INVERSE HIGGS EFFECT IN NONLINEAR REALIZATIONS

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In theories with nonlinear symmetry realization one can in a number of cases eliminate some of the original Goldstone and gauge fields by equating to zero the corresponding covariant Cartan forms; this is the inverse Higgs effect. We discuss the general conditions of applicability of the inverse Higgs effect in gauge and space-time symmetries, and we also consider some examples: the elimination of inessential gauge fields in chiral dynamics and a nonlinear realization of supersymmetry, the elimination of inessential Goldstone fields in spontaneously broken conformal and projective symmetries.

1. Introduction

In models with dynamical symmetry, Goldstone and gauge fields play a particular role. Because of the special interactions with these fields, invariance under some group of dynamical symmetry is achieved.

In a number of cases, some of the Goldstone and gauge fields introduced in accordance with the formal prescriptions are in fact inessential and "redundant" in the sense that they can either be eliminated by a redefinition of the remaining fields, or they can be expressed in terms of them. For example, in theories with spontaneously broken gauge symmetry the Goldstone fields are inessential variables since they can be eliminated from the invariant Lagrangian by a gauge transformation (Higgs effect [1-3]).

We wish to draw attention to the fact that in models with nonlinear realization of symmetry [4-8] one can eliminate the redundant Goldstone and gauge fields by imposing invariant conditions on the Cartan forms. Any Cartan form with homogeneous law of group transformation can be put equal to zero without affecting any of the invariant properties of the theory. Solving the resulting equations, one can express some of the original Goldstone and gauge fields in terms of the remainder. Those of the variables that cannot be eliminated in this way are the "true Goldstone" and "true gauge" fields.

We shall call this elimination of inessential fields by equating Cartan forms to zero the inverse Higgs effect since for gauge symmetries this effect is the opposite of the ordinary Higgs effect in three respects.

First, the two effects are essentially opposite in nature. The direct Higgs effect consists of the spontaneous occurrence of mass of the gauge fields $\mathscr{Z}_\mu^i(x)$ associated with the generators of the transformations under which the vacuum is noninvariant [1-3]. Another aspect of the effect is the existence of a gauge in which the Goldstone fields $\xi^i(x)$ disappear from the invariant Lagrangian (gauge $\xi^i(x) = 0$), and the field $\mathscr{Z}_\mu^i$ satisfies the condition

$$\mathscr{Z}_\mu^i(x)\,|_{\xi=0} = \frac{1}{f}\nabla_\mu\xi^i = \frac{1}{f}\partial_\mu\xi^i(x) + \mathscr{Z}_\mu^i(x)\,|_{\xi\neq 0} + O(\xi), \tag{1.1}$$

where $\nabla_\mu\xi^i$ is the covariant derivative of the field $\xi^i(x)$ and f is a constant. Thus, in the case of the direct Higgs effect the Goldstone field $\xi^i(x)$ is eliminated because of its "absorption" by the field $\mathscr{Z}_\mu^i(x)$. At the same time, in the case of the inverse Higgs effect, i.e., when one imposes the invariant condition

$$\nabla_\mu\xi^i = 0, \tag{1.2}$$

one eliminates the field $\mathscr{Z}_\mu^i(x)$, and this by solving the equation (1.2) can be expressed in terms of $\xi^i(x)$ and the true gauge field $\mathscr{V}_\mu^{\hat{a}}(x)$, which corresponds to the algebraic subgroup (which leaves the vacuum invariant).

Joint Institute of Nuclear Research, Dubna. Translated from Teoreticheskaya i Matematicheskaya Fizika, Vol. 25, No. 2, pp. 164-177, November, 1975. Original article submitted February 27, 1975.

Second, the two effects have different regions of application. The direct effect holds for invariant Lagrangians, i.e., when the gauge symmetry is broken only spontaneously. If to the invariant Lagrangian one adds a term of the type $\mathscr{Z}_\mu^i\mathscr{Z}_\mu^i+\mathscr{V}_\mu^\alpha\mathscr{V}_\mu^\alpha+\ldots$, which directly breaks the gauge symmetry, the Goldstone fields become physical and cannot be eliminated. Conversely, the inverse Higgs effect is constructive precisely when there is direct breaking of symmetry. In this case, the condition (1.2) leads to a model that reproduces all the consequences of broken gauge symmetry and, at the same time, does not require introduction of the field $\mathscr{Z}_\mu^i$.* Essential variables are the fields ξ^i and $\mathscr{V}_\mu^\alpha$.

Finally, the two effects are opposite in the sense that in the case of the direct Higgs effect the kinetic term of the Goldstone fields $\sim\nabla_\mu\xi^i\nabla_\mu\xi^i$ after their elimination can be transformed into the mass term of the fields $\mathscr{Z}_\mu^i|_{\xi=0}$, whereas after the inverse Higgs effect has been used the mass term of the fields $\mathscr{Z}_\mu^i$ leads to the kinetic term of the Goldstone particles.

The inverse Higgs effect can also be used to eliminate inessential Goldstone fields. This possibility can be observed, for example, in nonlinear realizations of a number of space-time symmetries [10].

We recall that nonlinear models with spontaneous symmetry breaking are nonrenormalizable and pretend only to a phenomenological description of the low-energy region.

The plan of the present paper is as follows. In the second section we give the general theory of the inverse Higgs effect for gauge fields and we consider two examples. For the first example, we briefly discuss the model of SU(2) × SU(2) algebra of fields with eliminated axial field [11]. Unfortunately, the predictions of this model do not agree with experiments. The second example is for a nonlinear realization of supersymmetry, which was considered by Volkov et al., [12, 13]. The inverse Higgs effect enables one to eliminate fields associated with generators of spinor translations [13].

In the third section we discuss the inverse Higgs effect for Goldstone fields in nonlinear realizations of space-time symmetries. We prove a necessary and sufficient criterion for a particular Goldstone field to be inessential. We consider two examples: nonlinear realization of conformal symmetry [5-7, 10] and nonlinear realization of the projective group isomorphic to the group SL(5, R). The first model reduces after application of the inverse Higgs effect to a nonlinear realization of scale symmetry (the essential field is a dilaton) [14]. The second model reduces to a nonlinear realization of the affine group $P_4 \odot GL(4, R)$, which was considered in [10] (the Goldstone tensor field $h_{\mu\nu}(x)$ corresponding to proper affine transformations is essential).

2. Inverse Higgs Effect for Gauge Fields

Let G be a dynamical symmetry group with the algebra

$$[Z_i, Z_k]=iC_{ikl}Z_l+iC_{ik\alpha}V_\alpha,\quad [V_\alpha, Z_i]=iC_{\alpha ik}Z_k,\quad [V_\alpha, V_\beta]=iC_{\alpha\beta\gamma}V_\gamma, \tag{2.1}$$

where C are structure constants. The generators V_α correspond to the vacuum stability subgroup H. In the case of space-time symmetries, the generators Z_i must include the generator P_μ of four-translations [7]. If G defines a supersymmetry [12, 13], some of the generators Z_i satisfy anticommutation relations.

The group G is realized by left shifts on the factor space G/H whose points are parametrized by the Goldstone fields $\xi^i(x)$ [4-8]:

$$G(\xi)=e^{i\xi_k Z_k}\xrightarrow{g} gG(\xi)=e^{i\xi_k' Z_k}e^{iU^\alpha(\xi,g)V_\alpha}. \tag{2.2}$$

The shift (2.2) induces a transformation of the fields $\xi^i(x)$:

$$\delta\xi^i(x)=\mathscr{F}^{ik}(\xi)\beta^k+\xi^m\alpha^\rho C^{m\rho i}, \tag{2.3}$$

where β^k and α^ρ are group parameters and $\mathscr{F}^{ik}(\xi)$ is a nonsingular matrix that is nonlinear in $\xi^i(x)$.

The other fields $\Psi(x)$, and also the covariant differentials of the fields $\xi^i(x)$ and $\Psi(x)$ are transformed in the group G with respect to representations of the group H with the function parameters $U^\alpha(\xi, g)$.

* The standard scheme with canonical field $\mathscr{Z}_\mu^i(x)$ corresponds to the different covariant condition $\tilde{\mathscr{Z}}_\mu^i(x)=(1/f)\nabla_\mu\xi^i$, where the field $\tilde{\mathscr{Z}}_\mu^i$ transforms by definition like $\nabla_\mu\xi^i$ (a condition of this type is used by Kawarabayashi and Kitakado [9] in SU(2) × SU(2) chiral symmetry to eliminate Goldstone fields from the gauge-invariant part of the Lagrangian). The formulations of the theory in terms of $\mathscr{Z}_\mu^i$ or $\tilde{\mathscr{Z}}_\mu^i$ are equivalent.

We shall be interested in the case of gauge symmetry, when the parameters β^k and α^ρ do not depend on the coordinates x_μ. The covariant differentials of the fields $\xi^i(x)$ and $\Psi(x)$, $\omega^i(d)$ and $D\Psi$, are determined by the standard expressions [4-8]

$$G^{-1}(\xi)[d+if(\mathscr{Z}^k Z_k+\mathscr{V}^\alpha V_\alpha)]G(\xi)=i\omega^k(d)Z_k+i\Theta^\alpha(d)V_\alpha. \tag{2.4}$$

$$D\Psi=d\Psi+i\Theta^\alpha(d)\overline{V}_\alpha\Psi, \tag{2.5}$$

where $\overline{V}_\alpha$ are the generators of the subgroup H in the Ψ representation. Then $\mathscr{Z}^k$ and $\mathscr{V}^\alpha$ are related to the gauge fields $\mathscr{Z}_\mu^k, \mathscr{V}_\mu^\alpha$ by

$$\mathscr{Z}^k=\mathscr{Z}_\mu^k(x)\,dx_\mu,\quad \mathscr{V}^\alpha=\mathscr{V}_\mu^\alpha(x)\,dx_\mu \tag{2.6}$$

and they have the transformation properties

$$\delta\mathscr{Z}^i=\mathscr{Z}^p\beta^k C^{pki}+\mathscr{V}^\alpha\beta^p C^{\alpha pi}+\mathscr{Z}^k\alpha^\rho C^{k\rho i}-\frac{1}{f}d\beta^i, \tag{2.7}$$

$$\delta\mathscr{V}^\rho=\mathscr{V}^\mu\alpha^\beta C^{\mu\beta\rho}+\mathscr{Z}^k\beta^l C^{kl\rho}-\frac{1}{f}d\alpha^\rho, \tag{2.8}$$

where f is a constant. For what follows, it is important that the Cartan form $\omega^i(d)$ transforms in the group G homogeneously. The differentials $\omega^i(d)$ and $D\Psi$ are related to the covariant derivatives $\nabla_\mu\xi^i$ and $\nabla_\mu\Psi$,

$$D\Psi=\nabla_\mu\Psi\,\omega_\mu^P(d),\quad \omega^i(d)=\nabla_\mu\xi^i\omega_\mu^P(d),$$

where $\omega_\mu^P(d)$ is the covariant Cartan form corresponding to the generator P_μ of four-translations. [Here and in what follows it is assumed that P_μ is transformed in H independently of the remaining generators Z_i. In the case of internal symmetries, $\omega_\mu^P(d) = dx_\mu$.]

We show that there exist nonlinear functions of the fields $\xi^i(x)$ and $\mathscr{V}_\mu^\alpha(x)$ which have the transformation properties (2.7) of the field $\mathscr{Z}_\mu^i$, for whose introduction there is thus no direct need. To construct invariant Lagrangians it is sufficient to have at ones disposal the fields ξ^i, $\mathscr{V}_\mu^\alpha$.

We impose the condition

$$\omega^i(\xi, d\xi, \mathscr{Z}, \mathscr{V})=0 \tag{2.9}$$

or

$$\nabla_\mu\xi_i=0\quad(\omega_\mu^P(d)\neq 0). \tag{2.9'}$$

By the homogeneity of the group transformation of the form $\omega^i(d)$, Eq. (2.9) is invariant under the action of the group G. The solution of this equation is found in the Appendix:

$$\tilde{\mathscr{Z}}^i(\xi,\mathscr{V})=-\frac{1}{f}(\mathscr{F}^{-1}(\xi))^{im}(d\xi^m+f\xi^p\mathscr{V}^\alpha C^{p\alpha m}). \tag{2.10}$$

Using the transformation laws (2.3) and (2.8), and also the Jacobi identities for the matrix $\mathscr{F}(\xi)$, which follow from the group properties of the transformation (2.3), one can show that the function $\tilde{\mathscr{Z}}^i(\xi, \mathscr{V})$ transforms in accordance with the law (2.7). It is also easy to prove the covariance of the expression (2.10) under a canonical substitution of the fields $\xi^i(x)$.

We emphasise that if some function $\tilde{\mathscr{Z}}^i(\xi, \mathscr{V})$ transforms in accordance with (2.7), then when it is substituted into the Cartan form $\omega^i(\xi, d\xi, \mathscr{Z}, \mathscr{V})$ the condition (2.9) is satisfied identically. This follows from the obvious properties

$$\tilde{\mathscr{Z}}^m(\xi, \mathscr{V})\,|_{\xi=0}=0,\ \omega^i(\xi, d\xi, \tilde{\mathscr{Z}}_m, \mathscr{V})\,|_{\xi=0}=f\tilde{\mathscr{Z}}^i|_{\xi=0}=0$$

and the existence of a gauge transformation g_0 such that $\omega^i(\xi, d\xi, \tilde{\mathscr{Z}}, \mathscr{V})\overset{g_0}{\mapsto}\omega^i(0, 0, 0, \mathscr{V}_0)$.

We have therefore proved

THEOREM 1. In nonlinear realizations of gauge symmetries one can always construct a function $\tilde{\mathscr{Z}}_\mu^i(\xi, d\xi, \mathscr{V})$ with the transformation properties of the gauge field $\mathscr{Z}_\mu^i$ (2.7) by equating to zero the covariant Cartan form $\omega^i(\xi, d\xi, \mathscr{Z}, \mathscr{V})$. Conversely if such a function exists then its substitution instead of the field $\mathscr{Z}_\mu^i$ into the form $\omega^i(\xi, d\xi, \mathscr{Z}, \mathscr{V})$ makes the latter vanish identically.

It must be emphasized that the gauge field $\mathscr{V}_\mu^\alpha$ cannot be eliminated since there do not exist Cartan forms with homogeneous law of group transformation whose equating to zero would lead to equations that are solvable for $\mathscr{V}_\mu^\alpha$. The field $\mathscr{V}_\mu^\alpha$ is essential and must be introduced as a canonical field.

The inessential fields $\mathscr{Z}_\mu^i$ need not be referred to at all.

We recall that the components of the covariant derivative $\nabla_\mu \xi_i$ that are irreducible with respect to H transform in the group G independently of one another. Therefore, conditions of the type (2.9′) can be imposed separately for each representation of the subgroup H contained in $\nabla_\mu \xi_i$. In other words, if desired one need not eliminate all the fields $\mathscr{Z}_\mu^i$ but only those belonging to a chosen representation of the subgroup H. Such a situation obtains, for example, in nonlinear realizations of space-time symmetries, in which it is meaningless to equate to zero the Cartan form $\omega_\mu^P(d)$ since it is used to construct the invariant element of four-volume. The gauge field associated with the translation subgroup cannot be eliminated and is therefore a true gauge field.

When not all the fields $\mathscr{Z}_\mu^i$ are eliminated, it is difficult to obtain general equations of the type (2.10). In one simple case, however, one can still use the expression (2.10).

Suppose the generators $Z_{k'}$ correspond to an invariant subgroup, i.e., they form an ideal of the algebra (2.1), $[Z, Z_{k'}]\sim Z_{\alpha'}$, $[V, Z_{k'}]\sim Z_{\alpha'}$. We denote the Goldstone and gauge fields corresponding to the remaining generators $Z_{k''}$ by $\xi_{k''}$ and $\mathscr{Z}_\mu^{k''}$. Then

THEOREM 2. The gauge field $\mathscr{Z}_\mu^{k''}$ can be expressed in terms of the fields $\xi_{k''}$ and $\mathscr{V}_\mu^\rho$ in accordance with the equation

$$\mathscr{Z}_\mu^{k''}=-\frac{1}{f}(\mathscr{F}^{-1}(\xi''))^{k''l''}(\partial_\mu\xi^{l''}+f\xi^{s''}\mathscr{V}_\mu^\rho C^{s''\rho l''}), \tag{2.11}$$

where $\mathscr{F}^{k''l''}$ is the matrix of the nonlinear transformation $\delta_\beta\xi^{l''}=\mathscr{F}^{l''m''}(\xi')\beta^{m''}$.

The expression (2.11) can be obtained in the same way as (2.10) by bearing in mind that the field $\xi^{l''}$ transforms independently of $\xi^{k'}$.

The elimination of gauge fields by imposing invariant conditions of the type (2.9) is essentially opposite to the Higgs effect [1-3]. The point is that in this case the field $\mathscr{Z}_\mu^i$ is eliminated in terms of the Goldstone fields $\xi^i(x)$, whereas in the Higgs effect the fields $\xi^i(x)$ are "absorbed" by the gauge field $\mathscr{Z}_\mu^i$ [choice of the "unitary" gauge $\xi^i(x) = 0$ in the invariant Lagrangian]. With allowance for this remark, it is natural to refer to the inverse Higgs effect.

In contrast to the ordinary Higgs effect, the inverse effect leads to nontrivial results if a term which directly breaks the gauge symmetry is added to the invariant Lagrangian. In this case, the use of conditions of the type (2.9′) and (2.9) enables one to retain all the restrictions of the broken gauge symmetry under the subgroup H and achieve invariance under nonlinear transformations of the group G with constant parameters without introducing the fields $\mathscr{Z}_\mu^i$. This invariance is now manifested only in the connections between the minimal and nonminimal (from the point of view of the subgroup H) interactions of the fields $\xi^i(x)$ and $\mathscr{V}_\mu^\alpha(x)$. Note that the kinetic term of the Goldstone particles can now appear only from the part of the breaking which is nonlinear in $\mathscr{Z}_\mu^i(\xi,\mathscr{V})$: $\mathscr{Z}_\mu^i(\xi,\mathscr{V})\mathscr{Z}_\mu^i(\xi,\mathscr{V})=f^{-2}\partial_\mu\xi^i\partial_\mu\xi^i+\ldots$.

The kinetic term of the gauge fields $\mathscr{V}_\mu^\alpha$ can be introduced by means of the covariant differential form of second order [10, 13] $R_{\mu\nu}^\alpha$,

$$\tfrac{1}{2}R_{\mu\nu}^\alpha(\omega_\mu^P(d_1)\,\omega_\nu^P(d_2)-\omega_\mu^P(d_2)\,\omega_\nu^P(d_1))=d_1\Theta^\alpha(d_2)-d_2\Theta^\alpha(d_1)-C^{\alpha\beta\gamma}\Theta^\beta(d_1)\,\Theta^\gamma(d_2),$$

where $\Theta^\alpha(d)$ is defined by the expansion (2.4)

Let us now consider two examples of the use of the inverse Higgs effect in gauge theories.

1. Model of SU(2) × SU(2) algebra of fields without the A_1 meson. In the review [11] a model of SU(2) × SU(2) algebra of fields is constructed in which a function $A_\mu(\pi,\rho)$ with appropriate transformation properties is used instead of the field of the A_1 meson. In the parametrization of the nonlinear sigma model the general equation (2.10) gives

$$A_\mu(\pi,\rho)=-\frac{Z_\rho^{-1/2}}{g_\rho}\frac{1}{\sqrt{f_\pi^2-\pi^2}}(\partial_\mu\pi-g_\rho\rho_\mu\times\pi), \tag{2.12}$$

where g_ρ and Z_ρ are the universal coupling constant and the renormalization constant of the ρ meson and $f_\pi \simeq 94$ MeV is the pion decay constant.

The invariant Lagrangian has the form

$$\mathscr{L}_{\text{inv}} = -\frac{1}{4}\frac{f_\pi^2}{f_\pi^2-\pi^2}\left[\rho_{\mu\nu}\rho_{\mu\nu} - \frac{1}{f_\pi^2}(\rho_{\mu\nu}\pi)(\rho_{\mu\nu}\pi)\right], \tag{2.13}$$

where

$$\rho_{\mu\nu} = \partial_\mu\rho_\nu - \partial_\nu\rho_\mu - g_\rho\rho_\mu\times\rho_\nu - g_\rho Z_\rho \mathbf{A}_\mu(\pi, \rho)\times\mathbf{A}_\mu(\pi, \rho). \tag{2.14}$$

The structure of the mass term $\mathscr{L}_{\text{Br}}$ of the field ρ_μ is uniquely determined by the requirement of chiral invariance and by the condition that the field ρ_μ be coupled to a conserved current:

$$\mathscr{L}_{\text{Br}} = -{}^1\!/_2 m_\rho^2(\rho_\lambda\rho_\lambda + Z_\rho \mathbf{A}_\mu(\pi, \rho)\mathbf{A}_\mu(\pi, \rho)). \tag{2.15}$$

The conserved axial current and vector current calculated by the Gell-Mann–Levi method satisfy the standard commutation relations of the SU(2) × SU(2) algebra of fields [15] despite the absence of the canonical field of the A_1 meson in the axial current.

Thus, the requirement of chiral invariance can be reconciled with the assumption of a coupling of the field ρ_μ to the conserved current without introducing the field of the A_1 meson. A choice between this model and the standard approach can be made only after their predictions have been compared with the experiments.

For the correct normalization of the kinetic term of the pions in (2.16) it is necessary that the following sum rule be satisfied [11]:

$$m_\rho^2 = g_\rho^2 f_\pi^2, \tag{2.16}$$

and this does not agree with the empirical Kawarabayashi–Suzuki–Riazuddin–Fayyazuddin relation [16]:

$$m_\rho^2 = 2g_\rho^2 f_\pi^2. \tag{2.17}$$

This discrepancy means essentially that in the present model the values of the partial widths of the ρ meson are much lower than the predictions of the ordinary approach [11].

A more serious shortcoming of the model, in our opinion, is the impossibility of constructing a gauge-invariant πN interaction, which, in the lowest order in π, would give the pseudovector coupling $\bar{N}\gamma_\mu\gamma_5\boldsymbol{\tau}N\partial_\mu\boldsymbol{\pi}$, which is needed to describe the p-wave part of πN scattering [17]. This is due to the vanishing of the covariant derivative of the pion.

Let us make a further remark concerning the Kawarabayashi–Suzuki–Riazuddin–Fayyazuddin relation. It is well known that it cannot be derived in the framework of only the assumption of ρ dominance and current algebra [18]. On the other hand, from the assumption of vanishing of the pion covariant derivative and the choice of the breaking in the form (2.15) the sum rule (2.16), which is incompatible with (2.17), necessarily follows. Therefore, for the fulfillment of the Kawarabayashi–Suzuki–Riazuddin–Fayyazuddin rule it is necessary that the covariant derivative of the pion be nonzero, i.e., that in the part of the axial current with quantum numbers 1^+ the A_1 meson be predominant (this condition is of course insufficient).

It is interesting that the present model is the limiting case $m_{A_1}\to\infty$ of the standard model with A_1 meson.

2. Elimination of gauge fields in supersymmetry model. In [13], Volkov and Soroka discussed the elimination of gauge fields in a nonlinear realization of supersymmetry [12]. In this case, the factor space is parametrized by Goldstone spinors Ψ_a and Ψ_a^+ (a is the subscript of a nonlinear internal symmetry), which correspond to spinor translations, and by ordinary coordinates x_μ, which serve as "Goldstone fields" of the subgroup of four-translations. The generators of the spinor translations are associated with the gauge fields Φ_a^μ, $\Phi_a^{\mu+}$ [13].

The generators P_μ form an ideal of the complete algebra, and therefore, in accordance with Theorem 2, the field Φ_a^μ can be expressed in terms of the Goldstone spinors and the gauge fields $\Omega_{\mu\nu}^\rho$ and V_a^ρ, which correspond to the Lorentz subgroup and the internal symmetry group. Applying (2.11), we obtain

$$\Phi_a^\mu = -\frac{1}{f}\left(\partial_\mu\Psi_a + i\frac{1}{2}f\Omega_{\rho\gamma}{}^\mu L_{\rho\gamma}{}^\Psi\Psi_a + ifV_b{}^\mu I_b{}^{ac}\Psi_c\right), \tag{2.18}$$

where $L_{\mu\nu}{}^\Psi$, I_a are the generators of the stability subgroup in the Ψ representation. The fact that the fields Φ_a^μ, $\Phi_a^{\mu+}$ are inessential was noted in [13].

3. Inverse Higgs Effect for Goldstone Fields

The solvability of Eqs. (2.9) for $\mathcal{Z}_\mu^i$ is associated with the fact that these fields enter the corresponding covariant derivatives linearly and additively.

If a covariant derivative contains a term that is additive with respect to a Goldstone field, it can also be eliminated and it can be expressed in terms of the true Goldstone fields by equating the covariant derivative to zero. This phenomenon occurs, for example, in nonlinear realizations of a number of space-time symmetries. We shall call the elimination of inessential Goldstone fields by means of conditions of the type (2.9) and (2.9′) the inverse Higgs effect by analogy with the corresponding effect for gauge fields. In this section, we discuss the elimination of Goldstone fields in nonlinear realizations of space-time symmetries.

Suppose such a symmetry is defined by the relations (2.1). The subgroup H contains the homogeneous Lorentz group. The generators Z_i must include the generator P_μ of four-translations [7, 8]. The remaining generators of nonlinear transformations and the related Goldstone fields will be denoted by Z_i' and ξ_i. Thus, the points of the factor space G/H are characterized by the coordinates x_μ and $\xi^i(x)$.

We shall restrict ourselves to the case of symmetries that satisfy two requirements: 1) the product of the representations of the subgroup H is completely reducible with respect to H, 2) the generators P_μ and Z_i' transform in H independently of one another.

With allowance for the second condition, the fields ξ_i transform in accordance with a linear representation D(h) of the group H, which can be decomposed into a direct sum of irreducible representations $R_N(h)$. The fields ξ_i are then decomposed onto subspaces ξ_{i_N} that are irreducible with respect to H. The derivative ∂_μ^x transforms in H in accordance with the representation $T^P(h)$ with respect to which the generator P_μ transforms.

In this case, $G(\xi)$ (2.2) has the form [7, 8]

$$G(x, \xi) = e^{ix_\mu P_\mu} e^{i\xi_i Z_i'}. \tag{3.1}$$

The Cartan forms $\omega^i, \omega_\mu^P, \Theta^\alpha$ are determined by the expansion

$$G^{-1}dG = e^{-i\xi_i Z_i'} iP_\mu dx_\mu e^{i\xi_i Z_i'} + e^{-i\xi_i Z_i'} de^{i\xi_i Z_i'} = i\omega^i Z_i' + i\omega_\mu^P P_\mu + i\Theta^\alpha V_\alpha. \tag{3.2}$$

Consider the commutator of P_μ with Z_i':

$$[Z_i', P_\mu] = iC^{i\mu t} Z_t' + \dots . \tag{3.3}$$

The possible terms $\sim V_\alpha$ and P_μ on the right-hand side of (3.3) are inessential for our analysis. In the Appendix we prove a theorem that enables one, given the structure of the commutator (3.3), to determine whether a particular H multiplet of Goldstone fields can be eliminated.

THEOREM 3. A field ξ_{i_N} can be expressed in terms of the remaining Goldstone fields and their derivatives if and only if:

a) the direct product $T^P(h) \otimes D(h)$ of representations contains the irreducible representation $R_N(h)$;

b) for some index $t \neq i_N$ the structure constants $C^{i_N(\mu t)_N}$ in the commutator (3.3) are nonzero [the combination of indices $\{\mu t\}_N$ refers to the representation $R_N(h)$].

This theorem determines the conditions under which the covariant derivative $\nabla_\mu \xi_t$ contains a term that is linear and additive in the field ξ_{i_N}. Bearing in mind that $C^{i_N(\mu t)_N} = \beta \delta^{i_N(\mu t)_N}$ by virtue of Schur's lemma and that $\beta \neq 0$ in accordance with condition b), and using the invariant equation

$$\nabla_{\{\mu}\xi_{t\}_N} = 0 \tag{3.4}$$

we can express the field ξ_{i_N} in terms of the true Goldstone fields:

$$\xi_{i_N} = -\frac{1}{\beta} \partial^{\{\mu}\xi^{t\}_N} + \dots . \tag{3.5}$$

The necessity of conditions a) and b) is proved in the Appendix.

Note that if the combination $\{\mu t\}_N$ can be formed in several inequivalent ways, then the most general covariant equation has the form

$$\sum a_{N'} \nabla_{\{\mu}\xi_{t\}'_N} = 0,$$

whence

$$\xi_{i_N} = -\frac{1}{\beta a_N} \sum a_N' \partial^{(\mu} \xi^{t)_{N'}} + O(\xi, x),$$

where a'_N are numerical coefficients, and the summation is over the representations R_N constructed in the inequivalent ways. If the invariant subspaces ξ_{i_k} include some that transform in accordance with R_N (i.e., ξ_{iN}, $\xi_{iN'}$ etc.) and at the same time the constants $C^i{}_{N'}{}^{(\mu t)}{}_N$, $C^i{}_{N''}{}^{(\mu t)}{}_N \ldots$ are nonzero, the eliminated field ξ_{iN} contains linearly the fields $\xi'_{iN}, \xi''_{iN} \ldots$:

$$\xi_{i_N} = -\frac{1}{\beta} \partial^{(\mu} \xi^{t)_N} - \frac{1}{\beta} \xi^{k_{N'}} C^{k_{N'}(\mu t)_N} + \ldots .$$

After the imposition of the conditions (3.4), not all the invariant kinetic terms of the fields ξ_t $(\sim \nabla_\mu \xi_t \nabla_\mu \xi_t)$ are independent. The "missing" terms can be constructed by using the differential covariant forms of second order. The contractions of these forms with respect to different pairs of indices contain terms $\sim \partial_\mu \xi_t \partial_\mu \xi_t$ and $\sim \xi_t \Box \xi_t$ (see, for example, [10]).

For two simple examples, let us consider the consequences of the inverse Higgs effect in space-time symmetries.

1. Nonlinear realization of conformal symmetry. Nonlinear realization of conformal symmetry with linearization on the homogeneous Lorentz group was discussed in [5-7, 10]. In this case, one introduces Goldstone fields $\Phi_\mu(x)$ and $\sigma(x)$ associated, respectively, with the generators of special conformal and scale transformations K_λ and D.

The commutator of K_ν with P_μ contains on the right-hand side the dilatation generator $[K_\nu, P_\mu] = -2i(\delta_{\mu\nu} D - L_{\mu\nu})$, and therefore in accordance with Theorem 3 the field $\Phi_\mu(x)$ can be expressed in terms of the true Goldstone field which is the dilaton $\sigma(x)$. Equating to zero the covariant derivative of the dilaton, $e^{-\sigma(x)}(\partial_\mu \sigma(x) - 2\Phi_\mu(x)) = 0$, we obtain [10]

$$\Phi_\mu(x) = {}^1\!/_2 \partial_\mu \sigma(x). \quad (3.6)$$

The fact that the field $\Phi_\mu(x)$ does not in reality have a Goldstone nature was noted in [7, 10].

Note that after the condition (3.6) has been used the part of the invariant action that depends only on the field $\sigma(x)$ can be constructed in such a way that it is identical to the corresponding part of the action in the case of nonlinear realization of only scale symmetry.

With regard to the interaction of the dilaton with the fields $\Psi(x)$, it is determined by the form of the covariant derivative $\nabla_\lambda \Psi(x)$ [10]:

$$\nabla_\lambda \Psi(x) = e^{-\sigma(x)} \partial_\lambda \Psi(x) + i e^{-\sigma(x)} \partial_\nu \sigma(x) L_{\lambda\nu}{}^\Psi \Psi(x), \quad (3.7)$$

where $L_{\lambda\nu}{}^\Psi$ are the generators of the Lorentz group for the representation Ψ. In Eq. (3.7), the first minimal (with respect to the nonlinear realization of the scale symmetry) term and the second nonminimal term are strictly related, which is the only "trace" of dynamical conformal symmetry that remains after elimination of the field $\Phi_\mu(x)$.

2. Nonlinear realization of the projective group. The projective group in four dimensions is isomorphic to the group SL(5, R), whose action on the coordinates x_μ is determined by the identification

$$x_\mu = y_\mu / y_5 \quad (\mu = 1, 2, 3, 4, \quad y_4 = i y_0, \; x_4 = i x_0),$$

where y_μ and y_5 are the coordinates of the five-dimensional space on which SL(5, R) acts linearly: $\delta y_i = a_{ik} y_k$ $(i, k = 1, 2, 3, 4, 5)$, $a_{ii} = 0$. Thus, $\delta x_\mu = a_{\mu\nu} x_\nu - a_{5\nu} x_\mu x_\nu - a_{55} x_\mu + a_{\mu 5}$, $a_{\mu\mu} = -a_{55}$. The parameters $a_{\mu\nu}$ correspond to transformations of the subgroup GL(4, R); $a_{\mu 5}$, to four-translations; and $a_{5\nu}$, to new projective transformations.

The algebra of the projective group contains 24 generators, which satisfy the relations

$$\frac{1}{i}[R_{\mu\nu}, R_{\rho\tau}] = \delta_{\mu\rho} L_{\tau\nu} + \delta_{\mu\tau} L_{\rho\nu} + (\mu \to \nu), \quad (3.8a)$$

$$\frac{1}{i}[R_{\mu\nu}, P_\rho] = \delta_{\mu\rho} P_\nu + (\mu \to \nu), \quad (3.8b)$$

$$\frac{1}{i}[R_{\mu\nu}, F_\lambda] = -\delta_{\mu\lambda} F_\nu + (\mu \to \nu), \quad (3.8c)$$

$$[F_\mu, F_\lambda]=0, \tag{3.8d}$$

$$\frac{1}{i}[F_\rho, P_\lambda]=-{}^1/_2(\delta_{\rho\lambda}R_{\mu\mu}+R_{\rho\lambda}+L_{\rho\lambda}), \tag{3.8e}$$

where we have omitted the trivial commutators that contain the generators $L_{\mu\nu}$ of the Lorentz group on the left-hand side. The generators P_λ, $L_{\mu\nu}$, $R_{\rho\nu}$ form the algebra of the affine subgroup $P_4 \odot GL(4, R)$ (20 generators). The F_λ's generate projective transformations.

Let us consider a nonlinear realization of the projective group with the homogeneous Lorentz group as stability subgroup. In this case, (2.2) can be represented in the form $G(\xi)=e^{ix_\mu P_\mu}\exp\{i^1/_2 h_{\mu\nu}R_{\mu\nu}\}$ $e^{iq_\rho F_\rho}$, where $h_{\mu\nu}(x)$, $q_\rho(x)$ are Goldstone fields.

The Cartan forms $\tilde{\omega}_\mu{}^P$, $\tilde{\omega}_{\mu\nu}{}^R$, $\tilde{\omega}_\mu{}^F$, $\tilde{\omega}_{\rho\nu}{}^L$ are associated with the Cartan forms found in [10] of the nonlinear realization of the affine group, $\omega_\mu{}^P$, $\omega_{\mu\nu}{}^R$, $\omega_{\mu\nu}{}^L$, by

$$\tilde{\omega}_\mu{}^P=\omega_\mu{}^P, \tag{3.9a}$$

$$\tilde{\omega}_{\mu\nu}{}^R=\omega_{\mu\nu}{}^R-(q_\nu\omega_\mu{}^P+q_\mu\omega_\nu{}^P)-2\delta_{\mu\nu}q_\lambda\omega_\lambda{}^P, \tag{3.9b}$$

$$\tilde{\omega}_\mu{}^F=dq_\mu+(q_\rho\omega_{\rho\mu}{}^R-q_\rho\omega_{\rho\mu}{}^L)-q_\mu(q_\nu\omega_\nu{}^P), \tag{3.9c}$$

$$\tilde{\omega}_{\mu\nu}{}^L=\omega_{\mu\nu}{}^L-(q_\mu\omega_\nu{}^P-q_\nu\omega_\mu{}^P). \tag{3.9d}$$

Since the commutator of F_ρ with P_λ in (3.8e) contains $R_{\mu\nu}$ on the right-hand side, the field $q_\rho(x)$ can, in accordance with Theorem 3, be expressed in terms of the true Goldstone field $h_{\mu\nu}(x)$.

The covariant derivative of the field $h_{\mu\nu}(x)$ has the form

$$\tilde{\nabla}_\lambda h_{\mu\nu}=\nabla_\lambda h_{\mu\nu}-q_\nu\delta_{\mu\lambda}-q_\mu\delta_{\nu\lambda}-2q_\lambda\delta_{\mu\nu}. \tag{3.10}$$

Solving the invariant equation

$$\tilde{\nabla}_\lambda h_{\mu\mu}+b\tilde{\nabla}_\mu h_{\mu\lambda}=0 \tag{3.11}$$

for $q_\lambda(x)$, we find the one-parameter family of solutions

$$q_\lambda(x)=\frac{1}{10+7b}(\nabla_\lambda h_{\mu\mu}+b\nabla_\rho h_{\rho\lambda}) \qquad b\neq-\frac{10}{7}. \tag{3.12}$$

With allowance for the condition (3.11), the covariant derivative of the field $\Psi(x)$ can be represented in the form

$$\tilde{\nabla}_\lambda\Psi\,\omega_\lambda{}^P=d\Psi+\frac{i}{2}(\tilde{\omega}_{\mu\nu}{}^L+\tilde{V}_{\mu\nu})L_{\mu\nu}{}^\Psi\Psi, \tag{3.13}$$

where

$$\tilde{V}_{\mu\nu}=[a_1(\tilde{\nabla}_\mu h_{\nu\lambda}-\tilde{\nabla}_\nu h_{\mu\lambda})+a_2(\delta_{\mu\lambda}\tilde{\nabla}_\tau h_{\tau\nu}-\delta_{\nu\lambda}\tilde{\nabla}_\tau h_{\tau\mu})]\omega_\lambda{}^P \tag{3.14}$$

and a_1 and a_2 are arbitrary parameters. At the same time, in the nonlinear realization of only affine symmetry the expression for the covariant derivative $\nabla_\lambda\Psi$ contains three arbitrary parameters [10]:

$$\nabla_\lambda\Psi\,\omega_\lambda{}^P=d\Psi+\frac{i}{2}(\omega_{\mu\nu}{}^L+V_{\mu\nu})L_{\mu\nu}{}^\Psi\Psi \tag{3.15}$$

and

$$V_{\mu\nu}=[c_1(\nabla_\mu h_{\nu\lambda}-\nabla_\nu h_{\mu\lambda})+c_2(\delta_{\mu\lambda}\nabla_\nu h_{\rho\rho}-\delta_{\nu\lambda}\nabla_\mu h_{\rho\rho})+c_3(\delta_{\mu\nu}\nabla_\rho h_{\rho\nu}-\delta_{\nu\lambda}\nabla_\rho h_{\mu\rho})]\omega_\lambda{}^P. \tag{3.16}$$

Comparing (3.13) and (3.15) and taking into account (3.9d), (3.10), and (3.12), we can show that if $\nabla_\lambda\Psi$ is to be covariant under the projective group it is necessary and sufficient for the parameters c_1, c_2, c_3 to satisfy the relation*

$$1+c_1-7c_2+10c_3=0. \tag{3.17}$$

Thus, after the inverse Higgs effect has been used the restrictions of the dynamical projective symmetry reduce to the relation (3.17) between the constants of the minimal and nonminimal interactions of the essential Goldstone field $h_{\mu\nu}(x)$ with the fields $\Psi(x)$ in the nonlinear realization of the affine group.

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* In [10] it was shown that if $c_2 = c_3 = 0$ and $c_1 = -1$ then $\nabla_\lambda\Psi$ becomes covariant under transformations of the conformal group and simultaneously under generally covariant transformations. This choice of the parameters c_1, c_2, c_3 is compatible with the condition (3.17), which is natural since the projective group is a subgroup of the generally covariant group.

4. Conclusions

It follows from our treatment that the direct and the inverse Higgs effects have a common feature, as well as the three opposite features. Namely, they both effectively reduce the original symmetry to a lower one. In the case of the inverse Higgs effect, the theory of the massive Yang–Mills fields $\mathscr{Z}_\mu^i$, $\mathscr{V}_\mu^\alpha$ is reduced to the theory of the fields $\mathscr{V}_\mu^\alpha$, the conformal symmetry is reduced to scale symmetry, and the symmetry under the projective group to dynamical affine symmetry. A "trace" of the higher symmetry remains in the form of relations between the constants of the minimal and nonminimal interactions of the essential fields in the form of sum rules of the type (2.16) and (3.17). Similarly, in the case of the ordinary Higgs effect after the Goldstone fields have been eliminated the Lagrangian is manifestly invariant only under the vacuum stability subgroup, and the restrictions of the total group are manifested in the form of relations between the different parameters of the Lagrangian.

We should emphasize that the main aim of this paper was to study general aspects of the inverse Higgs effect. The examples were illustrative. Application of the inverse Higgs effect to more realistic models will be considered separately.

We thank D. V. Volkov and B. M. Zupnik for helpful discussions and R. Lednitski for a valuable comment.

Appendix

We solve Eq. (2.9). We define a nonsingular matrix $A_{mt}(\xi)$:

$$e^{-i\xi_p Z_p}\frac{\delta}{\delta\xi_t}e^{i\xi_p Z_p} = iZ_m A_{mt}(\xi) + \ldots . \quad (A.1)$$

In (A.1) and the following equations, we shall be interested in the coefficients of only the generators Z_m.

Using the basic law of the nonlinear realizations (2.2), we readily find

$$e^{-i\xi_k Z_k}Z_t e^{i\xi_k Z_k} = Z_m A_{mn}(\xi)\mathscr{F}_{nt}(\xi) + \ldots , \quad (A.2)$$

$$e^{-i\xi_k Z_k}V_\alpha e^{i\xi_k Z_k} = Z_m A_{mn}(\xi)C^{p\alpha n}\xi_p + \ldots . \quad (A.3)$$

With allowance for the relations (A.1)-(A.3), the Cartan form ω^i determined by the expansion of (2.4) has the form

$$\omega^i(\xi, d\xi, \mathscr{Z}, \mathscr{V}) = A_{is}(\xi)\,(d\xi_s + f\mathscr{F}_{sp}(\xi)\mathscr{Z}_p + fC^{p\alpha s}\xi^p\mathscr{V}^\alpha). \quad (A.4)$$

Since the matrices A_{is}, $\mathscr{F}^{tp}$ are nonsingular, Eq. (2.9) can be solved for $\mathscr{Z}_p$. Then the expression (2.10) is obtained.

Proof of the necessity of conditions a) and b) of Theorem 3. Suppose there exists an analytic function $f_{i_N}(x, \xi_t, \partial_\nu, \xi_t)$ which transforms under the group G like the field ξ_{i_N}:

$$\delta f_{i_N} = \beta_{i_N} + O(\xi_t, x_\mu, \partial_\nu, \xi_t) \quad (t \neq i_N), \quad (A.5)$$

where β_{i_N} are the parameters of the transformation with generator Z'_{i_N}. It follows from the law (A.5) that the expansion of f_{i_N} in a series in ξ_t, x_μ, $\partial_\nu\xi_t$ must begin with terms of first order. From such a term, proportional to ξ_t or x_μ, an additive correction β_{i_N} in (A.5) cannot appear since the transformations of these fields contain the parameters β_{i_N} to higher orders in the Goldstone fields. Therefore, the additive correction can appear only from the term of first order in the derivatives. The function f_{i_N} transforms in the subgroup H in accordance with the representation $R_N(h)$, and therefore the term linear in the derivatives must also transform in accordance with $R_N(h)$. This is possible only if the representation product $T^p(h)\otimes D(h)$ contains the representation $R_N(h)$. The necessity of condition a) is proved.

Thus, $f_{i_N} = \lambda\partial^{(\mu}\xi^{t)_N} + \ldots$. Using the basic law of the nonlinear realizations (2.2) and the commutation relations (2.1), we can readily find the transformation of the field ξ_t in the lowest order in β_{i_N} and in the first nonvanishing order in the Goldstone fields, $\delta\xi_t = -\beta_{i_N}x_p C^{i_N p t} - {}^1/_2\beta_{i_N}\xi_k C^{i_N k t} + \ldots\ (t \neq i_N)$. Thus,

$$\delta f_{i_N} = -\lambda\beta_{i_N}C^{i_N(\mu t)_N} + \ldots . \quad (A.6)$$

It follows from Schur's lemma that $C^{i_N(\mu t)_N} = \beta\delta^{i_N(\mu t)_N}$. Comparison of (A.6) and (A.5) shows that $\lambda = -1/\beta$ and $\beta \neq 0$, i.e., we have proved the necessity of condition b) as well.

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