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Applicability of the Theory of Hollow Inclusions for Overcoring Stress Measurements in Rock

By

Bernard Amadei

University of Colorado, Boulder, Colorado, U. S. A.

Summary

Whenever solid or hollow inclusions are used as instrumented probes in overcoring techniques, "residual stresses" remain in the overcored rock sample and in the probes. When using such devices for computing the in-situ stress field components from measured strains or displacements, it is common practice to assume that the overcoring diameter is infinite and that there is a perfect bonding between the rock and the probes. The validity of these assumptions depends on the magnitude of the residual stresses at the rock-probe contact as compared to the tensile and shear strengths of the rock-probe bond material. It also depends on the distribution of residual stresses in the overcored sample.

In comparison to previous work, new expressions are proposed in this paper for the residual stresses associated with solid or hollow inclusion type stress probes in anisotropic ground. These expressions are presented in dimensionless form and are used to show that the distribution and magnitude of residual stresses depend on the isotropic-anisotropic rock character, the degree and type of rock anisotropy, the orientation of the rock anisotropy with respect to the hole in which the probes are located and the relative deformability of the rock with respect to the deformability of the material comprising the probes. The conditions that are required for neglecting the overcored sample diameter are also discussed. This is shown for rocks that can be described as isotropic, transversely isotropic and orthotropic materials.

Introduction

Several overcoring techniques have been proposed in the rock mechanics literature to measure the state of stress in-situ. Some of them use instrumented probes that can be described either as solid or as hollow inclusions. Such probes include the CSIRO (Worotnicki, G., Walton, R. J., 1976) and the LNEC (Rocha, M, et al., 1974) hollow inclusion cells, and the solid epoxy probes proposed by Rocha and Silverio (1969) and Blackwood (1977). As the probes and the rock have different deformability properties, an analysis is required to relate strains and/or displacements measured within the probes to the components of the in-situ stress field. This analysis must account for the elastic properties of the rock and the elastic properties and geometry of the inclusion. As far as hollow inclusions are concerned, such analysis already exists when the rock is modelled as an isotropic material (Duncan Fama, M. E., Pender, M. T., 1980) or as an anisotropic material (Amadei, B., 1983). The anisotropic analysis does not put any constraint on the type of anisotropy and on the orientation of the anisotropy with respect to the inclusion.

This paper begins with a solution for the elastic equilibrium of an anisotropic homogeneous medium bounded internally by an isotropic hollow inclusion of circular cross section and loaded at infinity by a three-dimensional stress field. The solution proposed by Amadei (1983) has been rederived and is presented in this paper in a new dimensionless form in terms of parameters describing the relative deformability of the medium with respect to the deformability of the inclusion. The problem of stresses induced by the inclusion in the medium is also discussed. The dimensionless-form character of the solution provides a valuable tool (i) for assessing the distribution and magnitude of residual stresses remaining after the overcoring of an isotropic or anisotropic rock sample containing a solid or a hollow inclusion type stress probe, and (ii) for discussing the conditions that are required for neglecting the diameter of the overcored sample when interpreting the data obtained from the probe in terms of in-situ stress components.

Equilibrium of an Anisotropic Medium Bounded Internally by a Hollow Isotropic Inclusion

Geometry, Definition and Solution of the Problem

Consider the equilibrium of an infinite, linearly elastic, anisotropic, continuous and homogeneous medium. The medium is bounded internally by a cylindrical surface of circular cross section which represents a borehole. The hole contains a hollow inclusion with outer and inner radii a and b respectively. The inclusion is assumed to be linearly elastic, isotropic, continuous and homogeneous and perfectly bonded to the anisotropic medium. Furthermore, both the hole and the inclusion are assumed to be infinitely long.

Consider the geometry of Fig. 1. Let x, y, z be a cartesian coordinate system with the z axis defining the longitudinal axis of the hole. The orientation of the hole and therefore the x, y, z coordinate system is defined with respect to a fixed arbitrary global coordinate system X, Y, Z. Similarly, the anisotropic medium has planes and/or axes of symmetry with respect to directions independent of the x, y, z directions. Thus, let x', y', z' be a system of cartesian coordinates attached to the anisotropy. From a practical point of view the x', y', z' coordinate system is attached to apparent planes of rectilinear anisotropy or symmetry in the anisotropic medium. In any case, x' is taken normal to the planes whereas y' and z' are contained within the planes. The orientation of the anisotropy and therefore the orientation of x', y', z' axes with respect to the X, Y, Z axes is also assumed to be defined by two angles β and ψ as shown in Fig. 2.

A general form of the constitutive relation of the anisotropic medium in the x, y, z coordinate system can be written as follows

$$(\varepsilon)_{xyz} = (A)\ (\sigma)_{xyz} \tag{1}$$

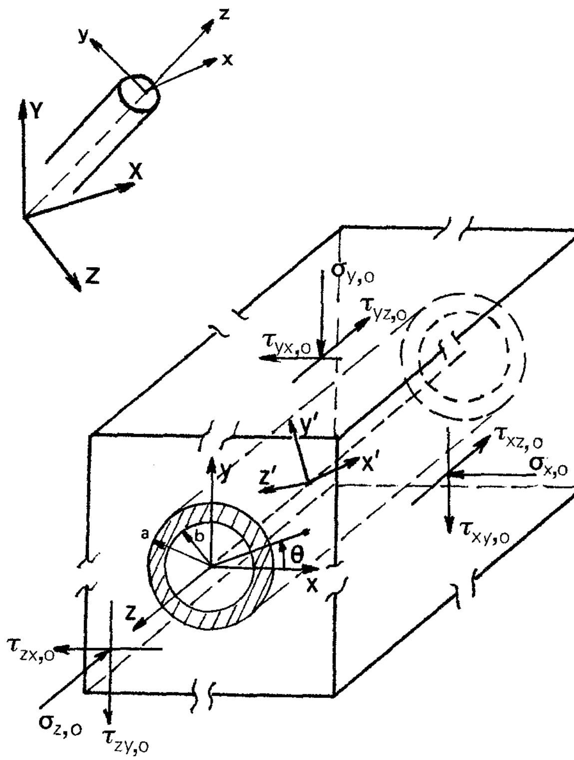

Fig. 1. Elastic equilibrium of an infinite anisotropic medium bounded internally by an isotropic hollow inclusion. Geometry of the problem

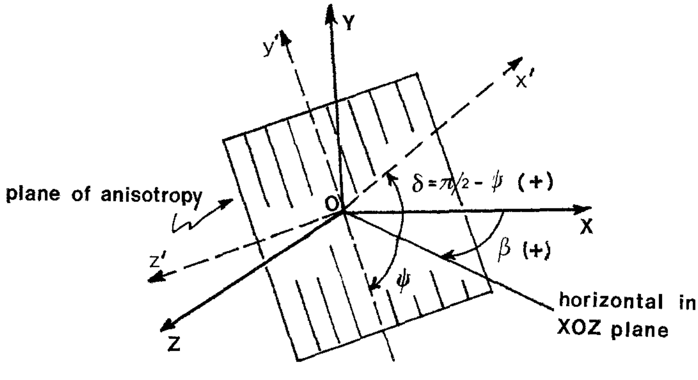

Fig. 2. Orientation of the coordinate system x', y', z' with respect to the global one X, Y, Z. Definition of angles β and ψ

where $(\varepsilon)_{xyz}$ and $(\sigma)_{xyz}$ are respectively (6×1) column matrix representations of the strain and stress tensors in the x, y, z coordinate system and (A) is a (6×6) compliance matrix whose components a_{ij} $(i, j=1$ to $6)$ can be calculated from those of the compliance matrix in the x', y', z' coordinate system. In general, matrix (A) has 21 distinct components. This number is further reduced if the anisotropic medium possesses any symmetry in the x, y, z coordinate system. This symmetry also exists in the inherent structure of the

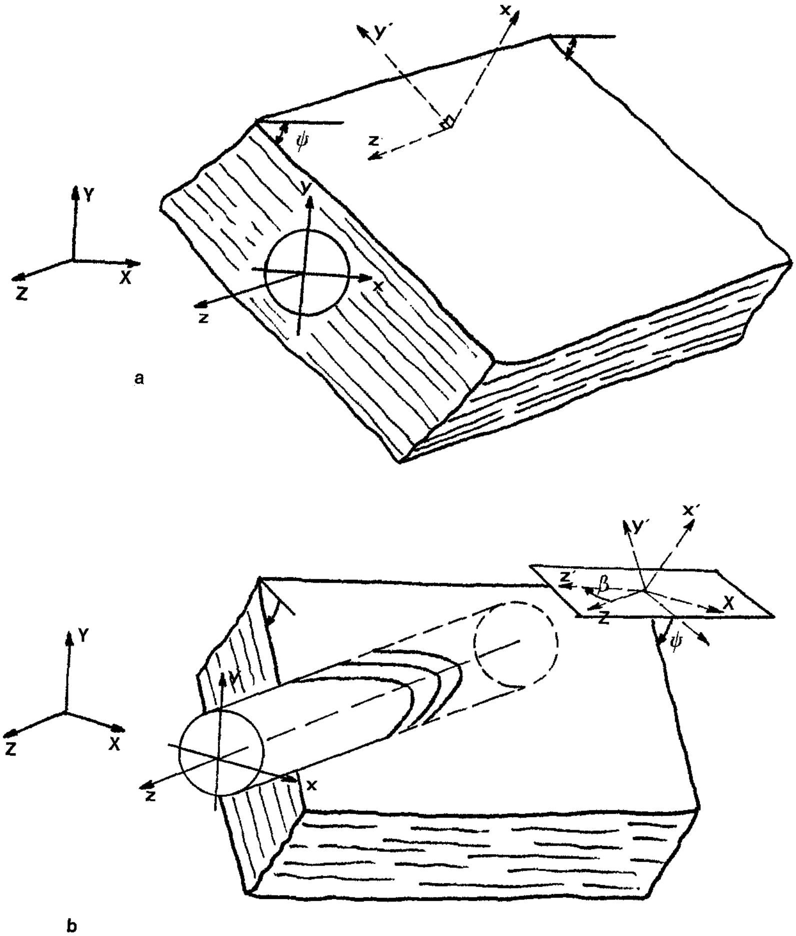

Fig. 3. Influence of orientation of anisotropy on the deformability properties in the x, y, z coordinate system. (a) Inclined transverse isotropy parallel to z axis, (b) inclined transverse isotropy non parallel to z axis

material. The number of distinct components a_{ij} is equal to 13 if there is a *plane of elastic symmetry* perpendicular to one of the three x, y, z axes; 9 if the medium is *orthotropic*, i. e., presents three planes of symmetry, each one being perpendicular to a coordinate axis, 5 if the medium is *transversely isotropic*, i. e., isotropic within a plane perpendicular to one of the three coordinate axes, 2 if the medium is *isotropic*. It is noteworthy that, except for the isotropic case, these numbers apply only when the coordinate system x, y, z is attached to the material symmetry directions associated with the x', y', z' axes. In any other coordinate system, the components of matrix (A) will depend on the compliances or elastic constants defined in the x', y', z' coordinate system and on the orientation of x, y, z with respect to x', y', z'. As an illustrative example, consider Fig. 3 [1]. In Fig. 3a the medium is transversely isotropic in the plane $y'z'$ of the $x'y'z'$ coordinate system and its deformability can be described by the following five elastic properties[2]

$$E_1;\ E_2 = E_3;\ \nu_{21} = \nu_{31};\ \nu_{32};\ G_{12} = G_{13} \tag{2}$$

where E_1, E_2, E_3 are Youngs moduli in the x', y', z' directions respectively; G_{12}, G_{13} are shear moduli in the $x'y'$, $x'z'$ planes respectively and ν_{ij} determines the ratio of strain in the j direction to the strain in the i direction due to a stress acting in the i direction. The ratios ν_{ij} and ν_{ji} are such that $\nu_{ij}/E_i = \nu_{ji}/E_j$. In the x, y, z coordinate system, the same medium has one plane of symmetry normal to the z axis and its deformability is now described by 13 elastic constants or compliances that depend on the five elastic properties defined in Eq. (2) and on the value of the angle ψ defining the inclination of the planes of transverse isotropy with respect to horizontal. These 13 coefficients reduce to 5 when ψ is equal to 0 or 90 degrees. Similarly, in Fig. 3b, the medium is again transversely isotropic in the x', y', z' coordinate system but its deformability in the x, y, z coordinate system is now described by 21 elastic constants or compliances since the planes of transverse isotropy are inclined with respect to the global coordinate system and the hole. These 21 terms depend on the five properties defined in Eq. (2) and on the values of the angles β and ψ defining the orientation of the planes of transverse isotropy with respect to the x, y, z axes. The same remarks would also apply if the medium was orthotropic in the x', y', z' coordinate system with the following nine elastic constants

$$E_1,\ E_2,\ E_3,\ G_{12},\ G_{13},\ G_{23},\ \nu_{21},\ \nu_{31},\ \nu_{32}. \tag{3}$$

The anisotropic medium of Fig. 1 is loaded at infinity by a three-dimensional stress field whose matrix representation (σ_0) is

$$(\sigma_0)^t = (\sigma_{x,0}\ \sigma_{y,0}\ \sigma_{z,0}\ \tau_{yz,0}\ \tau_{xz,0}\ \tau_{xy,0}). \tag{4}$$

In the present analysis, body forces are absent and allowance is made for a constant axial strain ε_{z0} to occur in both the anisotropic medium and the inclusion such that

$$\varepsilon_{z0} = a_{31}\,\sigma_{x,0} + a_{32}\,\sigma_{y,0} + a_{33}\,\sigma_{z,0} + a_{34}\,\tau_{yz,0} + a_{35}\,\tau_{xz,0} + a_{36}\,\tau_{xy,0} \tag{5}$$

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[1] In this example, the x, y, z axes are taken parallel to the global axes X, Y, Z for sake of clarity.

[2] The shear modulus G_{23} is not independent and can be expressed in terms of ν_{32} and E_2.

where a_{3i} ($i=1$ to 6) are the components of the third line of matrix (A) in Eq. (1).

The problem of the elastic equilibrium of an anisotropic linearly elastic medium bounded internally by an isotropic inclusion can be decomposed into two problems referred to as problems A and B as shown in Fig. 4. This decomposition is useful (i) for deriving a closed-form solution between the components of matrix (σ_0) and the components of strain, stress and displacement at each point in any cross section of the inclusion located far enough from its ends, and, (ii) for studying how far within the anisotropic medium the stress field is disturbed by the presence of the hollow inclusion.

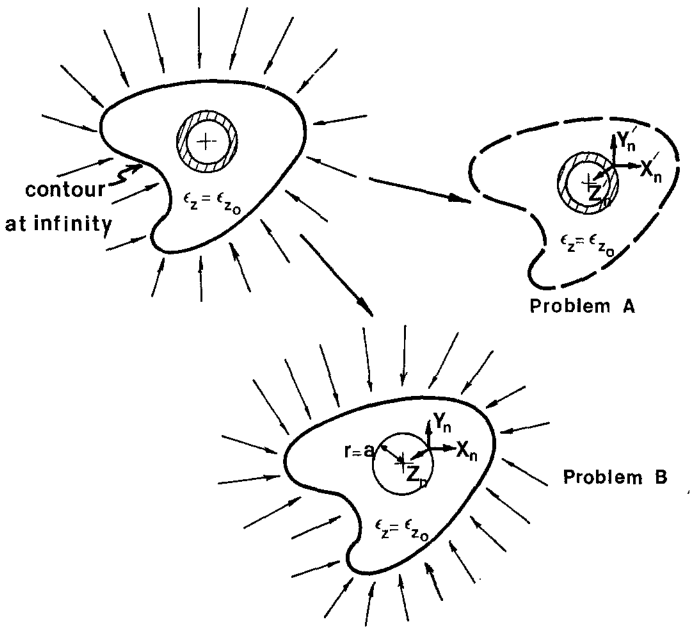

Fig. 4. Decomposition of the general problem into two problems A and B

In order to simulate the perfect bonding between the anisotropic medium and the inclusion, consider surface forces per unit area whose components in the x, y, z directions are equal to X_n', Y_n', Z_n' for problem A and equal to X_n, Y_n, Z_n for problem B such that

$$X_n' + X_n = 0, \ Y_n + Y_n' = 0, \ Z_n + Z_n' = 0. \tag{6}$$

Thus, problem A corresponds to the equilibrium of a linearly elastic, isotropic annulus loaded on its external contour by surface forces per unit area with components X_n', Y_n', Z_n'. Similarly, problem B is equivalent to the equilibrium of an infinite anisotropic body bounded internally by a hole of circular cross section loaded along its contour by surface forces per unit area with components X_n, Y_n, Z_n and loaded at infinity by a three-dimensional stress field. It is noteworthy that perfect bonding also implies that (i) at any point along the contact between the inclusion and the anisotropic medium, the radial, tangential and longitudinal displacement components, the radial stress σ_r, and the shear stress components τ_{rz}, $\tau_{r\theta}$ must be equal and, (ii) ε_{z0} must be the same in both materials.

Solutions to problems A and B expressed in terms of stress, strain and displacement components have been proposed by Amadei (1982) by assuming Fourier series type expression for X_n, Y_n, Z_n, and X_n', Y_n', Z_n', i. e.,

$$X_n = \sum_{m=1}^{N} (a_{mx} \cos m\theta + b_{mx} \sin m\theta)$$
$$Y_n = \sum_{m=1}^{N} (a_{my} \cos m\theta + b_{my} \sin m\theta)$$
$$Z_n = \sum_{m=1}^{N} (a_{mz} \cos m\theta + b_{mz} \sin m\theta) \tag{7}$$

where N is an arbitrary number and θ is an angle that assumes all values from zero to 2π for a complete circuit along the contour between the anisotropic medium and the inclusion. Using the perfect bonding condition along this contour, the $6N$ coefficients a_{mx}, b_{mx}, a_{my}, b_{my}, a_{mz}, b_{mz} $(m=1, N)$ and the difference between the rigid body rotation of the anisotropic medium and that of the inclusion, $w-w'$, can be expressed as linear functions of the components of matrix (σ_0) through the following equation[3]

$$(A_x)\,(X) = (C_x)\,(\sigma_0) \tag{8}$$

where (X) is a $(6N+1, 1)$ column matrix such that

$$(X)^t = (w-w' \; a_{1x}\; b_{1x}\; a_{1y}\; b_{1y}\; a_{1z}\; b_{1z} \;\ldots\; a_{Nx}\; b_{Nx}\; a_{Ny}\; b_{Ny}\; a_{Nz}\; b_{Nz}) \tag{9}$$

and (A_x), (C_x) are respectively $(6N+1,\ 6N+1)$ and $(6N+1, 6)$ matrices whose components depend on the elastic properties of the anisotropic medium, the orientation of the hole with respect to the direction of anisotropy, the Young's modulus, E, and Poisson's ratio, ν, of the inclusion as well as the geometry of the latter defined in terms of the ratio a/b.

Assume that the medium around the inclusion of Fig. 1 is *orthotropic* in the x', y', z' coordinate system with the nine elastic properties defined in Eq. (3). The orientation of one of the three planes of symmetry of the medium with respect to the y, x, z coordinate system is also assumed to be known and fixed. For that *fixed geometry*, the components a_{ij} of matrix (A) in Eq. (1) can be expressed as follows

$$a_{ij} = \frac{1}{E_1} F\left(\frac{E_1}{E_2}, \frac{E_1}{E_3}, \frac{E_1}{G_{13}}, \frac{E_1}{G_{12}}, \frac{E_1}{G_{23}}, \frac{E_1}{E_2}\nu_{21}, \frac{E_1}{E_3}\nu_{31}, \frac{E_1}{E_2}\nu_{23}\right) \tag{10}$$

where F is a linear function of some or all the terms in parentheses. Sub-

[3] See Eq. (5.27) in Amadei (1983).

stituting Eq. (10) into the expression for the components of matrices (A_x) and (C_x) and assuming given values for the geometry and Poisson's ratio of the inclusion, the components a_{ix}, b_{ix}, a_{iy}, b_{iy}, a_{iz}, b_{iz} ($i=1$ to N) that are the solution to Eq. (8) can be expressed as linear functions of the components of matrix (σ_0). A general form for these components is the following

$$\text{Component} = \frac{E}{E_1} [f_1 \, \sigma_{x,0} + f_2 \, \sigma_{y,0} + f_3 \, \sigma_{z,0} + f_4 \, \tau_{yz,0} + f_5 \, \tau_{xz,0} + f_6 \, \tau_{xy,0}] \tag{11}$$

where $f_1, f_2, \ldots, f_6$ are non-linear functions that depend on the following nine dimensionless parameters

$$\frac{E}{E_1}, \frac{E_1}{E_2}, \frac{E_1}{E_3}, \frac{E_1}{G_{13}}, \frac{E_1}{G_{12}}, \frac{E_1}{G_{23}}, \nu_{21}, \nu_{31}, \nu_{23}. \tag{12}$$

These quantities describe the relative deformability of the orthotropic medium with respect to the deformability of the inclusion. As far as $w-w'$ is concerned, Eqs. (11) and (12) still apply but the ratio E/E_1 in Eq. (11) only must be replaced by $1/E_1$. Substituting Eq. (11) into Eqs. (6) and (7) leads to expressions for the surface force components per unit area X_n', Y_n', Z_n', that have the general form of Eq. (11) as well as a dependency on the angle θ. These components can then be used as boundary conditions for problem A in Fig. 4. Using the analytical method proposed by Amadei (1982), a solution to that problem for the stress components in the r, θ, z coordinate system at any point in the inclusion can be expressed in a dimensionless form as follows

$$\begin{Bmatrix} \sigma_r \\ \sigma_\theta \\ \sigma_z \\ \tau_{\theta z} \\ \tau_{rz} \\ \tau_{r\theta} \end{Bmatrix} = \frac{E}{E_1} (Q_\sigma) \, (\sigma_0) \tag{13}$$

or in matrix form

$$(\sigma)_{r\theta z} = \frac{E}{E_1} (Q_\sigma) \, (\sigma_0) \tag{14}$$

(Q_σ) is a (6×6) matrix whose components depend on the nine parameters of Eq. (12), the Poisson's ratio and geometry of the inclusion, the orientation of the orthotropic planes of symmetry with respect to the inclusion and the coordinates $(r/a, \theta)$ of the point of interest. The strains at that point are obtained by multiplying both sides of Eq. (14) by the isotropic type compliance matrix for the inclusion. This leads to the following equation

$$(\varepsilon)_{r\theta z} = \frac{1}{E_1} (Q_\varepsilon) \, (\sigma_0) \tag{15}$$

Similarly, the radial, tangential, and longitudinal displacements at the point of interest can be expressed as follows

$$\begin{Bmatrix} \frac{u_r}{a} \\ \frac{v_\theta}{a} \\ \frac{w}{a} \end{Bmatrix} = \frac{1}{E_1} (Q_u)(\sigma_0) \tag{16}$$

In the last two equations, matrices (Q_ϵ) and (Q_u) are respectively (6×6) and (3×6) matrices whose components depend on the same parameters as the components of (Q_σ).

If the medium around the inclusion is now *transversely isotropic* with the five elastic properties defined in Eq. (2), then Eqs. (10) to (16) still apply but the nine terms in Eq. (12) must be replaced by the following

$$\frac{E}{E_1}, \frac{E_1}{E_2}, \frac{E_1}{G_{12}}, \nu_{21}, \nu_{23} \tag{17}$$

If the medium is now *isotropic* with two elastic properties $E_1 = E_2$ and $\nu_{21} = \nu_{23}$, then the five dimensionless quantities of Eq. (17) further reduce to two, e. g.,

$$\frac{E}{E_1}, \nu_{21}. \tag{18}$$

Induced Stress Field in the Anisotropic Medium and Limiting Cases

As shown in Fig. 4, the stress field in the anisotropic medium induced by the presence of the hollow inclusion is associated with the application of the surface force components per unit area X_n, Y_n, Z_n along the contour $r = a$ in problem B. According to Eq. (6), the expressions for X_n, Y_n, Z_n are identical in magnitude but opposite in sign to those associated with X_n', Y_n', Z_n'. If the medium is either orthotropic, transversely isotropic or isotropic as in the previous section, then using the analytical method proposed by Amadei (1982), and the dimensionless expression for the components of matrix (X) in Eq. (8), problem B can be solved for the stress components induced by X_n, Y_n, Z_n at any point in the anisotropic medium. These components have the general form of Eq. (14) i. e.,

$$(\sigma)_{r\theta z} = \frac{E}{E_1} (Q_\sigma^*)(\sigma_0). \tag{19}$$

The components of matrix (Q_σ^*) are of course different to those of matrix (Q_σ) but they depend on the same parameters.

The elastic equilibrium of an infinite anisotropic medium with a solid inclusion and the one of an infinite anisotropic medium with a hole containing no inclusion can be seen as two limiting cases of the general results discussed above. The former condition takes place when b vanishes whereas the latter one takes place when $a = b$ and when X_n, Y_n, Z_n vanish along the contour $r = a$.

Overcoring Techniques with Inclusion Type Instrumented Devices

Overcoring techniques have been used extensively in rock mechanics to measure the in-situ state of stress. These techniques can be classified as relief techniques, i. e., procedures that wholly or partially isolate a rock specimen from the stress field in the surrounding rock. Strain and/or displacement measurements on the specimen thus isolated are recorded in the vicinity of the point at which the state of stress has to be determined. Fig. 5 shows the three steps commonly involved when measuring stresses in-situ by overcoring. First, a large diameter hole is drilled to the required depth in the volume of rock in which stresses have to be determined. Then a small pilot hole is drilled at the end of the previous hole. An instrumented device is inserted in that hole. The device must be able to measure strains, displacements or both if required. Finally, the large diameter hole is resumed and resulting changes of strain and/or displacement within the instrumented device are recorded. Only overcoring techniques using devices that are positioned on the walls of the pilot hole are considered in this paper.

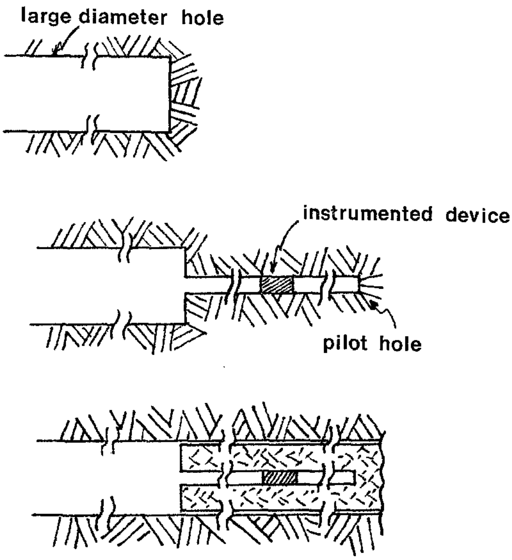

Fig. 5. Steps commonly involved in overcoring techniques with instrumented devices positioned on the pilot hole walls

Computation of in-situ stresses from overcoring measurements requires that an analytical solution exists between the strain and/or displacement measurements and the components of the in-situ stress field. Theoretically, that solution should account for rock properties such as homogeneity/heterogeneity, isotropy/anisotropy, continuous/discontinuous character and a possible non-linear rock behaviour. It should also account for the finite character of the overcored sample since the process of overcoring can be seen as cancelling the components of initial stress acting across a cylindrical surface in the rock.

As far as rock properties are concerned, no solution exists to date that accounts for combined "non ideal" rock characteristics such as heterogeneity, anisotropy, discontinuous character and non-linear behaviour. Instead, assumptions of homogeneity, continuity, isotropy and linear elasticity are commonly used in practice. Solutions that account for rock anisotropy instead of isotropy have been proposed in the literature with different degrees of assumption and sophistication (Amadei, B., 1982; Berry, D. S., Fairhurst, C., 1966; Becker, R. M., Hooker, V. E., 1967; Berry, D. S., 1968; Becker, R. M., 1968; Berry, D. S., 1970; Niwa, Y., Hirashima, K., 1971; Hirashima, K., Koga, A., 1977). Accounting for rock anisotropy when measuring stresses in-situ is particularly important when dealing with metamorphic rocks such as schist, slate, phyllite, gneiss or sedimentary rocks such as sandstone, shale or limestone. For such rocks, assuming isotropy may induce large errors in the in-situ stress determination (Amadei, B., 1982; Amadei, B., 1983; Amadei, B., and Walton, R. J., 1985/86).

Accounting for the size of the overcoring diameter in the analysis of in-situ stress measurements complicates the analytical solution. Therefore, it is common practice to neglect the finite character of the overcoring diameter in the solution by assuming that the process of overcoring is equivalent to applying a three-dimensional stress field at infinity. The components of that stress field are equal in magnitude but opposite in sign to that of the in-situ stress field. The validity of this assumption depends firstly, on the type of instrumented device used in the overcoring technique. If the device in contact with the pilot hole rock walls does not interfere with the deformation of the rock during overcoring, then the overcored sample will be completely free of strains and stresses after completion of overcoring. This is true regardless of its size and shape and therefore the overcoring diameter can be set equal to infinity. Devices that permit the strain relief without interference include the U. S. Bureau of Mines gage (Merrill, R. H., 1967) and the CSIR triaxial strain cell (Leeman, E. R., Hayes, D. J., 1966). If the instrumented device does interfere with the rock deformation and can be described as a hollow or solid inclusion probe perfectly bonded to the pilot hole walls, then overcoring does not produce a total relief since the presence of an inclusion results in the retention of residual stresses and strains within the inclusion and in the overcored sample in a region near the contact between the rock and the inclusion. Provided overcoring takes place beyond that region, the errors involved in neglecting the finite overcore diameter become insignificant and the overcoring diameter can be set equal to infinity. If this is not the case, the size of the overcoring diameter must be taken into account since it will influence the distribution of stresses and strains within the inclusion. Devices that induce residual stresses include the CSIRO (Worotnicki, G., Walton, R. J., 1976) or LNEC (Rocha, M., et al., 1974) hollow inclusion cells and solid epoxy probes such as the ones proposed by Rocha and Silverio (1969) and Blackwood (1977).

Whenever the overcoring diameter can be set equal to infinity, and the in-situ stress field components in an x, y, z coordinate system attached to the overcoring hole is represented by the components of matrix (σ_0) in Eq. (4), then the process of overcoring can be seen as applying a three-dimensional stress field equal to $-(\sigma_0)$ at infinity. Eqs. (15) and (16) can be used to relate respectively changes in strain or displacement measured during overcoring of a solid or hollow inclusion device, or directly on the walls of a pilot hole, with the components of the in-situ stress field. In these equations, (σ_0) must be replaced by $-(\sigma_0)$. Applications of these closed-form solutions for the analysis of data obtained with the CSIRO cell in isotropic or anisotropic rocks can be found in Amadei and Walton (in preparation).

Another problem associated with residual stresses induced by inclusion devices is the bonding between the rock and the inclusion. Interpretation of overcoring measurements in terms of in-situ stresses implies perfect bonding between the rock and the instrument. Since in-situ stress fields in rock are mostly compressive, compressive stresses will also be released during overcoring and the residual radial stress at the contact rock inclusion will be mostly tensile. If this stress is large enough and reaches the tensile strength of the bond between the rock and the inclusion, breaking and separation will take place and the in-situ stress measurement will be invalid. The same problem will arise if the residual shear stresses reach the shear strength of the bond material.

In view of the problems described above, a knowledge, or even just an appreciation, of the magnitude and distribution of residual stresses is required when interpreting in-situ stress measurement data obtained with inclusion instruments. This can be done by assuming first that the overcoring diameter is equal to infinity and by using Eqs. (15) or (16) to calculate from the overcoring measurements the components of the in-situ stress field defined by matrix (σ_0). Then Eq. (19) is used with (σ_0) replaced by $-(\sigma_0)$ to obtain the distribution of residual stresses in the rock modelled as an infinite medium. This distribution is then checked with respect to the following two simultaneous constraints:

(i) along the overcored sample outer surface, the radial component σ_r and the shear components, $\tau_{r\theta}$, τ_{rz}, of residual stresses must vanish (or be negligible),

(ii) the radial and shear residual stresses at the contact between the rock and the instrumented device must not reach the tensile or shear strengths of the bond material.

If both constraints are satisfied, using an infinite overcoring diameter to interpret the overcoring measurements is correct. Otherwise, the finite character of the overcoring diameter must be taken into account.

Two sets of parameters will influence the residual stress distributions when using inclusion type instrumented devices: (1) the type and geometry of the inclusion, and, (2) the deformability of the rock. The type refers to either solid or hollow inclusions whereas the geometry is defined for hollow inclusions only and refers to the ratio between their outer and inner radii. The deformability of the rock refers to the anisotropic rock character, the degree and type of rock anisotropy, the orientation of the anisotropy with respect to the hole in which the inclusion is located and the relative deformability of the rock with respect to the deformability of the inclusion. This is defined by the ratio E/E_1 and by the other dimensionless parameters

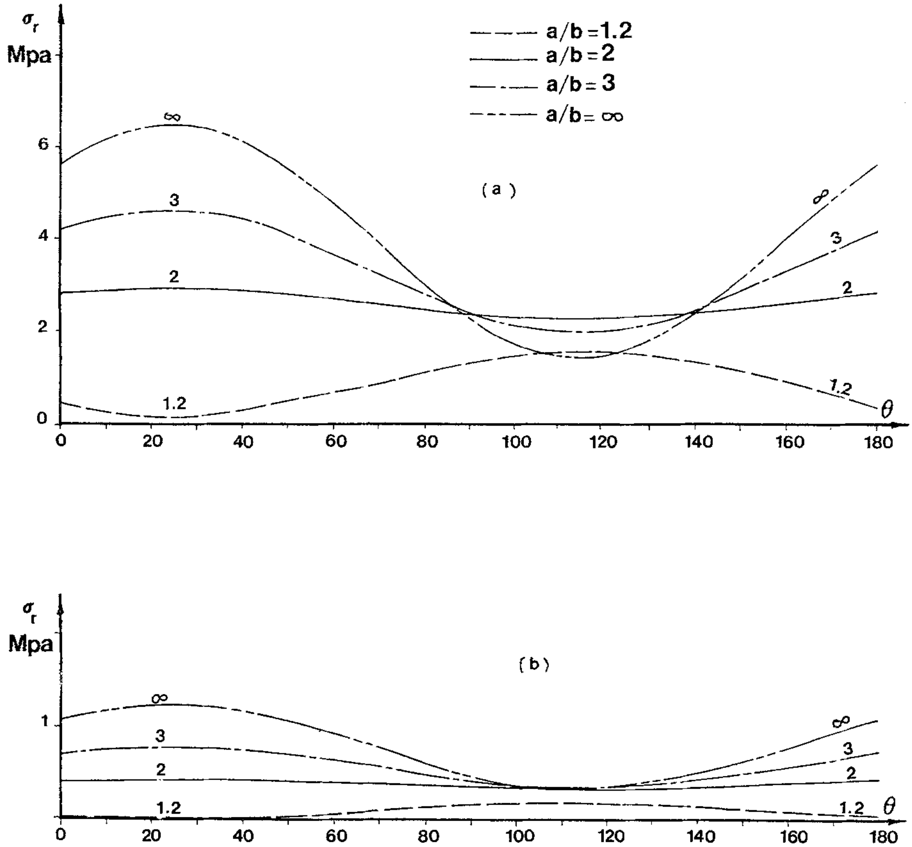

Fig. 6. Variation of the radial stress component σ_r at the contact inclusion, isotropic medium ($0 \leq \theta \leq 180^0$) when (a) $E/E_1 = 0.5$ and (b) $E/E_1 = 0.05$

of Eqs. (12), (17), (18), entering into the expressions of the components of matrix $(Q_\sigma{}^*)$ in Eq. (19). It is noteworthy that there are several ways to understand the variation of the ratio E/E_1. For instance, an increase of this ratio will take place if the rock becomes softer in the x' direction attached to the rock anisotropy, and/or the inclusion becomes stiffer. It should be kept in mind that the limiting case $E = 0$ and $a = b$ will induce no residual stresses in the rock since the inclusion vanishes.

In order to illustrate the previous remarks, two numerical examples are now presented with the geometry of Fig. 1. As a first example, consider an infinite isotropic medium with Young's modulus E_1 and Poisson's ratio equal to 0.25 that is loaded at infinity by a stress field with components $\sigma_{x,0}=7$ MPa; $\sigma_{y,0}=2.5$ MPa; $\sigma_{z,0}=5.8$ MPa; $\tau_{yz,0}=-1.8$ MPa; $\tau_{xz,0}=1.0$ MPa; $\tau_{xy,0}=2.6$ MPa[4]. The inclusion has a Poisson's ratio equal to 0.3, a Young's modulus E and a geometry defined by the ratio a/b. Two values of the ratio E/E_1 are considered (0.5, 0.05) and four values of a/b are considered (1.2, 2, 3, ∞). The case $a/b=\infty$ corresponds to a solid inclusion. Figs. 6a and 6b show respectively the variation of the radial stress component σ_r at the contact between the inclusion and the isotropic medium for the two values of E/E_1 and the four values of a/b considered above. σ_r varies between high and low values especially when $E/E_1=0.5$ and can be as large as some of the stress components applied at infinity. The magnitude of σ_r is strongly reduced as the ratio a/b decreases and/or when $E/E_1=0.05$. Fig. 7a shows the variation of the radial stress induced

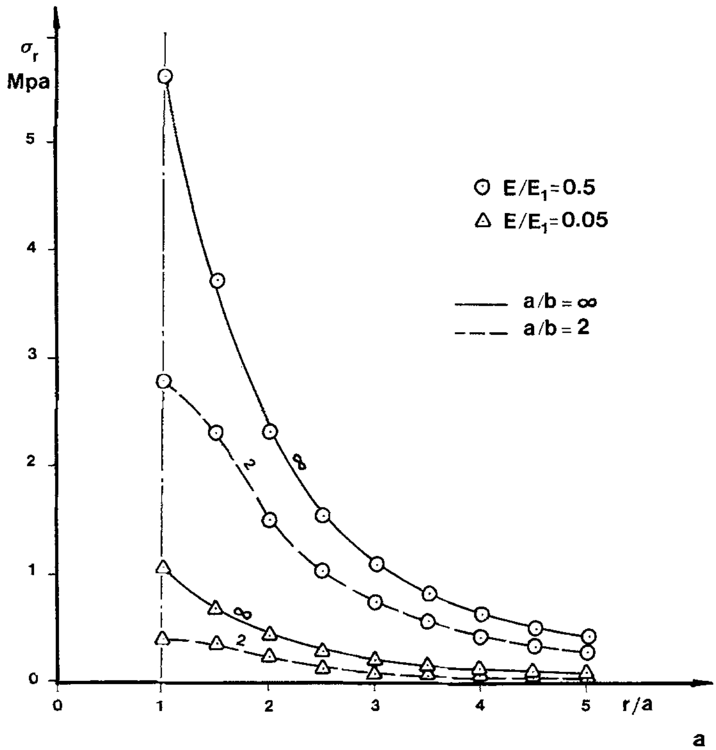

Fig. 7

by the inclusion in the isotropic medium along the x axis ($\theta=0^0$) of Fig. 1 for $a/b=2$ and infinity and for the two values of E/E_1 considered above. σ_r decays rapidly but is still non-negligible at a distance of five times the

[4] Stress field borrowed from Duncan Fama and Pender (1980).

radius of the hole, a, unless a/b is less than 2 and/or E/E_1 is equal to 0.05. Similar conclusions apply for the induced shear stress components τ_{rz}, $\tau_{r\theta}$ as shown in Figs. 7b and 7c. Let the medium and the inclusion considered

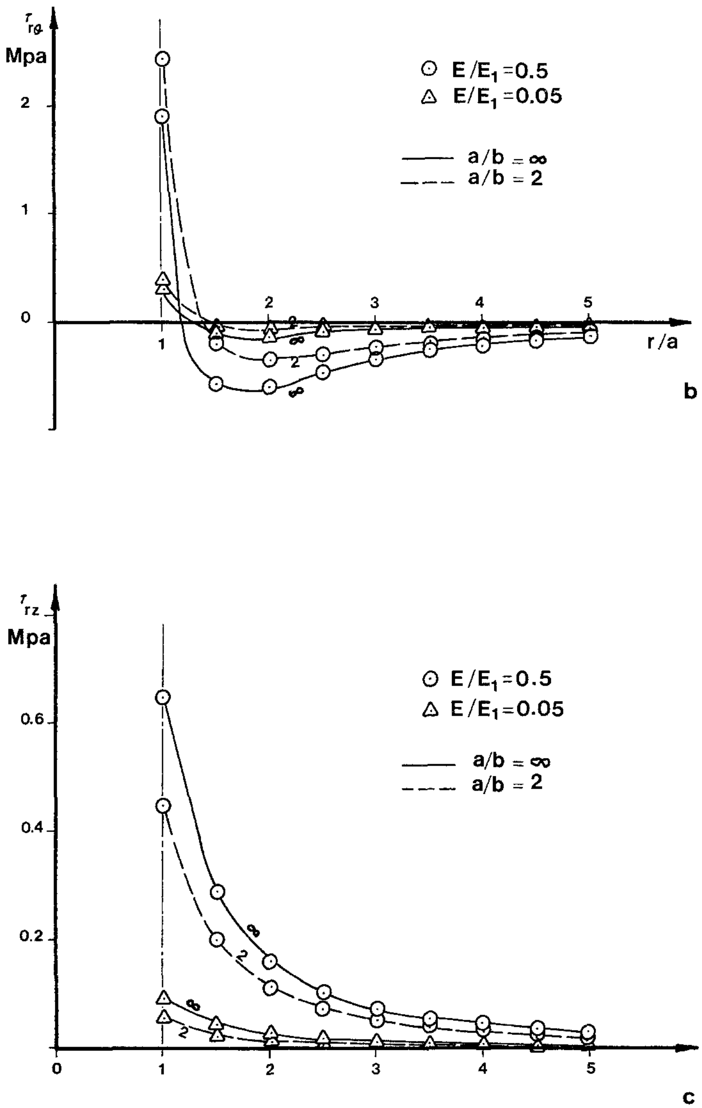

Fig. 7. Variation of stresses induced by an inclusion within an isotropic medium along the x axis of Fig. 1. (a) Radial stress σ_r, (b) shear stress $\tau_{r\theta}$ and, (c) shear stress τ_{rz}

above be respectively a rock and an instrumented probe that is to be overcored, and let the components of (σ_0) be those of the in-situ stress field. Constraints (i) and (ii), associated with the assumption of neglecting the size of the overcoring diameter, are most likely to be satisfied if for a solid probe the ratio between the Young's modulus of the rock to that of the probe material, E/E_1, does not exceed 0.05 or if for a hollow probe the same ratio is equal to 0.05 but the probe is thin walled (a/b less than 2). This conclusion was reached by assuming an overcoring diameter varying between three and five times the pilot hole diameter and a rock inclusion bond tensile strength ranging between 3 and 5 MPa. Similar conclusions about the adequacy of solid and hollow inclusions as overcoring instrumented devices have been proposed by Duncan Fama (1979) and Duncan Fama and Pender (1980) for rocks modelled as isotropic material sand by Amadei (1983) for rocks modelled as anisotropic materials.

As a second example, consider now a rock that can be modelled as an infinite transversely isotropic medium with a hollow inclusion that has the geometry and properties of a CSIRO cell. The planes of transverse isotropy correspond, for instance, to schistosity, foliation or bedding planes or to any apparent direction of rock symmetry. The orientation of these planes with respect to the inclusion is assumed to be known and to be defined by the angles β and ψ of Fig. 3b. Both angles vary between 0 and 90 degrees. The inclusion has fixed geometry and properties such that $E = 3500$ MPa, $\nu = 0.4$ and the ratio a/b is equal to 1.1875 (thin walled inclusion). The deformability of the rock is defined by the five elastic properties of Eq. (2). The domains of variation for the five parameters of Eq. (17) are arbitrarily defined as follows:

a) E/E_1 varies between 0.1 and 1. The first value corresponds to $E_1 =$ 35,000 MPa (defined as high modulus rock) and the second one to $E_1 = 3500$ MPa (defined as very low modulus rock).

b) ν_{32} and ν_{21} are equal to 0.25 and 0.27 respectively.

c) The domain of variation of E_1/E_2 depends on the values of ν_{21} and ν_{32}. According to Pickering (1970), the following condition must be satisfied for the strain energy to be positive.

$$\frac{E_2}{E_1}(1 - \nu_{32}) - 2\,\nu_{21}{}^2 > 0. \tag{20}$$

For the values of ν_{32} and ν_{21} considered above, this is satisfied if E_1/E_2 is less than 5.14. The following three values of E_1/E_2 are considered: 2, 1 and 0.5.

d) E_1/G_{12} is equal to 2.19, 4.38 and 8.75.

The stress field applied at infinity and defined by matrix (σ_0) has the same components as in the previous example.

An example of distribution of the radial stress component σ_r and the shear stress components $\tau_{r\theta}$, τ_{rz} at the contact between the rock and the inclusion is shown in Fig. 8 for fixed conditions of rock anisotropy. As far as the maximum value of σ_r along that contact is concerned, Fig. 9 shows its variation with the ratio E_1/E_2 and the angle β when $E/E_1 = 0.1$. The angle ψ, the ratio E_1/G_{12} and the other parameters are all fixed. σ_r increases with the angle β when E_1/E_2 is equal to 2 and decreases as β increases for E_1/E_2 equal to 1 and 0.5. Also shown in this figure is the increase of σ_r with E_1/E_2 for any fixed value of the angle β. This variation can also be seen as an increase of σ_r with the ratio E/E_2 since $E/E_2 = (E/E_1)\,(E_1/E_2)$ and the ratio E/E_1 is fixed. The same behaviour was observed

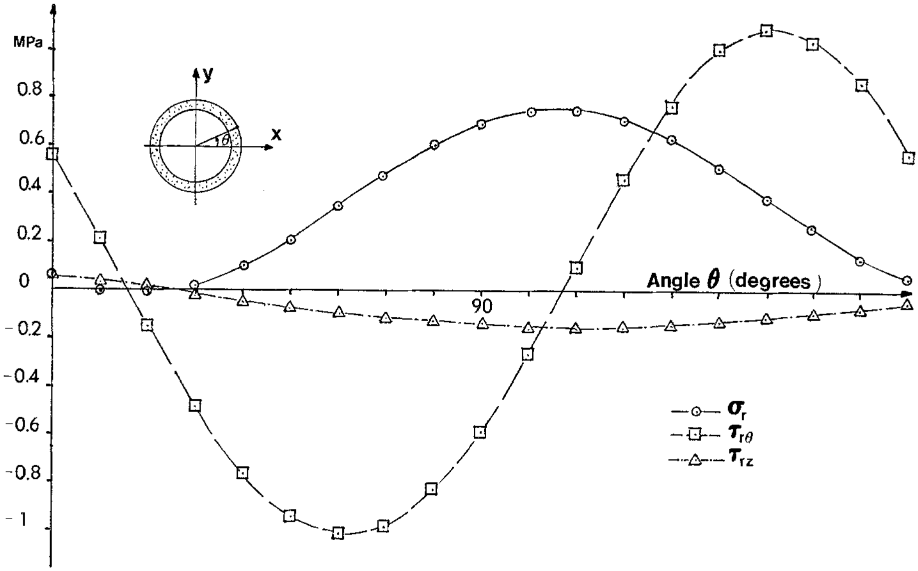

Fig. 8. Variation of the radial stress σ_r and shear stresses $\tau_{r\theta}$, τ_{rz} at the contact inclusion anisotropic medium ($0^0 \leq \theta \leq 180^0$) when $E_1/E_2=2$; $E_1/G_{12}=8.75$; $E/E_1=0.1$; $\nu_{21}=0.27$; $\nu_{32}=0.25$; $\psi=30^0$; $\beta=0^0$

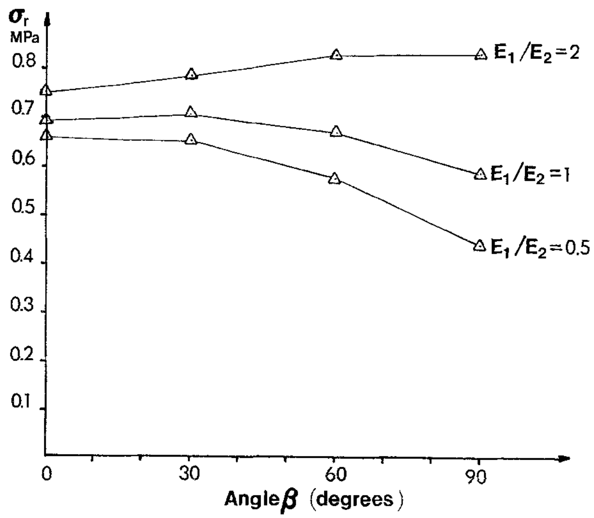

Fig. 9. Variation of the maximum value of the radial stress σ_r at the contact rock inclusion with the angle β for different values of E_1/E_2 and for $E_1/G_{12}=8.75$; $E/E_1=0.1$; $\nu_{21}=0.27$; $\nu_{32}=0.25$; $\psi=30^0$

for the other values of E/E_1 and E_1/G_{12} considered in this analysis but much higher values for the magnitude of σ_r were obtained for larger values of E/E_1. The increase of the maximum contact radial stress σ_r with E_1/E_2 was also observed when the angle ψ varies but the angle β, the ratios E/E_1 and E_1/G_{12} are fixed. This is shown in Fig. 10 when β is equal to 90 degrees. If the orientation angles β, ψ and the ratios E_1/E_2, E/E_1 are now

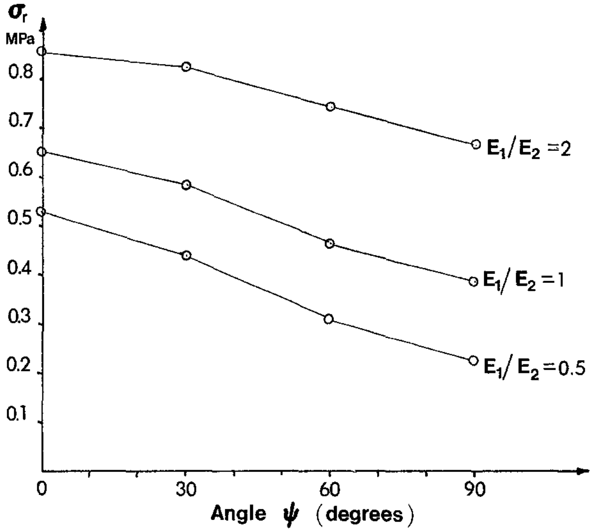

Fig. 10. Variation of the maximum value of the radial stress σ_r at the contact rock inclusion with the angle ψ for different values of E_1/E_2 and for $E_1/G_{12}=8.75$; $E/E_1=0.1$; $\nu_{21}=0.27$; $\nu_{32}=0.25$; $\beta=90^0$

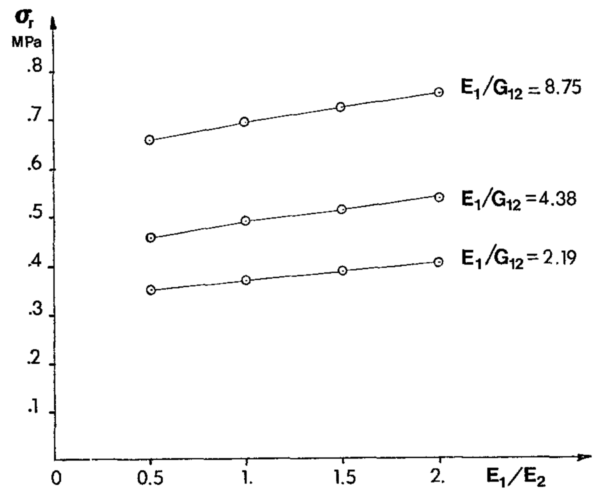

Fig. 11. Variation of the maximum value of the radial stress σ_r at the contact rock inclusion with the ratio E_1/E_2 for different values of E_1/G_{12} when $E/E_1=0.1$; $\nu_{21}=0.27$; $\nu_{32}=0.25$; $\beta=0^0$; $\psi=30^0$

fixed, then σ_r was found to increase with the ratio E_1/G_{12} or equivalently with E/G_{12} since the ratio $E/G_{12} = (E/E_1)\,(E_1/G_{12})$. This is shown in Fig. 11.

As far as the maximum value of the shear stress $\tau_{r\theta}$ at the rock-inclusion contact is concerned, it was found that its variations with the orientation angles β, ψ and the ratios E/E_1, E_1/E_2 and E_1/G_{12} follow the same patterns as for the maximum value of σ_r described above. In this numerical example, it was also found that the shear stress component τ_{rz} along the rock-inclusion contact is small and that its variations with the parameters considered above was negligible in comparison to the variations for σ_r and $\tau_{r\theta}$.

Figs. 9 to 11 show that for a transversely isotropic rock for which the planes of transverse isotropy have a fixed orientation with respect to the inclusion, the magnitude of both the maximum radial and shear stress

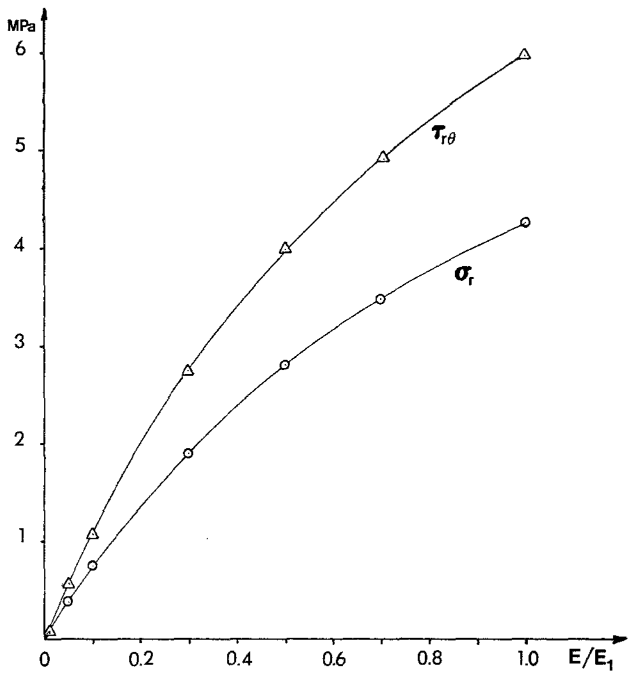

Fig. 12. Variation of the maximum values of the radial and shear stresses at the contact rock inclusion with the ratio E/E_1 when $E_1/E_2=2$; $E_1/G_{12}=8.75$; $\nu_{21}=0.27$; $\nu_{32}=0.25$; $\beta=0^0$- $\psi=30^0$

components at the contact between the rock and the inclusion depend on the values of E/E_1, E/E_2 and E/G_{12} instead of E/E_1 only for an isotropic rock. Fig. 12 shows the variation of these stress components when E/E_1, E/E_2 and E/G_{12} increase simultaneously. This was done by keeping E_1/E_2 and E_1/G_{12} constant and by increasing E/E_1. This figure indicates a drastic increase of the maximum values of the contact stresses with the ratio E/E_1.

The distributions of the radial stress σ_r and the shear stress components $\tau_{r\theta}$, τ_{rz} induced by the inclusion in the anisotropic rock were found to depend on the five parameters of Eq. (12) and on the angles β and ψ. Fig. 13(a) shows an example of distribution of σ_r along the y axis of Fig. 1 ($\theta = 90^0$) when E/E_1 varies between 0.1 and 1. The other four parameters and the orientation angles are all fixed. σ_r is found to decay very rapidly near the contact between the rock and the inclusion but is still non-negligible at a distance of five times the radius of the hole in which the inclusion is located

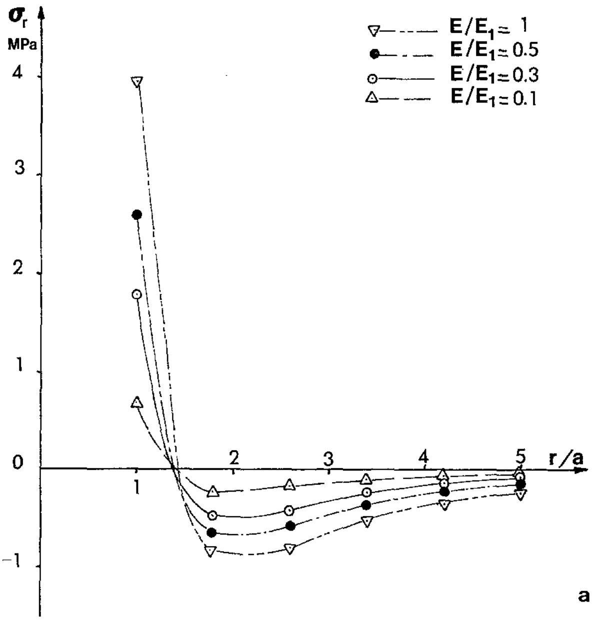

Fig. 13

unless E/E_1 is less than 0.3. Assuming that (i) the inclusion considered before is used as an overcoring instrumented probe, (ii) (σ_0) defines the in situ stress field, and (iii) the overcoring diameter ranges between three and five times the pilot hole diameter, then, the finite character of the overcored diameter could be neglected when measuring in-situ stresses if E/E_1 was less than 0.1. In other words, for a transversely isotropic rock with elastic properties such that $E_1/E_2 = 2$ and $E_1/G_{12} = 8.75$, the ratios E/E_2 and E/G_{12} will have to be respectively less than 0.2 and 0.875 for assuming the overcoring diameter to be at infinity. Similar conclusions apply for τ_{rz} and $\tau_{r\theta}$ whose distributions are shown in Figs. 13(b) and 13(c) respectively.

Conclusions

Analytical solutions have been proposed in the rock mechanics literature for the interpretation of overcoring measurements in terms of in-situ stresses in isotropic and anisotropic grounds. Whenever solid or hollow inclusions are used as instrumented probes, residual stresses and strains will remain in the overcored rock sample and in the probes. When using such devices for computing the in-situ stress field components, it is common practice to assume that the overcoring diameter is infinite and that there is

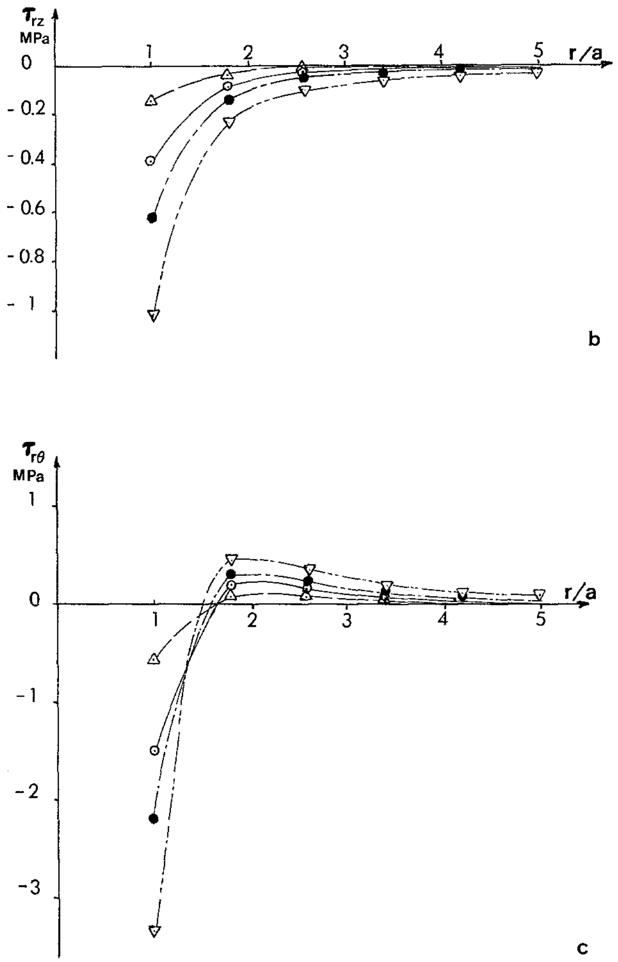

Fig. 13. Distribution of stresses induced by an inclusion in an anisotropic medium along the y axis of Fig. 1 for different values of E/E_1. (a) Radial stress σ_r, (b) shear stress τ_{rz}, (c) shear stress $\tau_{r\theta}$

$E_1/G_{12}=8.75$; $E_1/E_2=2$; $\nu_{21}=0.27$; $\nu_{32}=0.25$; $\beta=0^0$; $\psi=30^0$

a perfect bonding between the rock and the probes. However, the validity of these assumptions depends on the magnitude and distribution of the residual stresses. This can be assessed by using the closed form solutions presented herein. These solutions should be used with the following procedure: (i) the overcoring diameter is set equal to infinity and the in-situ stress field is determined from the overcoring strain and/or displacement measurements, (ii) the residual stresses are calculated at any point in the rock and along the contact rock probe. The residual stresses must satisfy two constraints: the radial and shear stress components of residual stresses σ_r, $\tau_{r\theta}$, τ_{rz} must vanish (or be negligible) along the overcoring diameter *and* the residual stresses at the rock-probe contact must not reach the tensile and the shear strengths of the rock probe bond material. If any one of these two constraints is not satisfied, then the finite character of the overcoring diameter must be taken into account when analyzing the overcoring measurements and the solution proposed by Amadei (1983) cannot be used.

The closed form solution proposed herein for the residual stress components show that they are linearly dependent on the components of the in-situ stress field and that their magnitude and distribution are also dependent on the isotropic-anisotropic character of the rock as well as on the relative deformability of the rock with respect to the deformability of the material comprising the instrumented inclusion. In the numerical examples proposed in this paper, this was expressed by the ratio E/E_1 for an isotropic rock and by the ratios E/E_1, E/E_2 and E/G_{12} for a rock modelled as a transversely isotropic material. For a rock with given isotropic or anisotropic elastic properties, solid inclusion type overcoring instrumented devices will have to be very soft with respect to the rock in order to keep the residual stresses in the overcored sample small in magnitude and distributed over a region near the contact between the rock and the instrumented device. If the device is a hollow inclusion, it will have to be both soft and thin walled for these conditions to be satisfied. This seems to apply regardless of the isotropic, anisotropic character of the rock. For given elastic properties and geometry of the overcoring instrumented device, isotropic rocks for which the ratio E/E_1 is large or transversely isotropic rocks for which E/E_1, E/E_2, E/G_{12} are large will be associated with higher residual stresses that are distributed over a larger domain in the overcored sample than for rocks for which the above ratios are small. A combination of soft rock, high modulus inclusion type instrumented device with inadequate geometry and high in-situ stress field may lead to unacceptable residual stresses in the overcored sample and in the device. For such a case, the overcoring diameter could not be neglected and separation could take place at the contact between the rock and the device.

The magnitude and distribution of residual stresses in the inclusion type instrumented devices and in the rock will also depend on the orientation of the pilot hole with respect to the directions of the principal in-situ stress field components. This was observed by Duncan Fama and Pender (1980) for isotropic rocks but has not been considered in the present analysis either for isotropic or for anisotropic rocks.

It is noteworthy that in order to use the analytical solutions presented in this paper, the deformability properties of the rock must be determined. This is particularly important when dealing with anisotropic rocks for which the directional character of these properties must also be assessed. Laboratory tests may be conducted on specimens located remotely from the site of the overcoring stress measurements. This may lead to errors in the determination of the rock deformability due to possible variations of the rock properties from one point to another in the rock mass. These variations can be eliminated by testing directly the overcored sample containing the instrumented device in a biaxial pressure chamber. The rock response in the biaxial test can then be analyzed in terms of isotropic or anisotropic elastic properties as proposed by Amadei and Walton (in preparation) for the CSIRO cell.

Finally, when drawing the previous conclusions, it is assumed that the overcoring process described in Fig. 5 does not create any stress disturbance effects. These are, for instance, stress concentrations near the end of the pilot hole, the large diameter hole or the overcoring grooves. A three-dimensional analysis conducted by Blackwood (1982) has shown that for isotropic rocks and solid inclusion stress instruments, these effects are negligible if some provisions are made regarding the geometry and position of the instruments in the pilot hole. Such an analysis is also needed for anisotropic rocks since both the degree of anisotropy and the orientation of anisotropy with respect to the pilot hole may now control the magnitude of overcoring stress disturbance effects.

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References

Amadei, B. (1983): Number of Boreholes to Measure the State of Stress in situ. Proc. 24th U. S. Rock Symp., Texas A & M Univ., pp. 87—98.

Amadei, B. (1982): The Influence of Rock Anisotropy on Measurement of Stresses in situ. Ph. D. thesis, Univ. of California, Berkeley. Also published by Springer-Verlag in: "Lecture Notes in Engineering", series under the title, "Rock Anisotropy and the Theory of Stress Measurements", (1983).

Amadei, B., Walton, R. J. (in preparation): Analysis of Data Obtained with the CSIRO Cell in Anisotropic Rock Masses. CSIRO report.

Becker, R. M. (1968): An Anisotropic Elastic Solution for Testing Stress Relief Cores. U. S. Bureau of Mines, RI. 7143.

Becker, R. M., Hooker, V. E. (1967): Some Anisotropic Considerations in Rock Stress Determinations. U. S. Bureau of Mines, RI. 6965.

Berry, D. S. (1968): The Theory of Stress Determination by Means of Stress Relief Techniques in Transversely Isotropic Medium. Tech. Report No. 5—68, Missouri River Division, Corps of Engineers, Omaha, Nebraska.

Berry, D. S. (1970): The Theory of Determination of Stress Changes in a Transversely Isotropic Medium Using an Instrumented Cylindrical Inclusion. Tech. Report No. MRD 1—70, Missouri River Division, Corps of Engineers, Omaha, Nebraska.

Berry, D. S., Fairhurst, C. (1966): Influence of Rock Anisotropy and Time Dependent Deformation on the Stress Relief and High Modulus Inclusion Techniques of in situ Stress Determination. In: Testing Techniques for Rock Mechanics, ASTM, STP. 402, Am. Soc. Testing Mats., pp. 190—206.

Blackwood, R. L. (1977): An Instrument to Measure the Complete Stress Field in Soft Rock or Coal in a Single Operation. Proc. Int. Symp. on Field Measurements in Rock Mechanics, Kovari, K. (ed.), Vol. 1, pp. 137—150, Zurich.

Blackwood, R. L. (1982): A Three-Dimensional Study of an Overcored Solid Inclusion Stress Instrument by the Boundary Integral Equation Method. Proc. 4th Int. Conf. in Australia on Finite Element Methods., pp. 109—113.

Duncan Fama, M. E. (1979): Analysis of a Solid Inclusion in situ Stress Measuring Device. Proc. 4th Cong. ISRM (Montreux), Vol. 2, pp. 113—120.

Duncan Fama, M. E., Pender, M. J. (1980): Analysis of the Hollow Inclusion Method for Measuring in situ Rock Stress. Int. J. Rock Mech. Min. Sci. *17* (No. 3), 137—146.

Hirashima, K., Koga, A. (1977): Determination of Stresses in Anisotropic Elastic Medium Unaffected by Boreholes from Measured Strains or Deformations. Proc. Int. Symp. on Field Measurements in Rock Mechanics, Kovari, K. (ed.), Vol. 1, pp. 173—182.

Leeman, E. R., Hayes, D. J. (1966): A Technique for Determining the Complete State of Stress in Rock Using a Single Borehole. Proc. 1st. Cong. ISRM (Lisbon), Vol. II, pp. 17—24.

Merrill, R. H. (1967): Three Component Borehole Deformation Gage for Determining the Stress in Rock. U. S. Bureau of Mines, RI 7015.

Niwa, Y., Hirashima, K. (1971): The Theory of the Determination of Stress in an Anisotropic Elastic Medium Using an Instrumented Cylindrical Inclusion. Memoirs of the Faculty of Eng., Kyoto Univ., Japan, Vol. 33, pp. 221—232.

Pickering, D. J. (1970): Anisotropic Elastic Parameters for Soils. Geotechnique *20* (No. 3), 271—276.

Rocha, M., Silverio, A. (1969): A New Method for the Complete Determination of the State of Stress in Rock Masses. Geotechnique *19* (No. 1), 116—132.

Rocha, M., Silverio, A., Pedro, J. O., Delgado, J. S. (1974): A New Development of the LNEC Stress Tensor Gauge. Proc. 3rd Cong. ISRM (Denver), Vol. II A, pp. 464—467.

Worotnicki, G., Walton, R. J. (1976): Triaxial Hollow Inclusion Gauges for the Determination of Rock Stress in situ. Proc. ISRM Symp. on Investigation of Stress in Rock and Advances in Stress Measurement. Supplement, pp. 1—8, Sydney.