1 + M_0)/f_+^2 + m_-(1 - M_0)/f_-^2 - n(1 - M_0^2)/2(f_+^2 + f_-^2) = C, \tag{12}$$

where the constant C depends on the ellipticity of the incident light

$$C(M_0) = m_+(1+M_0) + m_-(1-M_0) - n(1-M_0^2)/4.$$

Using these two integrals and transforming to new variables $u_\pm = (1 + M_0)f_\pm^2/\Phi(z, C)$, one can find (for arbitrary values of the parameters of the radiation and the medium) analytic dependences of the nondimensional widths of the partial beams $f_\pm$ on the propagation coordinate. The latter are expressed in terms of elliptical integrals (including also those of the third kind). The cumbersomeness, and also the abundance of partial cases, render the analytic solutions for $f_\pm$ in their general form not particularly revealing. We display them for the case, which is of practical importance, of linearly polarized light ($M_0 = 0$) incident to the medium (see Table 1). Given specific values of the nondimensional power of the incident radiation $P = |\sigma_1|LE_0^2/2$ and σ_1, and choosing the appropriate case from the far left column of the table, we find $f_\pm^2$, the normalized intensity $I_n = |\sigma_1|LI$ (and, consequently, also the effective width f_e) and the ellipticity M along the beam axis. Dependence of the generalized coordinate η on z for various values of $\sigma_{1,2}$ and P is given in the far right column of the table. The following notation has been used in the table:

$$C_0 = C(0), \quad \Phi_0(z) = \Phi(z, C_0), \quad a = \mathrm{sign}(\sigma_1)P - 4.$$

TABLE 1. Analytic Solution for $f_{\pm}^2$, I_n, and M in the Case of Incidence of Linearly Polarized Radiation into a Medium with Spatial Dispersion of Nonlinearity

P, σ_1	$f_{\pm}^2, I_n, M$	$P, \sigma_{1,2}$	$\eta(z)$
$\sigma_1>0$, $P>4$	$f_{\pm}^2=\left\{1\pm\frac{2\rho_1 P}{\sigma_1 a}\mathrm{sh}^2\left(\frac{\sqrt{a}\eta}{2}\right)\right\}\Phi_0/2$ $I_n=\frac{2P}{\left\{1-\frac{4\rho_1^2P^2}{\sigma_1^2a^2}\,{}^4\left(\frac{\sqrt{a}\eta}{2}\right)\right\}\Phi_0}$ $M=-2\rho_1 P\mathrm{sh}^2(\sqrt{a}\eta/2)/\sigma_1 a$	$\sigma_1+\sigma_2<0$ or $\sigma_1+\sigma_2>0$, $\frac{P(\sigma_1+\sigma_2)}{\lvert\sigma_1\rvert}<2$	$\eta=\sqrt{2/C_0}\times$ $\times\,\mathrm{arctg}(\sqrt{C_0}\times$ $\times z/\sqrt{2})$
$\sigma_1>0$, $P=4$	$f_{\pm}^2=\{1\pm 2\rho_1\eta^2/\sigma_1\}\Phi_0/2$ $I_n=2P/((1-4\rho_1^2\eta^4/\sigma_1^2)\Phi_0)$ $M=-2\rho_1\eta^2/\sigma_1$	$\sigma_1+\sigma_2>0$, $\frac{P(\sigma_1+\sigma_2)}{\lvert\sigma_1\rvert}=2$	$\eta=z$
$\sigma_1<0$ or $\sigma_1>0$, $P<4$	$f_{\pm}^2=\left\{1\mp\frac{2\rho_1P}{\lvert\sigma_1\rvert a}\sin^2\left(\frac{\sqrt{-a}\eta}{2}\right)\right\}\Phi_0/2$ $I_n=\frac{2P}{\left\{1-\frac{4\rho_1^2P^2}{\sigma_1^2a^2}\sin^4\left(\frac{\sqrt{-a}}{2}\eta\right)\right\}\Phi_0}$ $M=2\rho_1P\sin^2(\sqrt{-a}\,\eta/2)/\lvert\sigma_1\rvert a$	$\sigma_1+\sigma_2>0$, $\frac{P(\sigma_1+\sigma_2)}{\lvert\sigma_1\rvert}>2$	$\eta=\sqrt{2/C_0}\times$ $\times\,\mathrm{Arth}(\sqrt{C_0}\times$ $\times z/\sqrt{2})$

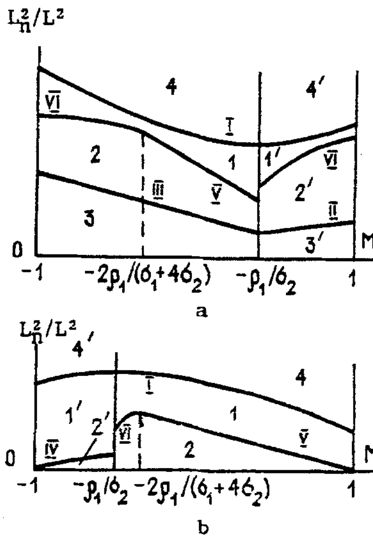

Fig. 1

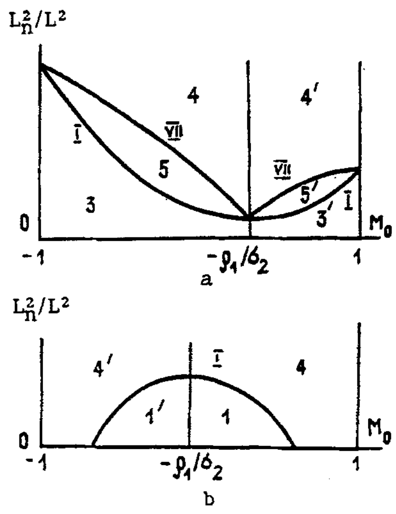

Fig. 2

Fig. 1. Regions of values of the radiation parameters (M_0 and $(L_n/L)^2$) corresponding to various regimes of beam propagation: a) $\sigma_1 > 0$, $\sigma_2 < 0$, $\sigma_1 + 2\sigma_2 > 0$, $4\sigma_2 + \sigma_1 > 2\rho_1$; b) $\sigma_{1,2} > 0$.

Fig. 2. Regions of values of the radiation parameters (M_0 and $(L_n/L)^2$) corresponding to various regimes of beam propagation: a) $\sigma_1 > 0$, $\sigma_1 + 2\sigma_2 < 0$; b) $\sigma_1 + 2\sigma_2 > 0$, $\sigma_1 < 0$.

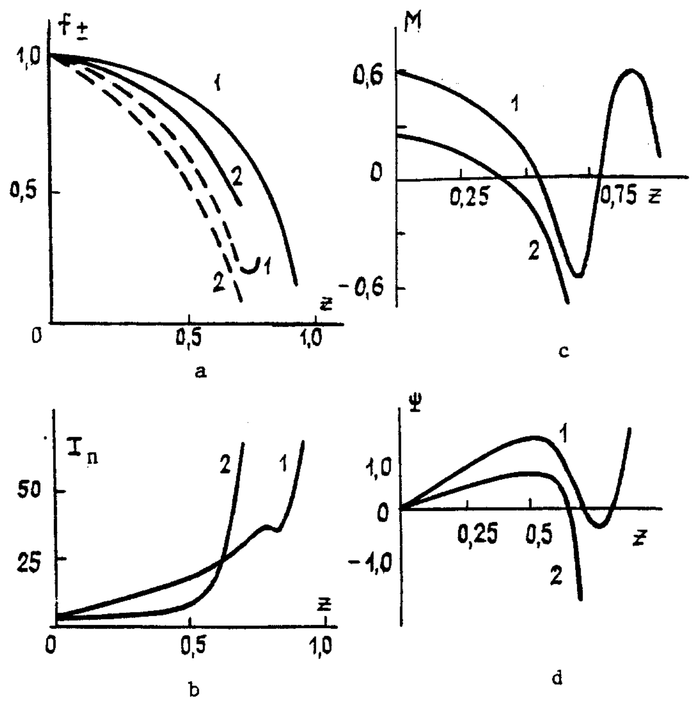

Fig. 3. Dependences of the parameters of the output radiation on the length of the nonlinear, gyrotropic medium (regions 1 and 2).

Substituting $f_\pm$ in Eq. (6), we obtain (after integration) the following expression for the rotation angle of the polarization ellipse along the beam axis:

$$\Psi(z) = \frac{1}{2\sqrt{a}}\left[\frac{\beta_+}{\sqrt{1-\alpha}}\,\mathrm{Arth}\left(\sqrt{1-\alpha}\,\mathrm{th}\left(\frac{\sqrt{a}}{2}\eta\right)\right) - \frac{\beta_-}{\sqrt{1+\alpha}}\,\mathrm{Arth}\left(\sqrt{1+\alpha}\,\mathrm{th}\left(\frac{\sqrt{a}}{2}\eta\right)\right)\right], \quad (13)$$

where $\alpha = 2\rho_1 P/|\sigma_1|a$ and $\beta_\pm = 1 + (\sigma_2 \pm \rho_1)P/|\sigma_1|$. In writting out Eq. (13), use has been made of the identity [16] $\mathrm{th}(ix) = i\tan x$, $\arctan(ix) = i\,\mathrm{Arth}(x)$.

Not dwelling in detail here on the threshold conditions, which are presented below for the general case of elliptical polarization of the incident light, we will consider by way of an example typical regimes in the absence of frequency dispersion ($\sigma_1 = 2\sigma_2$). In a defocusing medium ($\sigma_1 < 0$) $M(z)$ varies monotonically with increasing z, tending toward the constant value $M(z\to\infty) = -2\rho_1 P\sin^2(\pi\sqrt{P+4}/2\sqrt{3P+4})\,(|\sigma_1|\,(P+4))^{-1}$. In a focusing medium ($\sigma_1 > 0$) oscillatory as well as monotonic variation is possible along the beam axis. The former takes place only if $P < 4$, whereby the character of the oscillations is different for $P < 4/3$ and $P > 4/3$.

For $M_0 \neq 0$ it is more convenient to solve Eq. (9) numerically, at the same time finding $\Psi(z)$ with the help of Eq. (6).

4. Before displaying typical dependences of $I_n(z)$, $M(z)$, $\Psi(z)$, and $f_\pm(z)$, found by numerical methods, we will subdivide the space of radiation and medium parameters into regions with monotypical character of beam propagation.

Analysis of the equations for $u_\pm$, which are easily obtained from Eq. (9), with the help of integrals (11) and (12) shows that the main features of variation of the parameters of the propagating radiation (oscillatory or monotonic character of dependence of $M(z)$, presence of self-focusing or its absence), as a rule, are determined by the signs of the derivatives $d^2f_\pm/dz^2$ as $f_\pm/f_\mp \to 0$, and also by the sign of C. The indicated values become negative at powers of the incident radiation $P = L^2/L_n^2$ which exceed the following values, respectively:

$$P_2^\pm = (L/L_2^\pm)^2 = 4|\sigma_1|/((1 \pm M_0)(\sigma_1 \mp 2\rho_1)) > 0\,; \quad (14)$$

$$P_0 = (L/L_0)^2 = 2|\sigma_1|/(\sigma_1 + \sigma_2 - M_0^2\sigma_2 - 2M_0\rho_1) > 0\,. \quad (15)$$

Sometimes the sign of the quantity $D = K^2 + 8M_0Cm_-$ (where $K = M_0(1 + M_0)n/2 + (1 + M_0)m_+ + (1 - 3M_0)m_-$) also plays an important role, determining the region of variation of the variables $u_\pm$. It can be easily seen that $D = 0$ is a quadratic equation in P. It is possible to show that for those values of M_0 for which its discriminant is positive, two branches of the roots are necessarily located between P_2^+ and P_2^-, and that in finding the boundaries only the lower of these two branches $P_3 = (L/L_3)^2$ is important. In media with $\sigma_1 > 0$, $\sigma_1 + 2\sigma_2 < 0$ the threshold of self-focusing coincides with $P_4 = (L/L_4)^2$, determined by the condition

$$2\int_0^\infty \frac{dz}{\Phi(z, C)} = \int_0^{(1+M_0)/2} \left[\frac{1 + M_0 + 2M_0u_-}{(1 + M_0 - 2u_-)(2M_0Cu_-^2 + K(1+M_0)u_- - m_-(1+M_0)^2)}\right]^{1/2} du_-, \tag{16}$$

where the nmotation which was introduced above has been used.

Analysis of Eq. (6) permits us, in addition, to identify the regions with character of variations of $\Psi(z)$ known to be monotonic. In this case the following values of the power are the limiting values:

$$P_1^\pm = (L/L_1^\pm)^2 = |\sigma_1|/((1 \pm M_0)(\mp\rho_1 - \sigma_2)) > 0. \tag{17}$$

In the final analysis it is possible to determine five regions p (p = 1-5) in which the dependences $I_n(z)$, $M(z)$, and $\Psi(z)$ differ qualitatively. Their boundaries in the plane of the variables $(L_n/L)^2$ and M_0 for various values of the parameters of the nonlinear, gyrotropic medium are shown in Figs. 1 and 2. Curves I-VII In Figs. 1 and 2 represent schematically the dependences of $(L_0/L)^2$, $(L_1^\pm/L)^2$, $(L_2^\pm/L)^2$, $(L_3/L)^2$, and $(L_4/L)^2$, respectively, on the values of M_0 lying in the interval -1 to 1. Here only those parts of the curves have been kept which divide regions with substantially different regimes of beam propagation. Figure 1a depicts the cases $\sigma_1 > 0$, $\sigma_2 < 0$, $\sigma_1 + 2\sigma_2 > 0$, $4\sigma_2 + \sigma_1 > 2\rho_1$; Fig. 1b depicts the cases $\sigma_1 > 0$ and $\sigma_2 > 0$; Fig. 2a depicts the cases $\sigma_1 > 0$, $\sigma_1 + 2\sigma_2 < 0$, and $\sigma_1 + \sigma_2 + \rho^2/\sigma_2 > 0$; and Fig. 2b depicts the cases $\sigma_1 < 0$, $\sigma_1 + 2\sigma_2 > 0$, $\sigma_1 + \sigma_2 + \rho_1^2/\sigma_2 > 0$. Here it has also been assumed that $\rho_1 < |\sigma_2|$.

Region 1 is distinguished by pronounced oscillatory variation of M, and also by the largely nonmonotonic dependence of I_n and Ψ on sample length. Collapse of the partial beams in this region occurs simultaneously, although $\Delta f = f_+ - f_-$ for small z can be quite large. Region 2 is characterized by initial growth of Ψ, followed by abrupt decrease near the focus, while I_n and M vary monotonically. A partial focusing of the partial beams take place here (one of them collapses for finite values of the width of the other one), and Δf grows continuously with growth of z. Regions 3 and 5 are distinguished by monotonic variation of the rotation angle of the polarization ellipse, whose rate of change grows with approach to the focus. In region 3 the growth of I_n takes place during the entire course of the process of self-focusing, while in region 5 it is preceded by a moderate decrease. Here there takes place a focusing of one of the partial beams and defocusing of the other, and Δf grows monotonically. A general tendency towards decrease of intensity is characteristic of region 4. However, variation of $I_n(z)$ can also be nonmonotonic. Dependences of M and Ψ on sample length here have a largely oscillatory character, and, in the process of defocusing, the quasiperiod of the oscillations increases. Nonmonotonicity of variation of all characterteristics grows significantly with approach of the incident powers to the threshold values.

The indicated differences are illustrated by dependences, characteristic for regions 1-5, which were found by numerical methods, of the quantities $f_\pm$, I_n, M, and Ψ on the length of the nonlinear medium. They are shown in Figs. 3-5. The solid curves represent the dependences $f_+(z)$, and the dashed curves, $f_-(z)$. The numbers which label the curves are the numbers of the regions (two curves 4a and 4b belong to region 4). For all the figures, $\sigma_1 > 0$. For Fig. 3, $\sigma_2/\sigma_1 = 0.75$ and $\rho_1/\sigma_1 = 0.25$. Curves 1 and 2 correspond to the cases $P = 3.9$, $M_0 = 0.6$ and $P = 3.65$, $M_0 = 0.2$. Dependences 3-5 in Figs. 4 and 5 correspond to the following parameter values: 3) $\sigma_2/\sigma_1 = -0.35$; $\rho_1/\sigma_1 = 0.05$; $P = 2.5$, $M_0 = -0.6$; 4a) $\sigma_2/\sigma_1 = 0.75$; $\rho_1/\sigma_1 = 0.25$; $P = 1.66$; $M_0 = 0.6$; 4b) $\sigma_2/\sigma_1 = 0.75$; $\rho_1/\sigma_1 = 0.25$; $P = 0.9$; $M_0 = 0.2$; 5) $\sigma_2/\sigma_1 = -0.58$; $\rho_1/\sigma_1 = 0.25$; $P = 5.7$; $M_0 = 0.4$.

Regions p' are analogous to regions p with interchange of f_+ and f_-. Their existence is determined by the sign of the quantity

$$\Delta = \left(\frac{d^2f_+}{dz^2} - \frac{d^2f_-}{dz^2}\right)_{f_+=f_-=f} = \frac{P\sigma_2}{f^3|\sigma_1|}(M_0 + \rho_1/\sigma_2). \tag{18}$$

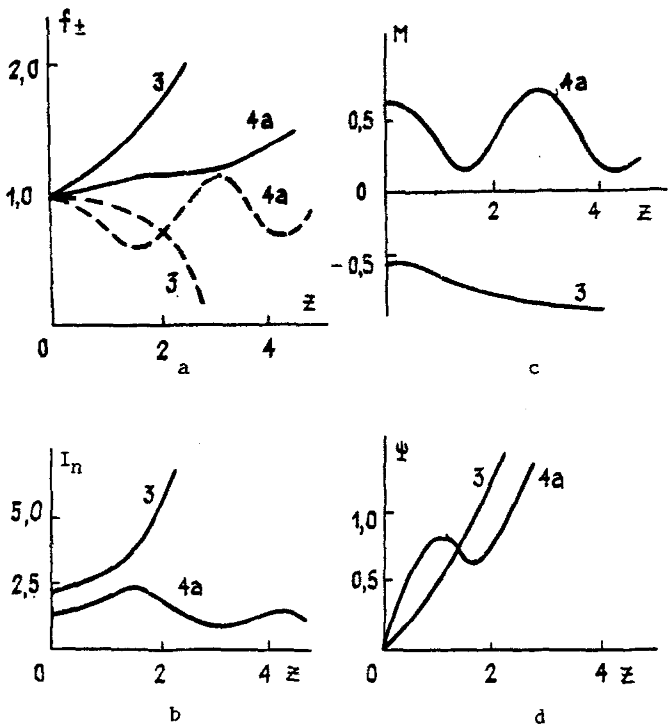

Fig. 4. Dependences of the parameters of the output radiation on length of the nonlinear, gyrotropic medium (regions 3 and 4).

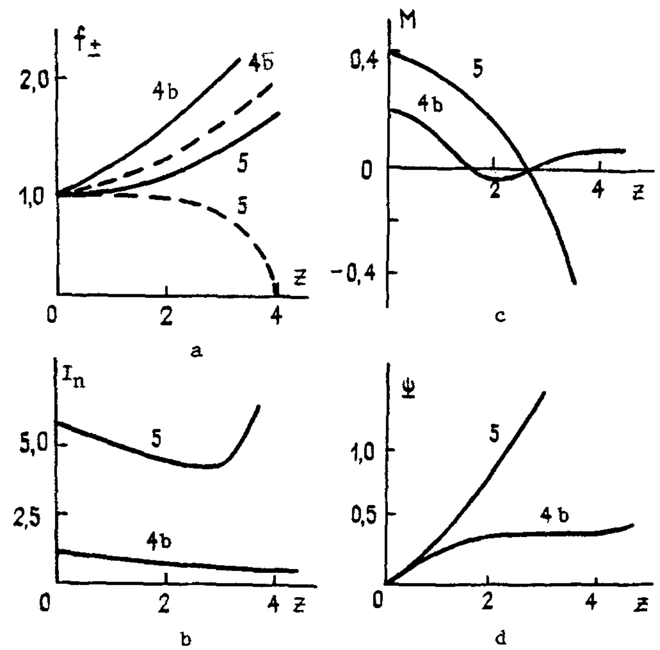

Fig. 5. Dependences of the parameters of the output radiation on length of the nonlinear, gyrotropic medium (regions 4 and 5).

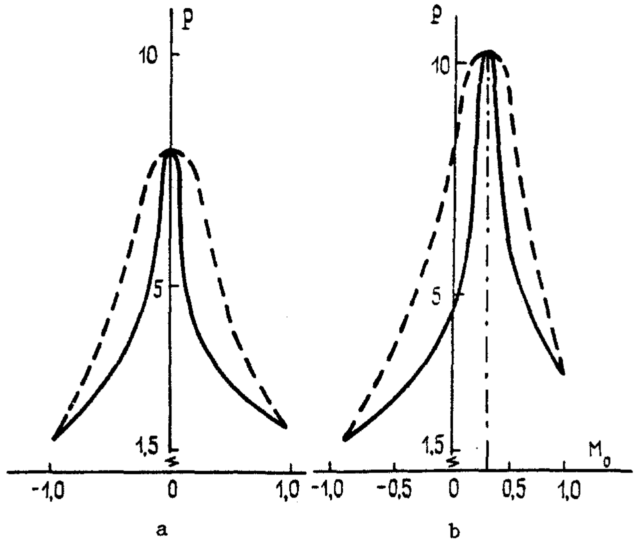

Fig. 6. Dependence of threshold power (solid curve) and P_0 (dashed curve) on degree of ellipticity of the incident light: a) $\rho_1 = 0$, b) $\rho_1/\sigma_1 = 0.2$.

In regions p' we have $\Delta < 0$. It can be easily seen that for $\rho_1 > |\sigma_2|$ regions of type p' are absent. We note also that the straight line $M_0 = -\rho_1/\sigma_2$ (for $\rho_1 < |\sigma_2|$) not only separates region p from region p', but also intersects the parabola I at its vertex.

If $\sigma_2 + \rho_1^2/\sigma_2 + \sigma_1 < 0$, then the vertex of parabola I in Fig. 2 will be located below the M_0 axis. Then for $\sigma_1 < 0$, $\sigma_1 + 2\sigma_2 > 0$, $\rho_1 < |\sigma_2|$ the picture will differ from Fig. 2b by the absence of regions 1 and 1'. Regions 4 and 4' become substantially larger. If $\sigma_1 > 0$, $\sigma_1 + 2\sigma_2 < 0$, then the location of the various regions will be analogous to that shown in Fig. 2a with the difference that regions 3 and 3' decrease and regions 5 and 5' osculate the M_0 axis. Curve VII, defined by condition (16), is the upper boundary of region 5 in Fig. 2a. It is also well known that it passes through the points with coordiantes $((\sigma_1 \pm 2\rho_1/2|\sigma_1|, \mp 1)$ and the vertex of parabola I. It was found numerically for a number of values of the parameters of the radiation and the medium (see below).

If $4\sigma_2 + \sigma_1 < 2\rho_1$, then parabola I in Fig. 1a intersects the straight line V ($L_n/L_2^- = 1$), which leads to the appearance between them of region 5. There also occurs a narrowing (with respect to M_0) of region 1 and the disappearance of region 2. In the case when $4\sigma_2 + \sigma_1 + 2\rho_1 < 0$, parabola I on the right side of Fig. 1a intersects straight line IV ($L_n/L_2^+ = 1$). Region 5' appears between I and IV, region 1' diminishes, and region 2' disappears.

Finally, if $\rho_1 > \sigma_2$, then there takes place still one more qualitative variation in Fig. 1b in addition to the absence of regions ρ': region two no longer osculates the M_0 axis (at $M_0 = 1$) and region 3 arises under it. Its upper boundary is determined by the relation $(L_n/L)^2 = (1 - M_0)(\rho_1 - \sigma_2)/|\sigma_2|$.

As immediately follows from Eqs. (1) and (9), there exist three values of the ellipticity for which for any P the beam propagates as a whole: ± 1, $-\rho_1/\sigma_2$, and which corresponds to the boundaries of regions with various propagation regimes. The question naturally arises as to how abruptly the transition takes place from one regime to another. Numerical investigations show that practically always it is smooth (some exceptions are associated with regions 3 and 3' for $\sigma_1 + 2\sigma_2 < 0$).

5. Self-focusing of light in a nonlinear, gyrotropic medium takes place in the regions lying below curve I in Figs. 1 and 2, and also in region 5. Consequently, the condition $L_n = L_0$, where L_0 is given by Eq. (15), determines the threshold of self-focusing for $\sigma_1 + 2\sigma_2 > 0$ and $\sigma_1 < 0$, $4\sigma_2 + \sigma_1 > 2\rho_1$, or $\sigma_1 < 0$. In this case, widespread opinion notwithstanding [2], the threshold power for linearly polarized radiation $P_0 = 2|\sigma_1|/(\sigma_1 + \sigma_2)$ does not depend on the spatial dispersion of nonlinearity. For $\rho_{0,1} = 0$ and $\sigma_2 = 3\sigma_1$, the dependence of P_0 on the ellipticity ($P_0 = 2/(4 - 3M_0^2)$) which we have found is in good agreement (especially for $|M_0| \leq 0.3$ and $|M_0| \geq 0.85$) with the results of [9], obtained on the basis of numerical integration.

If $\sigma_1 + 2\sigma_2 < 0$ and $\sigma_1 > 0$. Self-focusing of light can also take place for $L_n > L_0$ (see Fig. 2a, region 5). Figure 6 illustrates the form of the threshold curve ($\sigma_1 > 0$, $\sigma_2/\sigma_1 = -0.75$), given in this case by Eq. (16). Here solid lines represent the dependence (found by numerical methods) of the threshold power on M_0, the dashed lines represent $P_0(M_0)$, and the dot-dashed line correspond to $M_0 = -\rho_1/\sigma_2$. The curves in Fig. 6a are constructed for $\rho_1 = 0$ (the absence of spatial dispersion), and those in Fig. 6b for ρ_1/σ_2. Regions 5 and 5' lie between the dashed and the solid curves, while regions 4 and 4' lie below the latter. The threshold in this case strongly depends on the parameter which characterizes the spatial dispersion of the nonlinearity. For light which is linearly polarized at z = 0, the threshold decreases with growth of ρ_1.

In media whose parameters satisfy the inequalities $\sigma_1 > 0$, $\sigma_2 < 0$, $\sigma_1 + 2\sigma_2 > 0$, and $4\sigma_2 + \sigma_1 \mp 2\rho_1 < 0$, for certain values of M_0 self-focusing also occurs for powers $P < P_0$ (region 5 appears), and for radiation with initial ellipticity less then $1 + (\sigma_1 + 2\rho_1)/2\sigma_2$ (or greater than $-1 - (\sigma_1 + 2\rho_1)/2\sigma_2$) the threshold power is equal to $P_2^{\mp}$ (here $P_2^{\mp} < P_0$) and also depends on ρ_1.

6. Experiment is promising in the isotropic phase of cholesteric liquid crystals (CLC's) far from the mesophase transition temperature. Unfortunately, concrete data on ρ_1, σ_1, and σ_2 in these media are very limited. However, they can be estimated partly, using the well-established similarity of the microstructures of CLC's and nematic liquid crystals, which have been studied in greater detail [17]. Self-focusing in the latter has been investigated in detail [6]. It is easily realized experimentally, since the threshold powers are only a few hundred watts. Therefore self-focusing in CLC's, where, in contrast with nematics, $\rho_{0,1} \neq 0$, should also take place for practical powers of laser radiation. Nonlinear rotation of the polarization ellipse, whose magnitude varies from 10^{-5} radians [10] to tens of degrees [18], has been reliably determined by contemporary, high-sensitivity technology.

Stability of self-focusing of light in a nonlinear, gyrotropic medium with respect to spatial stratification of the beam into individual threads has been investigated in [19]. In this article it is shown that in the overwhelming majority of cases perturbations whose ellipticity is different from that of the main beam are the most stable.

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