MOTION RESISTANCE OF A PARTICLE SUSPENDED IN A TURBULENT MEDIUM

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The equation of motion of a particle in a turbulent fluid was considered by Tchen [1], who started from the known Basset-Boussinesq-Oseen equation for the accelerated motion of a particle in a stationary fluid, and then on the basis of intuitive considerations supplemented it by a term associated with the action of the pressure forces which arise with unsteady motion of the fluid. Tchen's result was criticized and developed further in the study of Corrsin and Lumley [2] who, using the same method, suggested the equation

$$\vartheta d_2 \frac{dv^i}{dt} = \vartheta d_1 \left(\frac{\partial u^i}{\partial t} - u^m \frac{\partial u^i}{\partial x^m} - \nu \Delta u^i\right) - \frac{1}{2}\vartheta d_1 \left(\frac{dv^i}{dt} - \frac{\partial u^i}{\partial t} - v^m \frac{\partial u^i}{\partial x^m}\right) - \vartheta (d_2 - d_1) g^i - 6\pi\mu a \left[(v^i - u^i) + \frac{a}{\sqrt{\pi\nu}} \int\limits_{-\infty}^{t} \left(\frac{dv^i}{dt} - \frac{\partial u^i}{\partial t} - v^m \frac{\partial u^i}{\partial x^m}\right)_{t=\tau} \frac{d\tau}{\sqrt{t-\tau}}\right]. \quad (1)$$

Here d_1 and d_2 are the densities of the fluid and of the particle material, ϑ and a are the volume and radius of the particle, μ and ν are the dynamic and kinematic viscosities, $u^i = u^i(t, x^m)$ is the fluid velocity, $v^i = v^i(t)$ is the particle velocity, g^i is the gravity force acceleration vector.

The equations in the form (1) lead to very strange results, which give rise to questions of their correctness. Actually, let us consider, for example, the steady motion of a particle in the field $u^i = u\delta^i_1$, $\partial u^i/\partial t = \partial u^i/\partial x^1 = 0$, assuming $v^i = v\delta^i_1$ and $k_1 = d_2$. It is easy to see that, according to (1), the quantity $v - u$ may take arbitrarily large positive or negative values depending on the magnitude and sign of Δu. Apparently the cause of such incongruities lies in the incorrect transition from the Basset-Boussinesq-Oseen equations, valid in the fluid-fixed (convective) coordinate system, to the corresponding equations in the fixed (laboratory) system. However this transition may be performed completely rigorously, using the known results of [3, 4], and the necessity for the artificial introduction of an additional "pressure force" disappears entirely.

Introducing the relative particle velocity $\omega^i = v^i - u^i$ in the convective coordinate system ξ^i, we write the Basset-Boussinesq-Oseen equation as

$$\vartheta d_2 \frac{Dw^i}{Dt} = -\frac{1}{2}\vartheta d_1 \frac{Dw^i}{Dt} - \vartheta (d_2 - d_1) g^i - 6\pi\mu a \left(w^i + \frac{a}{\sqrt{\pi\nu}} \int\limits_{-\infty}^{t} \frac{Dw^i}{Dt}\Big|_{t=\tau} \frac{d\tau}{\sqrt{t-\tau}}\right).$$

Here the symbol D/Dt denotes convective differentiation with respect to time ($\xi^i = \text{const}$). Using for the transition to the fixed coordinate system x^i the rigorously proved relations of [3, 4], we obtain

$$w^i \to v^i - u^i, \qquad \frac{Dw^i}{Dt} \to \frac{\partial w^i}{\partial t} + u^m \frac{\partial w^i}{\partial x^m} - w^m \frac{\partial u^i}{\partial x^m} = \frac{dv^i}{dt} - \frac{\partial u^i}{\partial t} - v^m \frac{\partial u^i}{\partial x^m},$$

$$\int\limits_{-\infty}^{t} \frac{Dw^i}{Dt}\Big|_{t=\tau} \frac{d\tau}{\sqrt{t-\tau}} \to \int\limits_{-\infty}^{t} \frac{\partial x^i}{\partial x'^k}\left(\frac{dv^k}{dt} - \frac{\partial u^k}{dt} - v^m \frac{\partial u^k}{\partial x^m}\right)_{t=\tau} \frac{d\tau}{\sqrt{t-\tau}}.$$

Here $x'^i(x'^m, t, \tau)$ are the coordinates of the medium element located at the time t at the point x^i, at the time $\tau \geq t$. The fluid motion is assumed known, therefore the derivatives $\partial x^i/\partial x'^k$ under the integral symbol may always be calculated.

Transferring to the right side of the transformed equation all the terms of the external force acting on the particle, we obtain in place of (1)

$$\vartheta d_2 \frac{dv^i}{dt} = -\frac{1}{2}\vartheta d_1 \left(\frac{dv^i}{dt} - \frac{\partial u^i}{\partial t} - v^m \frac{\partial u^i}{\partial x^m}\right) + \vartheta d_1 \left(\frac{\partial u^i}{\partial t} + v^m \frac{\partial u^i}{\partial x^m}\right) - \vartheta (d_2 - d_1) g^i - 6\pi\mu a \left[(v^i - u^i) + \frac{a}{\sqrt{\pi\nu}} \int\limits_{-\infty}^{t} \frac{\partial x^i}{\partial x'^k}\left(\frac{dv^k}{dt} - \frac{\partial u^k}{\partial t} - v^m \frac{\partial u^k}{\partial x^m}\right)_{t=\tau} \frac{d\tau}{\sqrt{t-\tau}}\right]. \quad (2)$$

The first term in the right side, as before, represents the inertia force associated with the added mass, the second term is the force of the excess pressure differential with acceleration of the fluid motion (which actually does not require a priori introduction), the third term is the gravity force with account for the Archimedes force, the fourth term is the linear resistance force with account for unsteady effects. For sufficiently small a the integral term in (2) may be dropped [1], after which these equations take a particularly simple form.

Equations (2) are valid under the same assumptions which have been made earlier in [1, 2] (smallness of the particle radius in relation to the microscale of the turbulence, potential nature of the external force, etc.). They are free of the intuitive nature of [1], and of the noted deficiencies of Eqs. (1). In passing, we note that the force due to the excess pressure differential introduced by Tchen is closer to the true situation than the analogous force introduced in [2].

The equations of [1, 2] have previously been widely used in the analysis of several problems of turbulent diffusion of discrete particles, etc. (see, for example, [5]). Thus, it appears that a reconsideration must be made of several results obtained on their basis.

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