SOLUTION OF THE PROBLEM OF THE FLOW AROUND A V-SHAPED WING WITH A STRONG SHOCK AT THE LEADING EDGE

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The influence of initial and boundary conditions on the numerical solution of the problem of the flow around a V-shaped wing with supersonic leading edges is considered.

Flow modes in the range of angles of attack close to the shock detached from the leading edge are investigated, and the realizability of the mode of flow around the V-shaped ring with a strong shock is shown.

Two solutions with an attached compression shock are possible in the supersonic gas flow around a wedge. Flow modes with weaker compression shocks are always realized in the simplest cases of flow around finite conical bodies, cones, triangular wings with shocks attached along the edges. An exact solution with a plane shock lying on the leading edges, which can be constructed in such a way that the shock in a plane perpendicular to the leading edge is strong [1], is obtained for a V-shaped wing. The possibility of realizing such a solution has been studied experimentally [2, 3]. It appears from the tests that a flow mode around the V-shaped wing, which is similar to the flow with a strong attached shock, actually occurs. There are no theoretical studies of this question.

G. G. Chernyi [4] first disclosed the existence of modes with a strong bow shock for plane triangular wings with subsonic leading edges.

The first flow mode shall be understood below to be the solution with a weak shock at the leading edge, and the second flow mode shall be the solution with a strong shock. The meaning of the terms "weak" and "strong" is the same as in the problem of the flow around a plane wedge.

The influence of the initial (zero approximation) and boundary conditions on the numerical solution is investigated below by using the numerical method in [5], the realizability of the second flow mode is shown, the mechanism for going from the first to the second mode is clarified, and an estimate is given of the domain of variation of the governing parameters of the problem for which the first and second flow modes are realized.

1. Investigation of the Influence of the Initial and Boundary Conditions on the Solution

There are neither strict mathematical formulations nor proofs of the existence and uniqueness for the majority of gasdynamics problems. The situation is analogous for numerical methods of their solution: investigations associated with the convergence to the desired solution and stability have been performed rigorously only for linear equations. An indirect indicator of the uniqueness of a solution when a numerical algorithm is used to solve nonlinear equations is the independence from the selection of the zero approximation, which should evidently correspond to the physical meaning of the problem. The influence of the initial flow field on the soloution (see [5]) is clarified in this section by the example of flow around a V-shaped wing.

Let us consider the flow around a V-shaped wing with $\gamma = 100°$, $\psi = 29°30'$, at an angle of attack $\alpha = 15°$ to a stream with $M_\infty = 3.95$ (Fig. 1) under three distinct initial conditions: 1) the flow behind a weak shock

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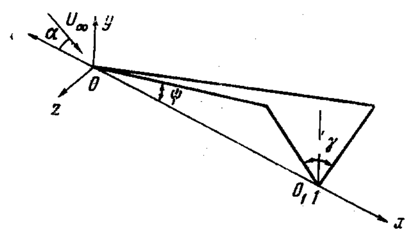

Fig. 1

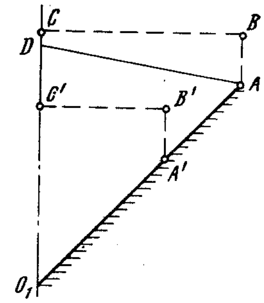

Fig. 2

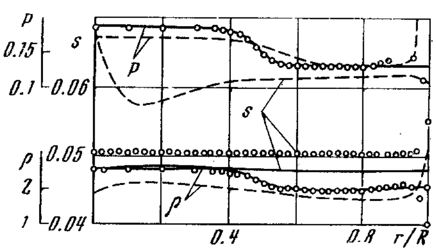

Fig. 3

at the leading edge; 2) the flow behind a strong shock at the edge; 3) the unperturbed flow. The computational domain $O_1A'B'C'$ for 1) (Fig. 2) has been selected in such a manner that the domain of influence of the plane of symmetry is inside it. A rectangular trapezoid O_1ABC (Fig. 2) has been selected as computational domain for the computations in 2) and 3), and it has been assumed that the stream parameters at the point A correspond to flow behind a strong shock for 2) and to the unperturbed stream for 3). The parameters at all the remaining points of the segment AB corresponded to the unperturbed stream. The segment BC is arranged in such a manner that the wake AD of the strong shock would lie below BC. The values of the stream parameters on the segments AB and BC did not vary during the computation.

The results of the computations are presented in Figs. 3 and 4, where the solid line corresponds to 1), the dashes to 2), and the open circles to 3). On the graphs p is the pressure referred to twice the velocity head of the unperturbed stream, ρ is the density referred to the unperturbed stream density, $s = \rho/\rho^{\varkappa}$, where $\varkappa = c_p/c_v = 1.4$, r is the distance between a point on the wing surface and the plane of symmetry, R is the distance between the leading edge and the plane of symmetry, and y is the distance between the point O_1 and a point O_1C on the axis of symmetry, $h = O_1A \cos(\gamma/2)$.

It is seen from the graphs presented that the results of computations for 1) and 3) are in good agreement in the whole flow field, where the accuracy of determining the stream parameters for 1) is higher as compared with 3). A sharp rise in p, ρ, s in the neighborhood of $r/R = 1$ for 3) is seen clearly in the graphs in Fig. 3, and this corresponds to a weak shock being formed at the leading edge.

Curves corresponding to 2) show that a solution different from the solution for 1) and 3) is obtained in this case, hence let us note the following singularities:

1) The location of the bow shock is similar for all the modifications computed, particularly in the neighborhood of the axis of symmetry.

2) The values of p, ρ, s are close to each other on and in some neighborhood of the bow shock in all three modifications.

3) The values of p in the uniform stream domain on the wing are similar in all the modifications.

4) A narrow zone of high gradients in s (the entropy layer) exists near the wing surface.

Actually, the solution of the following problem has been obtained in the computation of 2): Let us assume that a detached shock is formed in the flow around the leading edge, such that the stream parameters behind it on the wing surface at the point A equal the corresponding stream parameters behind a strong shock attached to the leading edge; let us assume further that the shock-layer thickness in the neighborhood of the point A is small and less than the value of the mesh spacing in the y-axis direction. Under these assumptions, the solution obtained for 2) is a solution of the problem of the flow around a V-shaped wing with slightly blunted leading edges.

Indeed, conditions corresponding to a strong shock are conserved at the point corresponding to the leading edge throughout the build-up process. Hence, there is a constant intensity perturbation on the edge for all the iterations, whereupon a domain of entropy-layer type is formed near the wing surface, which is

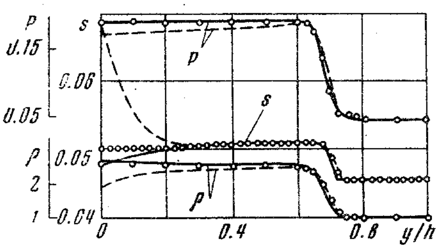

Fig. 4

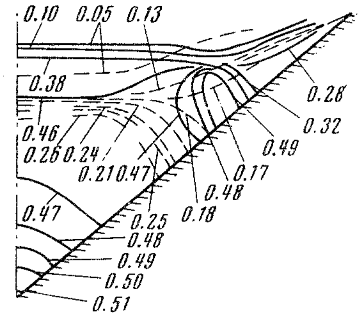

Fig. 5

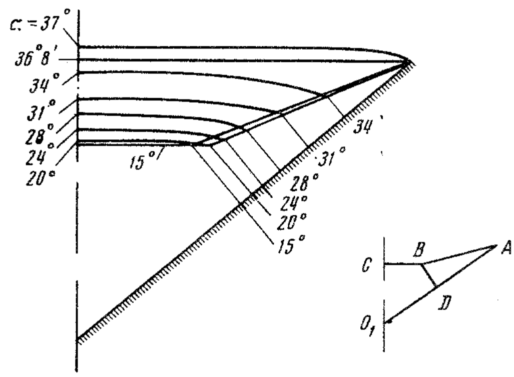

Fig. 6

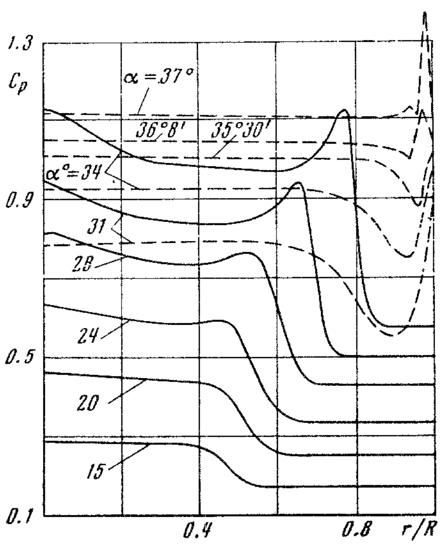

Fig. 7

caused by the discrepancy between the solution as a whole and the boundary conditions on the leading edge. The singularities of the solution noted above indeed become conceivable (compare with the results in [6], for example). Therefore, 2) to some degree simulates the flow around a V-shaped wing with blunt leading edges and permits making qualitative deductions about the flow picture.

Let us note that the solution obtained in 2) is not a flow with the formation of a strong shock at the edge, as the abrupt change in the stream parameters in the neighborhood of the leading edge indicates.

The analysis carried out on the dependence of the solution on the initial and boundary conditions shows that if the perturbations introduced artificially into the solution (because of the specific method of assigning the boundary conditions) are discarded, the solution is unique, where the first flow mode is realized for the considered values of γ, ψ, M_∞, α.

Let us emphasize the singularity in the behavior of the solution in the neighborhood of the right boundary, which is important to the subsequent analysis: If the values of the stream parameters on the right boundary do not correspond to the solution which is realized as a whole, then this discrepancy is manifested as an abrupt change in the values of the parameters near the right boundary of the computational domain, and which of the possible modes is realized can always be determined.

2. Passage from the First to the Second Flow Mode

Let us select a V-shaped wing with the angle $\gamma = 100°$, $\psi = 29°30'$ as the object of investigation and let us consider the flow around it at $M_\infty = 3.95$ at all angles of attack down to a detached bow shock (α_0). Let

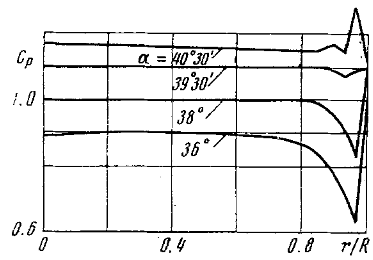

Fig. 8

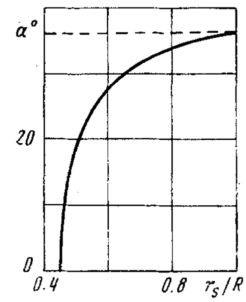

Fig. 9

α_M denote the angle of attack corresponding to the flow around a wing with a strong plane shock at the leading edges. In this case the stream behind the shock is uniform. For the wing under consideration $\alpha_M = 36°8'$, $\alpha_0 = 41°$.

Let us examine the isobars of the stream field around the wing for $\alpha = 20°$ (dashes) and $\alpha = 31°$ (solid lines, Fig. 5) and the shape of the shocks for various values of α (Fig. 6) obtained as the loci of points of maximum pressure gradient. It is seen from the figures presented that the flow for $\alpha < \alpha_M$ occurs with the formation of a Mach-type bow wave and an inner shock. As the angle of attack increases, the central part of the stream domain bounded by the central shock BC, the plane of symmetry O_1C, and the inner shock BD increases (Fig. 6) so that the inner shock is displaced to the leading edge and the central shock is displaced upwards.

In order to determine the flow modes based on the deductions in Sec. 1, the flow was analyzed by using the stream behind both a strong shock on the edge and behind a weak shock as the initial field. The results are shown in Fig. 7 as the diagram of the pressure coefficient distribution over the wing span $(C_p = 2(p - p_\infty / p_\infty V_\infty^2)$. It is seen in the graphs presented that a characteristic sharp diminution in C_p is observed near the leading edge for angles of attack $\alpha < \alpha_M$ (compare with Fig. 3), in a calculation from the strong shock parameters (dashes), while the behavior of the curve for $\alpha > \alpha_M$ is distinct from the behavior of the curve for $\alpha < \alpha_M$ and is characterized by fluctuations in the value of C_p near the right boundary. These fluctuations are explained by the small number of grid points behind the shock at the right boundary (just one) and are a numerical effect not related to the physical flow picture.

Presented in Fig. 8 are $C_p(\overline{r})$ diagrams obtained when using the parameters behind a strong shock as the initial field for a wing with $\gamma = 70°$, $\psi = 20°$, $M\infty = 6$ for diverse α ($\alpha_M = 39°30'$). The behavior of the curves is analogous to those of Fig. 7. It should be noted that the numerical solution for $\alpha = \alpha_M$ agrees with the exact solution with high accuracy.

As follows from the results in Sec. 1, the behavior of the $C_p(\overline{r})$ curves presented in Figs. 7 and 8 indicates the realization of the first flow mode for $\alpha < \alpha_M$ and the second for $\alpha \geq \alpha_M$.

It has been remarked above that the inner shock approaches the leading edge as α increases, as is seen well in Fig. 7. The position of the shock from r_s at the wing wall for $\gamma = 100°$, $\psi = 29°30'$, $M\infty = 3.95$ is illustrated in Fig. 9 as a function of the angle of attack. It is seen that as $\alpha \to \alpha_M$ the inner shock approaches the leading edge and $r_3 = R$ for $\alpha = \alpha_M$. As α increases further, the first flow mode is not realized since the inner shock tends to "depart" behind the edge.

The mechanism of second flow-mode formation becomes comprehensible: The second flow mode is a result of the inner shock wave "displacing" the weak shock being formed at the leading edge.

It is interesting to note that as α increases, not only the size of the central flow domain but also the qualitative nature of both the isobar field and the pressure diagram vary (Figs. 5 and 7). If the pressure diagram at the wall is monotone for moderate angles of attack $\alpha < 15$-$20°$, then as α increases the monotoneity is spoiled and a break occurs directly behind the inner shock front and a pressure rise is again observed in the neighborhood of the plane of symmetry.

Effects analogous to those noted above were also observed in experiment [3], namely, as α increased r_s increased, and an abrupt nonmonotoneity is characteristic for the spanwise pressure diagram for $\alpha < \alpha_M$ behind the inner shock front, while the quantity C_p along the span varies slightly for $\alpha \geq \alpha_M$ and is almost constant. This qualitative agreement with experiment indicates that the sharply nonmonotone nature of the behavior of $C_p(\bar{r})$ is explained not only by the effect of inner shock interaction with the boundary layer but also by the properties of the inviscid part of the flow.

It follows from the analysis conducted that, if a flow mode with a plane strong shock on the leading edges exists for M_∞ = const in the $0 < \alpha < \alpha_0$ range of angles of attack and the Mach number in the flow behind the wave is hence greater than one, then the flow occurs at $\alpha < \alpha_M$ with the formation of a weak shock at the edge. For $\alpha > \alpha_M$ the plane of the strong shock being formed at the edge is below the plane resting on the leading edge, hence the initial shock direction is into the wing. However, a numerical computation indicates the existence of a convex wave (Fig. 6). This says that for $\alpha > \alpha_M$ either a mode with a strong shock which has an inflection near the leading edge is realized and is not captured in the computation because of its nearness to the leading edge, or a mode with a detached wave is realized.

The same situation evidently holds also for α = const when M_∞ varies in the neighborhood of that number M_∞ where the flow with one plane strong shock is realized.

The passage from the first to the second flow mode has been considered above with viscosity effects neglected. In reality, because of the presence of a separation zone ahead of the inner shock and the appearance of a new shock within the flow in this connection, the passage from a flow with the formation of the first mode into the second occurs for $\alpha < \alpha_M$.

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