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Technical Note

A Model Describing Rock Cutting with Conical Picks

By

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Introduction

A number of models describing drag bit cutting of rock have been presented during the last decades. These models are attempts to describe the reality in simplified terms and mathematical expressions. The purpose is to gain a better understanding of the cutting process and to create a tool for design of picks and machines. Any model will almost certainly contain several simplifications, but a good model should still resemble the actual cutting of the rock.

The perhaps most commonly used model was presented by Evans (1962). The basics of this model is also used by Roxborough (1973), Finnie et al. (1977) and others. The model by Evans assumes a wedge which is pressed into a piece of rock or coal. The rock breaks in tension along a circular failure surface. The failure starts at the wedge tip, initial direction is tangential to the tip, and reaches the surface some distance in front of the wedge.

Nishimatsu (1972) proposes a slightly different geometry. The failure surface is plane and the stress varies along the plane according to a specified function. Mohr's failure criterion is assumed to be valid. From this the cutting force on the tool edge can be calculated.

In both of these models the cutting force is determined by the geometry of the pick and the chip, and by rock parameters. They also have two more basic assumptions in common. The first one has been indicated earlier, all broken chips will have the same geometry. This has been shown to be quite valid when the cutting situation is identical to the model assumption (Roxborough, 1973). The chips will often have the proposed shape. The second assumption is that the top rock surface is smooth and without previous cuts. This is of course necessary in order to obtain the specified chip geometry.

Both of these assumptions are not valid in continuous cutting. The rock surface is to a large extent affected by previous cuts. It is difficult to quantify the influence this will have on the cutting result. Even in a homogeneous rock there will be chips of many different shapes and sizes, since the models are not exact descriptions of actual cutting.

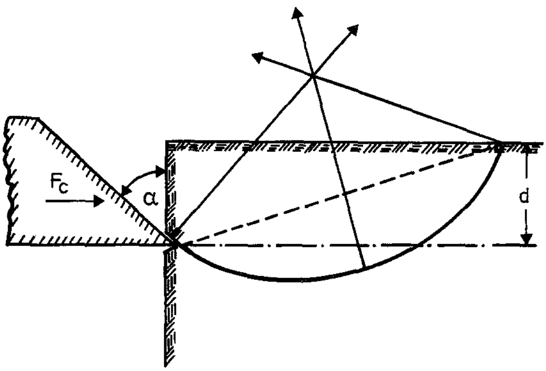

Fig. 1. Geometry of the model by Evans (1962)

The first part of this investigation deals with the validity of the geometrical assumption, i. e. chips of similar size and shape. In the second part, a model based upon measurements of actual cutting force is presented. These two parts are based upon two separate series of tests.

Test Equipment

All tests were performed in a full-scale cutting machine. The cuts were linear and parallel, and at spacings which were previously determined to be near optimum conditions with regard to specific energy. The tool is a single point-attack pick, Vimet W 47 in a CB 47 tool box. It is mounted so that the pick angle is the same as on a roadheader, but in line with the cutting direction. Normally it would be mounted with a skew angle of approximately 5^0 to give a symmetrical wear. Here the cutting length was so short that wear does not have to be considered. Pick forces were measured by a dynamometer. Pick velocity was 0.1 m/s.

A 1 m^3 block of concrete simulated a medium strength sedimentary rock. When mixed according to specifications it should have a compressive strength of 80—90 MPa. Tests on cubes, which is the standard procedure for concrete, indicated a compressive strength of 80 MPa. A test according to ISRM standards would probably have given a slightly higher value.

Chip Geometry Study

The basic idea of this test was to measure a large number of chips, and to determine if they can be considered as similar in shape. The rock surface was prepared before the test, with a number of cuts with the same spacing as in the test. Each cut was made in a previous groove, so the situation is quite different from the one in the models presented earlier.

The hypothesis to be tested in this experiment was:

The largest chips formed during one cut have the same shape.

From observations of the cutting result it is obvious that chips can have almost any size. The size is only limited by spacing between cuts and cutting depth. Therefore it was not considered meaningful to check any hypothesis regarding the size of chips.

In order to describe the chip shape mathematically we must place the chip in a coordinate system. The origin of coordinates is placed at the lowest point of the chip. We also assume that the chip has three orthogonal planes of symmetry, through the center of the chip. This assumption is not necessary for the following calculations, but it is supported by observations of broken chips in this test.

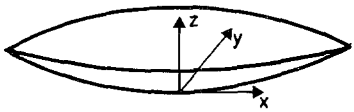

Fig. 2. General chip geometry

If the hypothesis about chips having the same shape is correct, it should be possible to describe the chips by a mathematical function. The lower surface of the chip is described by a general function in the coordinate system x, y, z.

$$z = ax^2 + bx + cy^2 + dy \tag{1}$$

where a, b, c, d are constants. By giving these constant different values, the chip surface can be "bent" to almost any shape. The chip size is such that

$2\,l$	= length	(x-direction)
$2\,w$	= width	(y-direction)
$2\,t$	= thickness	(z-direction)
V	= volume	

A few other equations were also tested, but they were not as flexible as Eq. (1). There are of course other equations that would be even better, but Eq. (1) was considered suitable for this investigation.

The chip volume can be calculated by solving the integral

$$V = 8 \int_0^l \int_0^w \int_z^t (1 \cdot dz)\, dx\, dy. \tag{2}$$

Using (1) and solving the integral gives

$$V = 8\, lw \left(t - \frac{al^2}{3} - \frac{bl}{2} - \frac{cw^2}{3} - \frac{dw}{2}\right). \tag{3a}$$

Rearranging the terms gives

$$\frac{l^2}{3}a+\frac{l}{2}b+\frac{w^2}{3}c+\frac{w}{2}d=t-\frac{V}{8\,lw}. \tag{3b}$$

If the largest chips have the same geometry they could all be described by Eq. (1), provided that the constants *a*—*d* are known. Eq. (3b) contains, apart from these constants, also four measurable parameters *l*, *w*, *t* and *V*. Theoretically it would be sufficient to measure length, weight, thickness and volume of four chips to determine the values of *a*—*d*. The volume was calculated from the weight of the chip and the density of the concrete. To be on the safe side we have measured the ten largest chips formed during a 1 m cut. This gives ten equations to determine the four unknown constants. Since there is a scatter in data, the chips do not look exactly the same, it is necessary to measure more than four chips. Otherwise the solution vector will vary a lot between the tests.

Using matrix notation for these ten equations, we get

$$A\cdot\bar{x}=\bar{y} \tag{4}$$

where A contains the terms $l^2/3$, $l/2$, $w^2/3$ and $w/2$,

$\bar{x}$ contains *a*—*d*,

$\bar{y}$ contains the terms $t-V/8\,lw$.

This can easily be solved and give values of *a*—*d* correct in the sense of least squares, so that the chip shape can be reconstructed from Eq. (1).

Table 1. *Chip Geometry Study Values of Constants a—d and Length of Residual Vector Related to Length of* $\bar{y}$

Test no.	a (mm^{-1})	b (−)	c (mm^{-1})	d (−)	$\lvert A\cdot\bar{x}-\bar{y}\rvert/\lvert\bar{y}\rvert$
1	0.018	−0.11	−0.07	1.36	0.66
2	−2.57	31.5	0.08	−1.14	18.41
3	0.02	−0.06	0.009	0.21	0.56
4	−0.037	1.18	0.09	−0.92	0.43
5	0.49	−10.00	−0.32	4.39	4.58
6	0.36	−7.33	−0.11	1.89	2.90
7	−0.16	3.99	0.17	−3.21	1.95
8	0.24	−7.82	0.14	−2.16	1.43
9	−0.83	4.13	−0.07	1.79	2.56
10	0.07	−0.64	0.23	−3.12	1.17

The quality of the solution can also be checked by calculating $\lvert A\cdot\bar{x}-\bar{y}\rvert$ which should be 0 if $\bar{x}$ is an exact solution. This will of course not occur here, but if the chips do have the same geometry, the length of the residual vector should not be far from zero.

Table 1 shows that only three out of ten measured cuts gave reasonable values of the length of residual vector. The ratio between length of residual vector and the length of $\bar{y}$ is less than 1. Only in two tests (no. 3 and 4) could the chip shape be reconstructed from the calculated values of *a*—*d* and Eq. (1). In all other tests the plot of Eq. (1) resulted in shapes that were totally unlike the actual chips.

There is of course the possibility that something has occurred in the procedure of solving the system of equations. A subtraction between almost equal numbers may lead to large errors in the final solution. However, it is not likely that this would happen in seven tests out of ten. Therefore we must conclude that the reason for residuals much larger than 0 is differences in chip geometry. We cannot in a mathematical respect, say that the largest chips have the same geometry.

Cutting Force Study

The breaking of small and large chip must in some way be connected to variations in the cutting force. It is reasonable to assume that the time between force peaks can be related to the chip length. This is also in agreement with the theoretical models presented earlier. These models however, assume chips of constant length, which means that the time between failures in the rock must be almost constant. The results presented above indicate that this may not be quite correct.

A simple way to check this is to measure the cutting force in detail over a section of the rock. The chipping events can be observed quite easily, and the force-time curve will also give other useful information.

The cutting force (in the cutting direction) was recorded both by a computer and a digital oscilloscope at the same time. Most of the data evaluation was done by the computer. The oscilloscope was used only for measuring the time between consecutive failures in the rock. It would have been very difficult to design a computer program that could handle all eventualities. The force curve is usually very staggered in a random fashion, only in some parts quite cyclic and regular. The decisions on where failures actually occur could best be done manually.

The curves consist of two frequencies added to one another. The high frequency at 250—400 Hz is a natural resonance frequency of the pick and holder. It does not affect the rock breaking and will not be discussed further in this paper. The lower frequency is usually around 20—50 Hz, but may be much lower. This frequency is directly related to rock breaking events. It is therefore dependent on cutting speed. A higher speed will give a higher frequency, but the relationship is not necessarily linear.

The time, or distance along the cut, between failures in this rock is measured on plots of the cutting force. The curve is not always so easy to evaluate as the ones in Fig. 3. The computer calculates mean and average peak force during one cut. A peak value is defined as a value where five samplings on each side are lower than the current value. Ten samplings are approximately equal to 1.25 mm along the cut. This will give some errors but comparisons with a few manually calculated values indicate that the error is less than 10%.

Input energy is calculated as a numerical integration of the cutting force — displacement curve. This gives the actual energy coming from the tool to the rock. It eliminates the efficiency of the rest of the cutting machine.

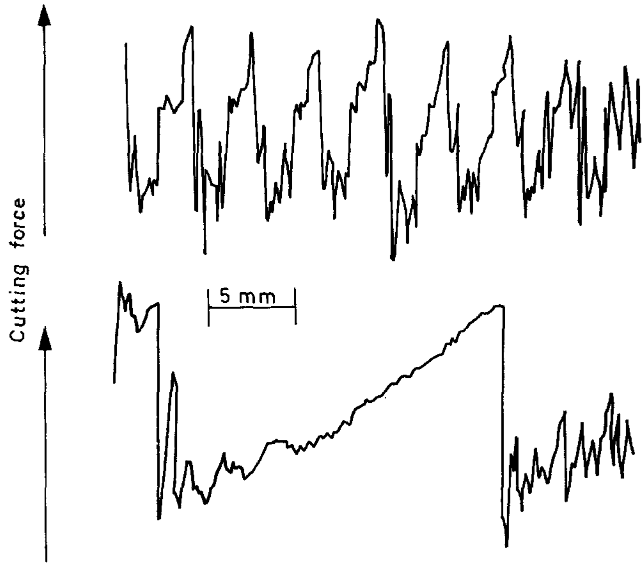

Fig. 3. Examples of cutting force curves with short and long intervals between fractures in the rock

The test consisted of 20 cuts at different depth and spacing. The depth was varied between 2—7 mm and the spacing was 22 mm (12 cuts) and 34 mm (8 cuts). Optimum spacing with respect to specific energy is around 25 mm for this combination of pick and rock.

Table 2. *Cutting Force Study Combinations of Parameters, Each Set Tested Twice*

Spacing (mm)	Cutting depth (mm)							
	2	3	3.7	4	5	5.3	6	7
22	×	×		×	×		×	×
34	×		×			×		×

The distance between force peaks varies a lot. Generally the distance is shorter than expected. Very few are longer than 10 mm, which should be compared with the longest chips of about 30—40 mm. When all measured distances between failures are placed in classes of 0—1, 1—2, 2—3 ... mm, we can see in Fig. 4 that the largest number is around 2—3 mm. Still, very few values are smaller than 1 mm.

The distribution is in fact very similar to a Poisson distribution which is well known in many statistical applications. The time between random and independent events often follows a Poisson distribution. Two good examples are the time between calls coming to a switchboard or customers entering a shop. The Poisson distribution depends only on one parameter, i. e. the mathematical expectation (mean) of x. The probability function is

$$Pr\,(x) = e^{-m}\, m^x/x \qquad (x=0, 1, 2, \ldots) \tag{5}$$

where m = mean.

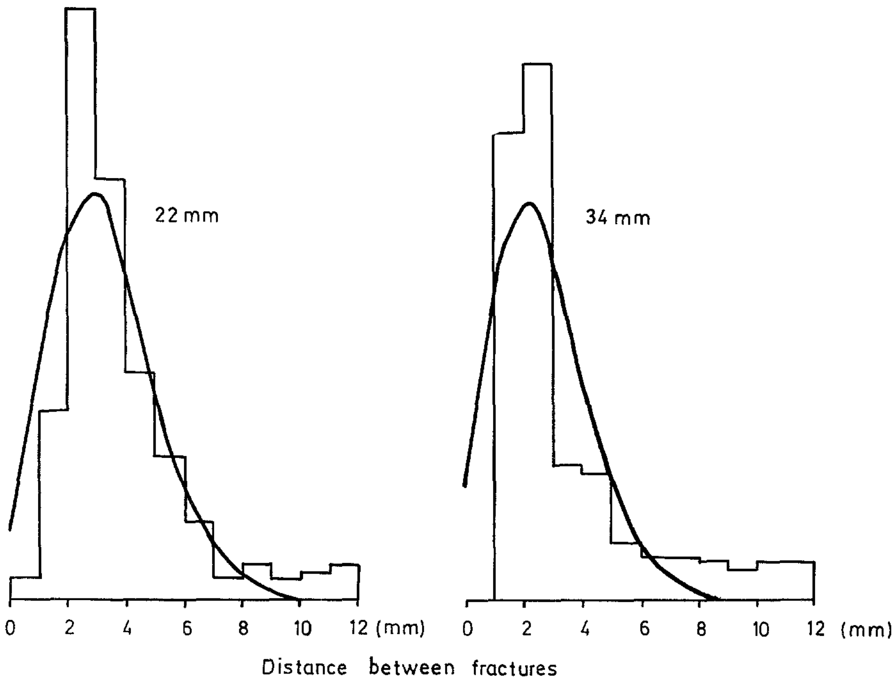

Fig. 4. Columns showing number of observation for different distances between fractures (left −22 mm spacing, right −34 mm spacing). Smooth curve is a Poisson distribution with the same mean as the columns, and scaled so that their areas are the same. All observations larger than 12 mm have been placed in the interval 11—12 mm

An Energy Model

From observations of the cutting force we can create a simplified model. The force curve has a saw-tooth shape where all peaks are equally high, and the force drops to zero after each failure. The distance between the peaks varies according to a Poisson distribution.

If the assumed curve is correct, the relationship between input energy and cutting force will be very simple.

$$E = L_t \cdot F_p/2 \tag{6}$$

where E = energy,
L_t = total cutting length,
F_p = peak force (average)

and

$$F_m = F_p/3 \tag{7}$$

where F_m = mean force.

Mean and peak forces can now be calculated using only the input energy per unit length of cut. We can compare the calculated values with those measured during the cuts. The comparison (Fig. 6) can only verify the model assumption itself, since the energy is calculated from the measured cutting force. If the model is valid it can be used for calculating pick forces from input energy. A much larger test in different rocks is necessary before a practical use, to check the possible influence of rock parameters on the validity.

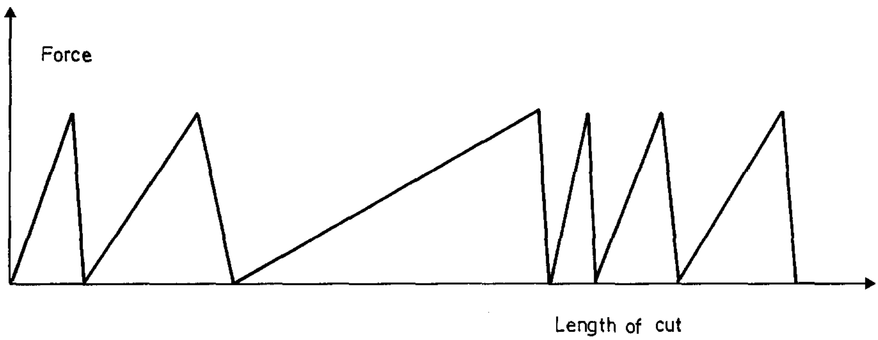

Fig. 5. Simplified cutting force

The measured and predicted mean and peak forces coincide very well in this test. From that we can draw the conclusion, that the simplification in Fig. 5 is quite valid.

Discussion

The fact that the time between failures in the rock follows a Poisson distribution allows us to draw some conclusions. The failures occur at random and are independent of each other. It is not possible to predict the size or shape of an individual chip from the previous cutting history. We must treat the cutting process statistically and describe it as a probability function. The formation of a single chip is often very much different from what we can call the average behaviour. A model which deals with a single chip will therefore not be sufficient to describe the whole cutting process.

The cutting has often been described as an interaction between local crushing in front of the pick and breaking of large chips. A rather large part of the crushing can be described as breaking of small chips. The process is quite similar for all chip sizes. When the force has reached a certain level, almost independent of chip size, a chip will break off. The chip size is only determined by conditions in the rock, ahead of the pick.

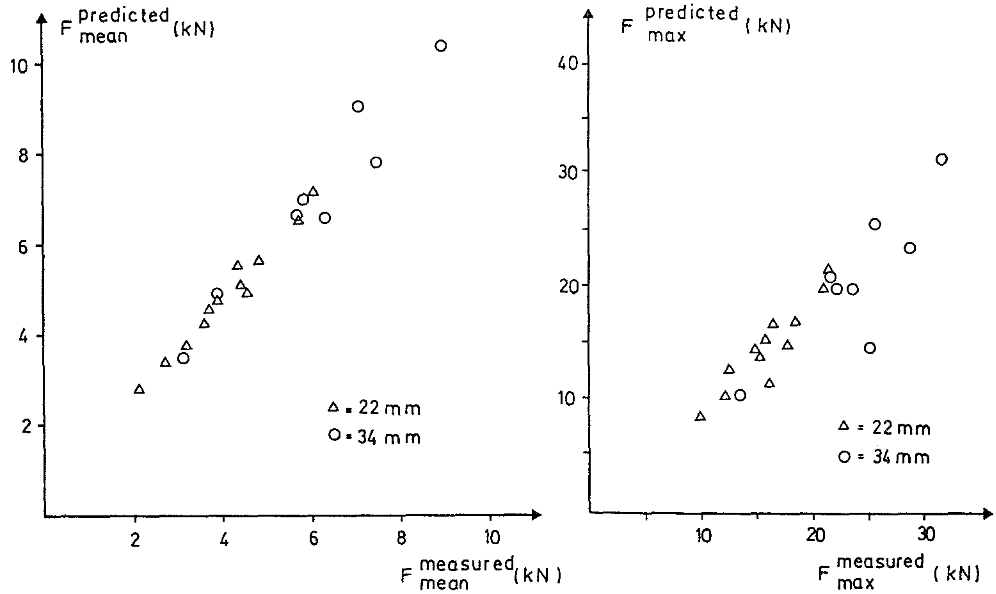

Fig. 6. Predicted versus measured peak and mean cutting force

The picks should be given a natural resonance frequency which is far from the most dominating fracture frequency. This latter frequency depends mainly on cutting velocity and rock type. A good design in this respect may lead to reduced vibrations.

The presented model assumes that the force drops to zero after each broken chip. This is probably valid only for sharp or nearly sharp picks. The whole curve will be lifted upwards for a blunt tool. Equations (6) and (7) will no longer be valid, but they can easily be modified for this new situation.

With some simplifications the fragmentation process can be described as a sequence with the following steps.

1. The tip of the pick comes in contact with the rock and is pressed into the surface with local crushing as a result. When the rock strength balances the pick force, the penetration will stop for an instant. Elastic energy is stored in the pick and tool box as the load increases rapidly.

2. At some higher force level the pick will penetrate further into the rock. A portion of the stored elastic energy will be consumed in local crushing. After that, the load will increase rapidly with no penetration of the pick.
3. A chip breaks off and the stored energy is used to propagate the crack. The force drops to zero if the pick moves without rock contact. Very soon the pick again comes in contact with the rock, and a new cycle starts.

These three steps come in a very rapid succession. The average distance between failures in this test was 3 mm which corresponds to 1.5 milliseconds at a pick velocity of 2 m/s. It should also be noted here that this description in many respects resembles button indentations in rock. This should not come as a surprise although ripping has been considered quite different from for example tunnel boring. It only shows that the indentation process is of fundamental importance in most types of mechanical rock fragmentation.

Acknowledgement

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References

Evans, I. (1962): A Theory on the Basic Mechanics of Coal Ploughing. Proc. Int. Symp. of Min. Res., Vol. 2. Pergamon Press, p. 761.

Finnie, I., Streit, R., Foote, L. (1977): Relationship of Coal Properties and Machine Parameters to Continuous Mining Machine Cutting Rates. NTIS PB-297291, p. 38.

Gehring, K. (1973): Möglichkeiten zur Beurteilung des Arbeitsverhaltens von Werkzeugen zur schneidenden Gesteinsbearbeitung. Berg- und Hüttenmännische Monatshefte *118*, no. 10, 319—327.

Nishimatsu, Y. (1972): The Mechanics of Rock Cutting. Int. J. Rock Mech. Min. Sci. *9*, 261—270.

Roxborough, F. F. (1973): Cutting Rocks with Picks. The Mining Eng., June 1973, 445—452.

Rånman, K. E. (1983): Pick Forces on Roadheaders. University of Lulea, Report 1983: 54 T, p. 31.