ASYMPTOTIC STATES OF MASSIVE PARTICLES INTERACTING WITH THE GRAVITATIONAL FIELD

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It is shown that, as in the case of quantum electrodynamics, infrared divergences do not arise in the interaction of massive particles with the gravitational field if the scattering operator and the state space on which it is defined are modified.

1. Introduction

The aim of the present paper is to apply the general method for defining the scattering operator when there are infrared singularities [1] to the interaction of a massive field with the gravitational field. The latter, being a self-interacting massless field, has infrared singularities on its own account. The Yang–Mills field, for example, also has such singularities. However, Weinberg [2] has noted that these singularities do not lead to additional difficulties because of the specific interaction with the gravitational field. The additional diagrams for the interaction with the fictitious particles [3, 4] which arise when gauge fields are quantized also do not lead to infrared singularities [5]. Therefore in what follows we shall omit the self-interaction terms in the Lagrangian for the gravitational field and in the total Lagrangian of the massive field we shall retain only the terms that are linear in the coupling constant.*

In the second section we recall the formalism of the quantization of the gravitational field in the linear approximation, using formulas that are, in our opinion, somewhat simpler than the traditional expressions [7].

The asymptotic Hamiltonian and the operator describing the asymptotic dynamics of a massive scalar field interacting with the gravitational field are obtained in the third section.

In the fourth section we use the asymptotic operator to define the space of asymptotic states on which the S matrix is defined.

The finiteness of the matrix elements for the scattering operator is established in the fifth section by concrete calculations. The calculations of this section are similar to those of Chung [8]. At the same time, we use Weinberg's results [2] to take into account the infrared divergences from the corrections for the massive fields that are virtual in the gravitational field.

The methods and many of the formulas employed in this paper are analogous to the case of the interaction of charged particles with the electromagnetic field [1]. Therefore, in the majority of cases we shall merely give the final results and only go into details when the particular features of the problem distinguish it from electrodynamics.

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* Note that for the gravitational field in the first-order formalism one can write down an exact interaction Lagrangian that contains only a single vertex [6].

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2. Quantization of the Gravitational Field

We take the Lagrangian density for the gravitational field in the form

$$\mathcal{L}(x)=\frac{1}{\varkappa^2}\sqrt{-g}g^{\mu\nu}(x)\{\Gamma^{\alpha}_{\mu\nu}(x)\Gamma^{\beta}_{\alpha\beta}(x)-\Gamma^{\alpha}_{\mu\beta}(x)\Gamma^{\beta}_{\alpha\nu}(x)\},$$

where $g=\det g_{\mu\nu}(x)=\det\sqrt{-g}\,g^{\mu\nu}(x)$; $g^{\mu\nu}(x)$ is the inverse of the tensor $g_{\mu\nu}(x)$; $\Gamma^{\alpha}_{\mu\nu}(x)$ are the Christoffel symbols; $\varkappa^2=16\pi G$, where G is the gravitational constant [9].

We introduce the variables $h_{\mu\nu}(x)$ which describe the linearized gravitational field: $g_{\mu\nu}(x)=\eta_{\mu\nu}+\varkappa h_{\mu\nu}(x)$, where $\eta_{\mu\nu}$ is the Minkovski metric tensor with diagonal $(1,-1,-1,-1)$. We rewrite the Lagrangian density in the variables $h_{\mu\nu}(x)$, retaining the terms of zeroth order in $\varkappa$:

$$\mathcal{L}_0(x)=\frac{1}{4}\left\{\partial^{\mu}h^{\alpha\beta}(x)\,\partial_{\mu}h_{\alpha\beta}(x)-\frac{1}{2}\partial^{\mu}h_{\alpha}{}^{\alpha}(x)\,\partial_{\mu}h_{\beta}{}^{\beta}(x)-2\left(\partial_{\alpha}h^{\mu\alpha}(x)-\frac{1}{2}\partial^{\mu}h_{\alpha}{}^{\alpha}(x)\right)^2\right\}.$$

The indices are raised and lowered with $\eta_{\mu\nu}$. Now the Lagrangian $\mathcal{L}_0(x)$ is singular [10], and therefore, using Fermi's method, we go over to the density

$$\mathcal{L}_0'(x)=\mathcal{L}_0(x)+\frac{1}{2}\left(\partial_{\alpha}h^{\mu\alpha}(x)-\frac{1}{2}\partial^{\mu}h_{\alpha}{}^{\alpha}(x)\right)^2=\frac{1}{8}\{\partial^{\mu}h^{\alpha\beta}(x)\,(\eta_{\alpha\sigma}\eta_{\beta\lambda}+\eta_{\alpha\lambda}\eta_{\beta\sigma}-\eta_{\alpha\beta}\eta_{\sigma\lambda})\,\partial_{\mu}h^{\sigma\lambda}(x)\}.$$

This density is not singular and the method of canonical quantization leads to the commutation relations

$$[h_{\mu\nu}(x),h_{\sigma\lambda}(x')]=i\omega_{\mu\nu,\sigma\lambda}D(x-x')$$

and the causal Green's function

$$\langle T(h_{\mu\nu}(x),h_{\sigma\lambda}(x'))\rangle=\omega_{\mu\nu,\sigma\lambda}\frac{i}{(2\pi)^4}\int\frac{e^{ik(x-x')}}{k^2+i\varepsilon}dk,$$

where $D(x-x')$ is the invariant D function and we have introduced the following convenient abbreviation: $\omega_{\mu\nu,\sigma\lambda}=\eta_{\mu\sigma}\eta_{\nu\lambda}+\eta_{\mu\lambda}\eta_{\nu\sigma}-\eta_{\mu\nu}\eta_{\sigma\lambda}$.

In the usual manner, we define the operators of creation and annihilation:

$$h_{\mu\nu}(x)=\frac{1}{(2\pi)^{3/2}}\int\frac{d\mathbf{k}}{(2k_0)^{1/2}}(a_{\mu\nu}(\mathbf{k})e^{-ikx}+a^{+}_{\mu\nu}(\mathbf{k})e^{ikx}) \tag{1}$$

with the commutation relations

$$[a_{\mu\nu}(\mathbf{k}),a_{\sigma\lambda}{}^{+}(\mathbf{k}')]=\omega_{\mu\nu,\sigma\lambda}\delta(\mathbf{k}-\mathbf{k}'), \tag{2}$$

which lead to an indefinite metric on the corresponding Fok state space $\mathcal{H}_{F,\,gr}$.

It is convenient to distinguish two groups of independent operators; the unphysical

$$a_{\lambda}(\mathbf{k})=\left\{a_{0i}(\mathbf{k});\ \frac{1}{2^{1/2}}a_{\alpha}{}^{\alpha}(\mathbf{k})\right\};\quad i=1,2,3;\ \lambda=1,\ldots,4$$

and the physical

$$a_{\lambda}(\mathbf{k})=\left\{\frac{1}{2}(a_{00}(\mathbf{k})+a_{33}(\mathbf{k}));\ a_{in}(\mathbf{k});\ \frac{1}{2}(a_{11}(\mathbf{k})-a_{22}(\mathbf{k}))\right\};$$
$$i\neq n;\quad \lambda=5,\ldots,10.$$

All the operators with different indices commute; the commutator of the physical operators with identical indices is +1 and that of the unphysical operators is −1. This division is adapted to a local basis in the space of second-rank symmetric tensors. The components of this basis $\varepsilon^{\lambda}_{\mu\nu}(\mathbf{k})$, $\lambda=1,\ldots,10$, are orthonormalized in the metric $\omega_{\mu\nu,\alpha\beta}$, and $\|\varepsilon^{\lambda}_{\mu\nu}(\mathbf{k})\|^2=-1$ for $\lambda=1,\ldots,4$, and $\|\varepsilon^{\lambda}_{\mu\nu}(\mathbf{k})\|^2=1$ for $\lambda=5,\ldots,10$.

The metric on the Fok space is connected with the form $\omega_{\mu\nu,\alpha\beta}$. Let us consider, for example, the single-particle state $|\Psi\rangle=\int d\mathbf{k}\Psi^{\mu\nu}(\mathbf{k})a^{+}_{\mu\nu}(\mathbf{k})|0\rangle$; on the one hand, its norm is

$$\langle\Psi|\Psi\rangle=\int d\mathbf{k}\Psi^{\mu\nu*}(\mathbf{k})\,\omega_{\mu\nu,\alpha\beta}\Psi^{\alpha\beta}(\mathbf{k}),$$

and, on the other hand, using the decomposition $\Psi_{\mu\nu}(\mathbf{k})=\sum_{\lambda=1}^{10}\varepsilon^{\lambda}_{\mu\nu}(\mathbf{k})\Psi_{\lambda}(\mathbf{k})$, we obtain

$$\langle\Psi|\Psi\rangle=-\sum_{\lambda=1}^{4}\int|\Psi_{\lambda}(\mathbf{k})|^2d\mathbf{k}+\sum_{\lambda=5}^{10}\int|\Psi_{\lambda}(\mathbf{k})|^2d\mathbf{k}. \tag{3}$$

We now introduce an additional condition, the analog of the Lorentz condition for the electromagnetic field:

$$\left(k^{\mu} a_{\mu\nu}(\mathbf{k}) - \frac{1}{2} k_{\nu} a_{\alpha}{}^{\alpha}(\mathbf{k})\right) | \Psi\rangle = 0$$

(the harmonic condition). Written down for single-particle or coherent-states, it reduces to the requirement $k^{\mu}\Psi_{\mu\nu}(\mathbf{k}) = 0$ and in the components $\Psi_{\lambda}(\mathbf{k})$ we have $\Psi_{\lambda}(\mathbf{k}) - \Psi_{\lambda+4}(\mathbf{k}) = 0, \lambda = 1, 2, 3, 4$. This result, together with formula (3), shows that the metric on the subspace of physical states is definite and has two independent components. It is shown in [7] that the spin corresponding to this field is two.

On the subspace of physical states there acts a group of gauge transformations whose operators $U(\Lambda_{\mu})$ commute with the operator of the additional condition

$$U(\Lambda_{\mu}) = \exp\left\{\int d\mathbf{k}\left[\left(k^{\mu}\Lambda^{\nu} - \frac{1}{2}\eta^{\mu\nu}k^{\alpha}\Lambda_{\alpha}\right) a^{+}_{\mu\nu}(\mathbf{k}) - \left(k^{\mu}\Lambda^{\nu} - \frac{1}{2}\eta^{\mu\nu}k^{\alpha}\Lambda_{\alpha}\right)^{*} a_{\mu\nu}(\mathbf{k})\right]\right\}.$$

Carrying out the factorization of the physical states with respect to equivalence generated by the group of gauge transformations, we obtain a Hilbert space of states with a positive-definite metric. This definition corresponds to factorization of the space of physical states with respect to the set of vectors of vanishing norm.

3. Asymptotic Dynamics

We shall base our subsequent treatment on the example of the interaction of a massive scalar field with the gravitational field. The exact Lagrangian for the scalar field

$$\mathcal{L}(x) = \frac{1}{2}\sqrt{-g}\,(g^{\mu\nu}(x)\,\partial_{\mu}\varphi(x)\,\partial_{\nu}\varphi(x) - m^2\varphi^2(x))$$

in the approximation linear in $h_{\mu\nu}(x)$ takes the form

$$\mathcal{L}(x) = \mathcal{L}_0(x) + \mathcal{L}_{\text{int}}(x),$$

where

$$\mathcal{L}_0(x) = \frac{1}{2}(\partial^{\alpha}\varphi(x)\,\partial_{\alpha}\varphi(x) - m^2\varphi^2(x)), \quad \mathcal{L}_{\text{int}}(x) = -\frac{\varkappa}{2}h^{\mu\nu}(x)\,T_{\mu\nu}(x)$$

and

$$T_{\mu\nu}(x) = \partial_{\mu}\varphi(x)\,\partial_{\nu}\varphi(x) - \frac{1}{2}\eta_{\mu\nu}(\partial^{\alpha}\varphi(x)\,\partial_{\alpha}\varphi(x) - m^2\varphi^2(x))$$

is the energy–momentum tensor.

We now wish to construct the asymptotic Hamiltonian in accordance with [1]. In the interaction Hamiltonian

$$H_{\text{int}} = \frac{\varkappa}{2}\int d\mathbf{x}\,h^{\mu\nu}(x)\,T_{\mu\nu}(x),$$

we must use the decomposition (1) for $h_{\mu\nu}(x)$ and the formula

$$\varphi(x) = \frac{1}{(2\pi)^{3/2}}\int\frac{d\mathbf{p}}{(2p_0)^{1/2}}(b(\mathbf{p})\,e^{-ipx} + b^{+}(\mathbf{p})\,e^{ipx}),$$

to separate out the terms which are candidates for infrared singularities by studying the behavior of the coefficient functions in the limit $|t| \to \infty$. The relevant terms are those that contain one creation operator and one annihilation operator of a massive particle since the function in the argument of the exponential function which determines the dependence on the time:

$$\exp\{\pm i(\sqrt{\mathbf{p}^2 + m^2} - \sqrt{(\mathbf{p} - \mathbf{k})^2 + m^2} \pm k_0)t\},$$

vanishes for $k = 0$ for all p. As a result, the asymptotic energy operator takes the form

$$H_{\text{as}}(t) = H_0 + V_{\text{as}}(t),$$

where

$$V_{\text{as}}(t) = \frac{\varkappa}{2(2\pi)^{3/2}}\int\frac{d\mathbf{k}}{(2k_0)^{1/2}}\frac{d\mathbf{p}}{p_0}\,b^{+}(\mathbf{p})\,b(\mathbf{p})\,p^{\mu}p^{\nu}\left(a_{\mu\nu}(\mathbf{k})\,e^{-i\frac{kp}{p_0}t} + a_{\mu\nu}(\mathbf{k})\,e^{i\frac{kp}{p_0}t}\right).$$

It is natural to regard the expression

$$T_{\mu\nu}(k,t)=\int\frac{d\mathbf{p}}{p_0}\rho(\mathbf{p})\,p^\mu p^\nu e^{i\frac{\mathbf{kp}}{p_0}t},\quad \rho(\mathbf{p})=b^+(\mathbf{p})\,b(\mathbf{p})$$

as the analog of the classical energy–momentum tensor of particles of mass m distributed with momentum density $\rho(\mathbf{p})$.

The operator $U_{as}(t)$, which describes the asymptotic dynamics, is a solution of the Schrödinger equation

$$i\frac{d}{dt}U_{as}(t)=H_{as}(t)\,U_{as}(t),$$

and it can be written down exactly:

$$U_{as}(t)=\exp\{-iH_0t\}\exp\left\{-i\int^t \mathrm{V}_{as}(\tau)\,d\tau-\frac{1}{2}\int^t d\tau\int^\tau ds Q(\tau,s)\right\},$$

where

$$Q(\tau,s)=[V_{as}(\tau),V_{as}(s)]=\frac{\varkappa^2}{(2\pi)^3}\int\frac{d\mathbf{p}}{2p_0}\frac{d\mathbf{q}}{2q_0}\rho(\mathbf{p})\rho(\mathbf{q})\,[2(pq)^2-m^4]\int\frac{d\mathbf{k}}{2k_0}\left(e^{-ik\left(\frac{p}{p_0}\tau-\frac{q}{q_0}s\right)}-e^{ik\left(\frac{p}{p_0}\tau-\frac{q}{q_0}s\right)}\right).$$

We rewrite $U_{as}(t)$ with allowance for the fact that $Q(\tau, s)$ commutes with H_0 and $V_{as}(t)$:

$$U_{as}(t)=e^{-iH_0t}e^{i\Phi(t)}e^{R(t)},$$

where

$$R(t)=\frac{\varkappa}{2(2\pi)^{3/2}}\int\frac{d\mathbf{k}}{(2k_0)^{1/2}}d\mathbf{p}\,\frac{p^\mu p^\nu}{pk}\left(a_{\mu\nu}(\mathbf{k})\,e^{-i\frac{\mathbf{kp}}{p_0}t}-a^+_{\mu\nu}(\mathbf{k})\,e^{i\frac{\mathbf{kp}}{p_0}t}\right)\rho(\mathbf{p}),$$

and $\Phi(t)$ is the phase operator calculated in the appendix to [1]:

$$\Phi(t)=\frac{\varkappa^2}{16\pi}\int d\mathbf{p}\,d\mathbf{q}:\rho(\mathbf{p})\,\rho(\mathbf{q}):\frac{(pq)^2-\frac{1}{2}m^4}{((pq)^2-m^4)^{1/2}}\int^t\frac{d\tau}{|\tau|}.$$

The first of two cofactors in $U_{as}(t)$ serve to define the S matrix and the operator $\exp\{R(t)\}$ participates in the description of the space of asymptotic states.

4. Space of Asymptotic States

The space of asymptotic states may be constructed either by means of the operator $\exp\{R(t)\}$ or by means of any other operator W^+ that possesses the property that $\exp\{R(t)\}W$ is a unitary operator on the Fok state space H_F, where

$$\mathcal{H}_F=\sum_{n=0}^{\infty}\mathcal{H}_n\otimes\mathcal{H}_{F,\,gr}.$$

Here $\mathcal{H}_n$ is the n-particle subspace of massive particles and $\mathcal{H}_{F,\,gr}$ is the Fok space for gravitons.

It is explained in detail in [1] how every such operator W maps the Fok subspace of the infinite tensor product of von Neumann – Hilbert spaces onto another separable subspace on which the scattering operator is defined.

We shall now take W to be the following operator:

$$W=\exp\left\{\frac{\varkappa}{2(2\pi)^{3/2}}\int\frac{d\mathbf{k}}{(2k_0)^{1/2}}f(\mathbf{k})\,d\mathbf{p}\,\frac{p^\mu p^\nu}{pk}(a^+_{\mu\nu}(\mathbf{k})-a_{\mu\nu}(\mathbf{k}))\,\rho(\mathbf{p})\right\},$$

where f(k) is a form factor equal to unity in the neighborhood of small k. Now W commutes with the massive-particle number operator and it follows that the space of asymptotic states $\mathcal{H}_{as}$ can be decomposed into subspaces with fixed number of massive particles:

$$\mathcal{H}_{as}=\sum_{n=0}^{\infty}\mathcal{H}_{as,\,n}.$$

Note that $\mathcal{H}_{as,\,0}$ coincides with $\mathcal{H}_{F,\,gr}$ on which W reduces to the identity operator.

The space $\mathscr{H}_{as,n}$ can be described by specifying how W acts on the set of states

$$|\Psi(\mathbf{p}_1, \ldots, \mathbf{p}_n)\rangle = b^+(\mathbf{p}_1) \ldots b^+(\mathbf{p}_n)\,|\Phi\rangle; \quad |\Phi\rangle \in \mathscr{H}_{F,gr}.$$

On the infinitesimal subspaces formed by such states the action of W reduces to a shift of the operators $a_{\mu\nu}(\mathbf{k})$ and $a^+_{\mu\nu}(\mathbf{k})$ by the function

$$\frac{\varkappa}{2(2\pi)^{3/2}} \frac{f(\mathbf{k})}{(2k_0)^{1/2}} \sum_{i=1}^{n} \frac{p_i^{\alpha} p_i^{\beta}}{p_i k}\, \omega_{\alpha\beta,\,\mu\nu},$$

which is not square-integrable in k in the neighborhood of zero; thus W defines a representation of the commutation relations (2) that is not equivalent to the Fok representation. We shall denote the space of this representation by $\mathscr{H}(p_1, \ldots, p_n)$. For $\mathscr{H}_{as}$ we then obtain the decomposition

$$\mathscr{H}_{as} = \sum_{n=0}^{\infty} \int \mathscr{H}(p_1, \ldots, p_n) \prod_{i=1}^{n} \frac{d\mathbf{p}_i}{(2p_{i0})^{1/2}}.$$

Among the different admissible W one can choose an operator that commutes with the operator of the additional condition that distinguishes the physical subspace in $\mathscr{H}_{F,gr}$. Such an operator W' maps $\mathscr{H}'_{F,g}$ onto the space of asymptotic states, which possesses a definite metric. As in [1], one can show that this definition of $\mathscr{H}_{as}$ and $\mathscr{H}_{as}'$ is Lorentz and gauge invariant.

5. Finiteness of the Matrix Elements of the Scattering Operator

We shall demonstrate the absence of infrared singularities for the scattering operator on states belonging to $\mathscr{H}_{as}$ by taking a specific process. Now, on the one hand, we wish to see how the phase operator which we include in the S matrix [1] acts but, on the other hand, we do not wish to make the formulas too cumbersome. We shall therefore take this process to be the annihilation of two massive particles. The corresponding final state belongs to $\mathscr{H}_{F,gr}$ and the initial state to $\mathscr{H}_{as,2}$:

$$|\Psi_{\text{in}}\rangle = Wb^+(\mathbf{p})\,b^+(\mathbf{q})\,|0\rangle = b^+(\mathbf{p})\,b^+(\mathbf{q}) \exp\left\{\frac{\varkappa}{2(2\pi)^{3/2}} \int \frac{d\mathbf{k}}{(2k_0)^{1/2}} f(\mathbf{k}) \left(\frac{p^{\mu}p^{\nu}}{pk} + \frac{q^{\mu}q^{\nu}}{qk}\right)(a^+_{\mu\nu}(\mathbf{k}) - a_{\mu\nu}(\mathbf{k}))\right\}|0\rangle$$

or, reducing the operator with $a^+{}_{\mu\nu}(\mathbf{k})$ and $a_{\mu\nu}(\mathbf{k})$ to normal form

$$|\Psi_{\text{in}}\rangle = b^+(\mathbf{p})\,b^+(\mathbf{q})\,N^{-1/2} \exp\left\{\frac{\varkappa}{2(2\pi)^{3/2}} \int \frac{d\mathbf{k}}{(2k_0)^{1/2}} f(\mathbf{k}) \left(\frac{p^{\mu}p^{\nu}}{pk} + \frac{q^{\mu}q^{\nu}}{qk}\right) a^+_{\mu\nu}(\mathbf{k})\right\}|0\rangle,$$

where

$$N = \exp\left\{\frac{\varkappa^2}{(4\pi)^3} \int \frac{d\mathbf{k}}{k_0} \left(2\,\frac{2(pq)^2 - m^4}{pk\,qk} + \frac{m^4}{(pk)^2} + \frac{m^4}{(qk)^2}\right)\right\}.$$

The action of the phase operator on $|\Psi_{\text{in}}\rangle$ reduces to multiplication by the phase factor:

$$e^{i\Phi(t)}|\Psi_{in}\rangle = \exp\left\{i\,\frac{\varkappa^2}{16\pi} \frac{2(pq)^2 - m^4}{((pq)^2 - m^4)^{1/2}}\, \operatorname{sign} t \ln \frac{|t|}{t_0}\right\}|\Psi_{in}\rangle.$$

The phase factor that arises from the virtual corrections [2] is

$$\exp\left\{-\,i\,\frac{\varkappa^2}{16\pi} \frac{2(pq)^2 - m^4}{((pq)^2 - m)^{1/2}} \ln \frac{\lambda}{\Lambda}\right\},$$

where λ is the vanishing mass of the graviton. Since the phase operator occurs on the right in the S matrix, we have $t \to -\infty$ and sign $= -1$. Setting $|t| = \lambda^{-1}$, we see that these factors annihilate each other.

The presence in the initial state of soft gravitons, defined by the operator

$$\exp\left\{\frac{\varkappa^2}{2(2\pi)^{3/2}} \int \frac{d\mathbf{k}}{(2k_0)^{1/2}} f(\mathbf{k}) \left(\frac{p^{\mu}p^{\nu}}{pk} + \frac{q^{\mu}q^{\nu}}{qk}\right) a^+_{\mu\nu}(\mathbf{k})\right\},$$

means that the matrix element $\langle\Psi_{\text{out}}|Sb^+(\mathbf{p})b^+(\mathbf{q})|0\rangle$ takes on the factor

$$\exp\left\{\frac{\varkappa^2}{(4\pi)^3} \int \frac{d\mathbf{k}}{k_0} f(\mathbf{k}) \left(2\,\frac{2(pq)^2 - m^4}{pk\,qk} + \frac{m^4}{(pk)^2} + \frac{m^4}{(qk)^2}\right)\right\}.$$

From the virtual corrections for this matrix element [2] we have

$$\exp\left\{-\frac{\varkappa^2}{(4\pi)^3}\int\frac{d\mathbf{k}}{k_0}\left(\frac{2(pq)^2-m^4}{pk\,qk}+\frac{m^4}{2(pk)^2}+\frac{m^4}{2(qk)^2}\right)\right\}.$$

The product of these two factors and the normalization factor $N^{-1/2}$ shows that the infrared divergences cancel.

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