188, 1470 (1929).
4. V. Fock, "The Dirac wave equation and Riemannian geometry," Zh. Russ. Fiz.-Khim. Ova, Chast Fiz., **62**, No.2, 133 (1930).
5. A. Lichnerowicz, Ann. Inst. H. Poincaré, **1A**, 233 (1964).
6. M. Markov, Phys. Z. Sowjetunion, **10**, 773 (1936).
7. N. P. Konopleva and V. N. Popov, Gauge Fields [in Russian], Atomizdat, Moscow (1972).
8. J. Schwinger, Phys. Rev., **82**, 914 (1951).
9. B. F. Plybon, J. Math. Phys., **12**, 57 (1971).
10. B. A. Levitskii, Izv. Vyssh. Uchebn. Zaved. Fiz., No.7, 152 (1977).
11. B. A. Levitskii and Yu. A. Yappa, Teor. Mat. Fiz., **48**, 227 (1981).
12. B. A. Levitskii, Vestn. Leningr. Univ. Fiz. Khim., No. 4, 7 (1973).
13. N. A. Chernikov and N. S. Shavokhina, "Quantization of a spinor field in a spherical world," Preprint R2-6351 [in Russian], JINR, Dubna (1972).
14. L. D. Faddeev, Usp. Fiz. Nauk, **136**, 431 (1982).
15. V. A. Fock, The Theory of Space, Time, and Gravitation, Oxford (1964).

DIAGRAM REPRESENTATION OF THE SINGLE-PHONON PROPAGATOR IN THE QUASIPARTICLE–PHONON MODEL OF NUCLEI

G. Kyrchev

A diagram technique is proposed for the quasiparticle–phonon model of nuclei. It is shown that restriction to single-phonon and two-phonon components in the wave functions of the excited states of even–even deformed nuclei is equivalent to summation of a definite class of single-loop diagrams. The contribution of a two-loop diagram is calculated. Further, closure of the hierarchy of equations for the Green's functions yields explicit approximations that lead to summation of the considered class of graphs.

1. Introduction

The construction of the quasiparticle–phonon model (QPM) of nuclei, the basic propositions of which are given in the review [1], is based on a concrete form model Hamiltonian and on the concept of a many-component operator wave function (which is assumed expanded in a series with respect to the operators in terms of which the Hamiltonian is expressed). The solutions of QPM equations derived from the variational principle uniquely determine the excitation spectrum and the coefficients of the operator wave function, which plays an important part in the QPM. However, some modern nuclear models use the language of propagators and representations of them by means of diagrams [2-5].

The theoretical justification [6] and comparison [7] of the nuclear models now firmly established in the theory of nuclear structure become ever more topical problems. To compare the QPM with models that use the language of graphs, it is important to establish what diagrams are taken into account in the framework of the QPM. This question was raised in particular in the recent papers [8, 9]. In this connection, it is necessary to give an alternative derivation of the QPM equations based on the formalism of Green's functions [10-12]. For the QPM, this formalism was used in [13], in which it was shown that the hierarchy of QPM equations is equivalent to a hierarchy of equations for the corresponding Green's functions.

In the present paper, which is methodological in nature, a problem with operator wave function containing single-phonon and two-phonon components [1, 14-16] (we shall call this problem the "model"), which is solved in the framework of the QPM by means of a variational principle in explicit form, is translated into the language of diagrams. For this model, the object that contains all the physical information is the perturbed propagator (the interaction Hamiltonian of the QPM plays the part of the perturbation), for which one can construct a diagram expansion by the standard method. Comparison with the equations derived from the variational principle [1, 14-16] make it possible to separate uniquely the class of diagrams contained

Joint Institute for Nuclear Research, Dubna. Translated from Teoreticheskaya i Matematicheskaya Fizika, Vol.53, No.2, pp.260-270, November, 1982. Original article submitted January 6, 1982.

in the model, which is the aim of the present paper. One can arrive at the same answer by writing down the equations for the two-time Green's functions and, closing them in a consistent manner, solving them in the corresponding approximation.

The paper is arranged as follows. The problem is formulated in Sec.2. In Sec.3, we analyze the diagram series of the single-phonon propagator, which makes it possible to deduce which graphs are taken into account in the model [1,14-16]. In Sec.4, the same result is obtained by approximate solution of the hierarchy of equations for the Green's functions. The adequacy of the Green's function formalism for the model is noted in Sec.5.

2. Formulation of the Problem

The model of an even–even deformed nucleus with wave function of the form

$$|\Psi_n(K^\pi)\rangle=\Big[\sum_{i=1}^{m_0}C_i^n(\bar{g}_0)Q^+_{\bar{g}_0 i}+\frac{1}{\sqrt{2}}\sum_{gg'}D^n_{gg'}(\bar{g}_0)Q_g^+Q_{g'}^+\Big]|\Psi_0\rangle \tag{1}$$

has been studied in detail in a number of papers [1,14-16]. The notation is as follows: n is the number of the root of the excited state with given values of the spin projection K onto the symmetry axis of the nucleus and parity π; $g\equiv(\lambda\mu i)\equiv(\bar{g}i)$; Q_g^+ is the operator of creation of a phonon with multipolarity $\bar{g}$ with number i; m_0 is the number of phonons of multipolarity $\bar{g}_0$; $Q_g|\Psi_0\rangle=0$.

When considering the model with wave function of the form (1) it is convenient to express the QPM Hamiltonian solely in terms of the phonon operators* $Q_g^+Q_g$. Under the assumption of validity of the quasi-boson approximation the QPM Hamiltonian can be given the form

$$H_M=H_V+H_{Vq}, \tag{2}$$

$$H_V=\sum_g \omega_g Q_g^+Q_g, \tag{3}$$

where the energies ω_g of the noninteracting phonons are obtained from the solution of the secular equations of the random phase approximation (RPA) [1,14],

$$H_{Vq}=-\sum_{gg'g''}\Big\{\Big[\frac{1}{\sqrt{2}}U_g^{g'g''}Q_{g'}^+Q_{g''}^+Q_g+\frac{1}{4}V^{gg'g''}Q_g^+Q_{g'}^+Q_{g''}^+\Big]+[\text{h.c.}]\Big\}. \tag{4}$$

The symmetry properties of $U_g^{g'g''}$ and $V^{gg'g''}$ are

$$U_g^{g'g''}=U_g^{g''g'},\qquad V^{gg'g''}=V^{g'g''g}=V^{g''g'g}=V^{g'gg''}. \tag{5}$$

By virtue of (4), $U_g^{g'g''}=\frac{1}{\sqrt{2}}\langle\Psi_0|Q_gH_{Vq}Q_{g'}^+Q_{g''}^+|\Psi_0\rangle$, i.e., $U_g^{g'g''}$ is the amplitude of transition of the single-phonon state g into the two-phonon state $(g'g'')$. The explicit form of $U_g^{g'g''}$ is given in [14,15] and is not needed in what follows.

For the normalized wave function (1), the variational principle gives the following system of equations [1,16]:

$$(\omega_{\bar{g}_0 i}-\eta_n)C_i^n(\bar{g}_0)-\sum_{gg'}U_{\bar{g}_0 i}^{gg'}D^n_{gg'}(\bar{g}_0)=0,\qquad(\omega_g+\omega_{g'}-\eta_n)D^n_{gg'}(\bar{g}_0)-\sum_{i=1}^{m_0}U_{\bar{g}_0 i}^{gg'}C_i^n(\bar{g}_0)=0. \tag{6}$$

The secular equation for the energies of the excited states is obtained [1,15,16] in the form

$$d(\bar{g}_0;\eta_n)\equiv\det\Big\|(\omega_{\bar{g}_0 i}-\eta)\delta_{ii'}-\sum_{gg'}\frac{U_{\bar{g}_0 i}^{gg'}U_{\bar{g}_0 i}^{gg'}}{\omega_g+\omega_{g'}-\eta_n}\Big\|=0. \tag{7}$$

Solving (6), we can readily show that

---

* For this purpose, it is necessary in the interaction Hamiltonian of the quasiparticle–phonon model, which has the structure $\alpha^+\alpha(Q^{++}Q)$, to express operators of the type $\alpha^+\alpha$ in terms of phonon operators (see [14], p.391).

$$(C_i^n)^2 = -\left\{\frac{\partial}{\partial\eta}[d(\bar g_0;\eta)/M^{ii}(\bar g_0;\eta)]_{\eta=\eta_n}\right\}^{-1}, \tag{8}$$

where $M^{ii}(\bar g_0;\eta)$ is the corresponding minor of the determinant in (7).

Equations (6)-(8) will be essentially used in the following exposition.

The point of departure for constructing the diagram representation of the single-phonon propagator in the QPM can be taken to be the well-known formula [11,12]:

$$G_{ii'}(\bar g_0; t-t') = -i\left\langle \Psi_0 \left| T\left\{Q_{\bar g_0 i}(t)\, Q^+_{\bar g_0 i'}(t') \exp\left[-i\int_{-\infty}^{+\infty} d\tau H_{Vq}(\tau)\right]\right\}\right| \Psi_0\right\rangle_{\rm con}. \tag{9}$$

Here, $Q_g^+(t)$ and $Q_g(t)$ are the operators of creation and annihilation of phonons in the interaction representation, i.e.,

$$Q_g(t) = e^{iH_Vt}Q_g e^{-iH_Vt} = e^{-i\omega_g t}, \qquad Q_g^+(t) = e^{iH_Vt}Q_g^+ e^{-iH_Vt} = e^{i\omega_g t}, \tag{10}$$

T is the time-ordering operator, $H_{Vq}=e^{iH_Vt}H_{Vq}e^{-iH_Vt}$, and the subscript con indicates that only connected diagrams are taken into account.

Our aim is to establish what graphs are taken into account in the framework of the model with the wave function (1). The prescription for answering this question is given by the following discussion. The considered model corresponds to a propagator (we denote it by G_{ii}^M) satisfying two requirements: the equation for the poles of the Fourier transform of G_{ii}^M is identical to the secular equation (7), and the residues of the Fourier transform of G_{ii}^M at the poles must be equal to $(C_i^n)^2$ from (8). It is clear that the diagram series for G_{ii} must contain a sequence of graphs, summation of which gives G_{ii}^M. This will be the required class of diagrams. To separate them, it is necessary to develop an appropriate graphical technique.

3. Diagram Analysis of the Single-Phonon Propagator

If in Eq. (9) we set $H_{Vq}=0$, we obtain the unperturbed propagator*

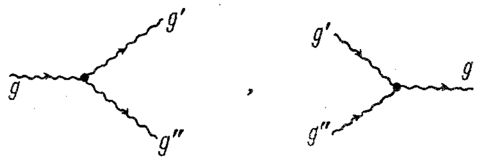
$$G_{ii'}^{(0)}(\bar g_0; t-t') = -i\langle\Psi_0|T\{Q_{\bar g_0 i}(t)\,Q^+_{\bar g_0 i'}(t')\}|\Psi_0\rangle \equiv -i\overline{Q_{\bar g_0 i}(t)\,Q^+_{\bar g_0 i'}}(t'). \tag{11}$$

From (11), using (10), we find its explicit form:

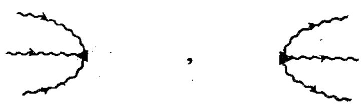
$$G_{ii'}^{(0)}(\bar g_0;\ t-t') = \begin{cases} -i\delta_{ii'}e^{-i\omega_{\bar g_0 i}(t-t')}, & \text{if } t>t', \\ 0 & \text{if } t<t'. \end{cases} \tag{12}$$

The expansion of the exponential in Eq. (9) generates a diagram series of perturbation theory if we use Wick's theorem [11,12] and introduce the following convention: the time-ordered pairings $\overline{Q_g(t)\,Q^+_{g'}}(t') = iG_{ii'}^{(0)}(\bar g; t-t')\delta_{\bar g\bar g'}$ are associated with the lines

(they are oriented by virtue of (12)), the matrix elements $U_g^{g'g''}$ are associated with the vertices



and the quantities $V^{gg'g''}$ are represented in the form



* The phonon vacuum $|\Psi_0\rangle$ in Eq. (9) is assumed to be nondegenerate, and $\langle\Psi_0|\Psi_0\rangle = 1$.

From these elements we shall construct the diagrams that contribute to $G_{ii'}(\bar{g}_0; t-t')$ from (9).

It is clear from Eqs. (4), (9), and (10) that the corrections δG of odd order to $G^{(0)}$ are equal to zero, i.e., $\delta G_{ii'}^{(2n+1)}(\bar{g}_0; t-t')=0$, and we therefore rewrite Eq. (9) in the form

$$G_{ii'}(\bar{g}; t-t')=G_{ii'}^{(0)}(\bar{g}_0; t-t')+\sum_{n=1}^{\infty}\delta G_{ii'}^{(2n)}(\bar{g}_0; t-t')\equiv G_{ii'}^{(0)}(\bar{g}_0; t-t')+(-i)\sum_{n=1}^{\infty}\frac{(-i)^{2n}}{2n!}\int_{-\infty}^{+\infty}d\tau_1\dots d\tau_{2n}\times$$
$$\langle\Psi_0|T\{Q_{\bar{g}_0 i}(t)Q^{+}_{\bar{g}_0 i'}(t')H_{Vq}(\tau_1)\dots H_{Vq}(\tau_{2n})\}|\Psi_0\rangle_{\mathrm{con}}. \tag{13}$$

The factor 1/2n! can be omitted and we can consider only index free diagrams [12]. We consider in somewhat more detail the corrections to $G_{ii'}(\bar{g}_0; t-t')$ of the second (n = 1) and fourth (n = 2) orders. Taking into account the expressions (4), (5), and (11) and using Wick's theorem, we obtain

$$\delta G_{ii'}^{(2)}(\bar{g}_0; t-t')=i^5\sum_{\bar{g}\bar{g}'jj'}U_{\bar{g}_0 i}^{gg'}U_{\bar{g}_0 i'}^{gg'}\int_{-\infty}^{+\infty}d\tau_1 d\tau_2 G_{i'i'}^{(0)}(\bar{g}_0; \tau_1-t')G_{j'j'}^{(0)}(\bar{g}'; \tau_2-\tau_1)G_{jj}^{(0)}(\bar{g}; \tau_2-\tau_1)G_{ii}^{(0)}(\bar{g}_0; t-\tau_2)+$$
$$i^5\sum_{\bar{g}\bar{g}'jj'}V^{gg'\bar{g}_0 i}V^{gg'\bar{g}_0 i'}\int_{-\infty}^{+\infty}d\tau_1 d\tau_2\, Q_{i'i'}^{(0)}(\bar{g}_0; \tau_2-t')G_{jj}^{(0)}(\bar{g}; \tau_2-\tau_1)G_{jj}^{(0)}(\bar{g}'; \tau_2-\tau_1)G_{ii}^{(0)}(\bar{g}_0; t-\tau_1), \tag{14}$$

where it is assumed that $t>\tau_2>\tau_1>t'$.

We now make a Fourier transformation:

$$\delta G_{ii'}^{(2)}(\bar{g}_0; \eta)=\int_{-\infty}^{+\infty}\delta G_{ii'}^{(2)}(\bar{g}_0; t-t')e^{i\eta(t-t')}d(t-t').$$

We substitute here the expression for $\delta G_{ii'}^{(2)}(\bar{g}_0; t-t')$ from (14) and, following [12], make an inverse Fourier transformation on the functions $G^{(0)}$ in (14):

$$G_{ii'}^{(0)}(\bar{g}_0; t-t')=\int_{-\infty}^{+\infty}G_{ii'}^{(0)}(\bar{g}_0; \eta)e^{-i\eta(t-t')}\frac{d\eta}{2\pi}.$$

As a result, we obtain

$$\delta G_{ii'}^{(2)}(\bar{g}_0; \eta)=iG_{i'i'}^{(0)}(\bar{g}_0; \eta)\sum_{gg'}U_{\bar{g}_0 i}^{gg'}U_{\bar{g}_0 i'}^{gg'}\int\frac{d\eta_2 d\eta_3}{2\pi}G_{jj}^{(0)}(\bar{g}; \eta_2)\delta(\eta_2+\eta_3-\eta)G_{j'j'}^{(0)}(\bar{g}'; \eta_3)G_{ii}^{(0)}(\bar{g}_0; \eta)+$$
$$iG_{i'i'}^{(0)}(\bar{g}_0; \eta)\sum_{\bar{g}\bar{g}'jj}V^{\bar{g}_0 i'gg'}V^{\bar{g}_0 igg'}\int_{-\infty}^{+\infty}\frac{d\eta_2 d\eta_3}{2\pi}G_{jj}^{(0)}(\bar{g}; \eta_2)\delta(\eta_2+\eta_3+\eta)G_{j'j'}^{(0)}(\bar{g}'; \eta_3)G_{ii}^{(0)}(\bar{g}_0; \eta) \tag{15}$$

(the δ functions express the conservation law for the energy at each vertex). The Fourier transforms are calculated with allowance for (12):

$$G_{ii'}^{(0)}(\bar{g}; \eta)=\delta_{ii'}(\eta-\omega_{\bar{g}i}+i\varepsilon)^{-1}, \quad \varepsilon\to 0. \tag{16}$$

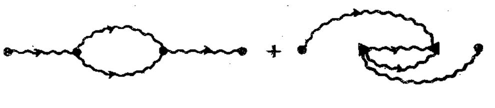
Substituting (16) in the upper expression, we finally obtain

$$\delta G_{ii'}^{(2)}(\bar{g}_0; \eta)=[\hat{G}^{(0)}(\bar{g}_0; \eta)\hat{\Pi}^{U+V}(\bar{g}_0; \eta)\hat{G}^{(0)}(\bar{g}_0; \eta)]_{ii'}, \tag{17}$$

where $\hat{G}^{(0)}(\bar{g}_0; \eta)$ is the matrix composed of the quantities (16), and $\Pi^{U+V}(\bar{g}_0; \eta)$ is the matrix

$$\Pi_{ii'}^{U+V}(\bar{g}_0; \eta)=\Pi_{ii'}^{U}+\Pi_{ii'}^{V}=\sum_{gg'}\frac{U_{\bar{g}_0 i}^{gg'}U_{\bar{g}_0 i'}^{gg'}}{\eta-\omega_g-\omega_{g'}}+\sum_{gg'}\frac{V^{\bar{g}_0 i'gg'}V^{\bar{g}_0 igg'}}{\eta+\omega_g+\omega_{g'}}. \tag{18}$$

Recalling the conventions introduced above, we can represent (17) graphically in the form

$\delta G_{ii'}^{(2)}(\bar{g}_0; \eta)=$  (19)

The graphical equivalent of $\Pi_{ii'}^{U}(\bar{g}_0; \eta)$ is a skeleton diagram, the so-called phonon loop

$$\Pi^U_{ii'}\ (\bar{g}_0;\ \eta) = $$  . (20)

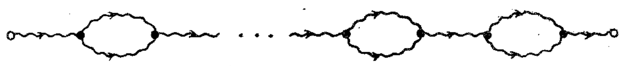
In the fourth order, there will be a nonvanishing contribution to $\delta G^{(4)}_{ii'}$ from, in particular, the matrix element with the contractions

$$\langle \Psi_0 \,|\, T\{Q^-_{g_0 i}(t)\, Q^+QQQ^+Q^+QQ^+QQQ^+Q^+QQ^+_{g_0 i'}(t)\}\,|\,\Psi_0\rangle_{\text{con}}. \tag{21}$$

Proceeding as above, we can readily show that (21) makes a contribution to $\delta G^{(4)}_{ii'}(\bar{g}_0;\ \eta)$ equal to $[\hat{G}^{(0)}\hat{\Pi}^U\hat{G}^{(0)}\hat{\Pi}^U\hat{G}^{(0)}]_{ii'}$, or, in diagram language,

 . (22)

In the higher orders, it is possible to have the diagram

 . (23)

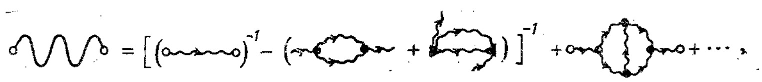
It is readily verified that the analytic equivalent of the diagram (23) is the expression $[\hat{G}^{(0)}\hat{\Pi}^U\hat{G}^{(0)}\ldots \hat{\Pi}^U G^{(0)}]_{ii'}$, which is generated by the term that arises in the 2n-th order (see Eq. (13)), where the operators are contracted as in (21). The fact that the structure of the expression is such is in no doubt (see, for example, [12]), and we show that the resulting factor in the 2n-th order is equal to unity. Indeed, the $\left(\frac{1}{\sqrt{2}}\right)^{2n}$ is compensated by 2^n because of the symmetry of $U^{g'g''}_g$ (see (5)), and from the contractions the factor i^{3n+1} arises; with each $\Pi_{ii'}$ there is associated* the factor $(-i)$, giving altogether $(-i)^n$. It is now obvious that $(-i)(-i)^{2n}i^{3n+1}(-i)^n=1$.

The V terms of the Hamiltonian H_{Vq} generate a ladder of graphs:

 . (24)

To this diagram there corresponds the expression $[\hat{G}^{(0)}\hat{\Pi}^V\hat{G}^{(0)}\hat{\Pi}^V G^{(0)}\ldots\hat{\Pi}^V\hat{G}^{(0)}]_{ii}$.

Collecting together the geometric progression (see (19), (22)-(24)), we obtain

 (25)

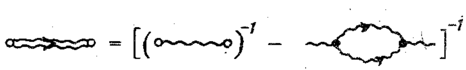
where the total propagator $G_{ii'}(\bar{g}_0;\ \eta)$ is represented by a thick phonon line. Equation (25) concretely is

$$G_{ii'}(\bar{g}_0;\ \eta) = ([(\hat{G}^{(0)})^{-1} - (\hat{\Pi}^V + \hat{\Pi}^V)])^{-1}_{ii'} + \text{many-loop corrections}. \tag{26}$$

It is readily seen that if $\hat{\Pi}^V = 0$ and many-loop diagrams are not taken into account then it follows from (26) with allowance for (18) that

$$G_{ii'}{}^M(\bar{g}_0;\ \eta) = -(-1)^{i+i'}M^{ii'}(\bar{g}_0;\ \eta)/d(\bar{g}_0;\ \eta), \tag{27}$$

where $M^{ii'}(\bar{g}_0;\ \eta)$ and $d(\bar{g}_0;\ \eta)$ are the same functions that were introduced earlier (see Eqs. (7) and (8)). It can be seen from (27) that the equation for the poles of $G_{ii'}{}^M(\bar{g}_0;\ \eta)$ is identical to the secular equation (7) and that $\operatorname{Res} G_{ii}{}^M(\bar{g}_0;\ \eta) = (C_i{}^n)^2$ from (8), i.e., $G^M_{ii'}\ (\bar{g}_0;\ \eta)$ is the propagator corresponding to the model (1). Its diagram representation is

 , (28)

---

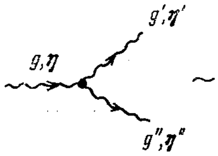
* This follows from the fact that $\int\limits_{-\infty}^{+\infty} \frac{d\eta_1}{2\pi}\, G^{(0)}_{ii}(\bar{g};\ \eta_1)\, G^{(0)}_{i'i'}(\bar{g}';\ \eta-\eta_1) = -i(\eta-\omega_g-\omega_{g'}+i\varepsilon)^{-1}$.

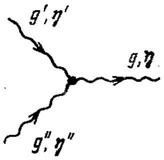
which corresponds to summation of the ladder of the diagrams (23).

To calculate the contribution of the two-loop diagram in (25), we use the correspondence rules that hold in the η representation:

1) 

$\sim i(\eta-\omega_g+i\varepsilon)^{-1}$;

2) 

$\sim U_g^{g'g''} \sim$


,

where the energy conservation law must be satisfied at each vertex: $\eta=\eta'+\eta''$;

3) if the line (g, η) is internal, it is necessary to perform the integration $\int \frac{d\eta}{2\pi}$ and the summation $\sum_g$. In the given case, we have

$$\sum_{gg'ffh} U_{\bar{g}_0 i}^{gg'} U_g^{ff'} U_h^{g'f} U_{\bar{g}_0 i}^{hf'} \int_{-\infty}^{+\infty} \frac{d\eta'\, d\eta''}{(2\pi)^2} (\eta-\eta'-\omega_g+i\varepsilon)^{-1}\times$$
$$(\eta'-\omega_{g'}+i\varepsilon)^{-1}(\eta''-\omega_f+i\varepsilon)^{-1}(\eta-\eta'-\eta''-\omega_{f'}+i\varepsilon)^{-1}(\eta'+\eta''-\omega_h+i\varepsilon)^{-1}.$$

Integrating, for the contribution of the two-loop graph we obtain the result

$$\sum_{gg'ff'h} U_{\bar{g}_0 i}^{gg'} U_g^{ff'} U_h^{g'f} U_{\bar{g}_0 i}^{hf'} (\eta-\omega_g-\omega_{g'})^{-1}(\eta-\omega_{g'}-\omega_f-\omega_{f'})^{-1}(\eta-\omega_g-\omega_h)^{-1}.$$

The denominator $(\eta-\omega_q-\omega_f-\omega_f)^{-1}$ in the upper formula is related to the three-phonon intermediate state of the considered graph. Diagrams of such type can be taken into account in the QPM if three-phonon components are included in the operator wave function.

With regard to diagrams of the type (24), they are related, as will be seen in what follows, to allowance for correlations in the ground state.

4. Approximate Solution of the Chain of Equations for the Green's Functions

We introduce the Green's functions

$$G_{ii'}^{-;+}(\bar{g}_0; t-t')=-i\Theta(t-t')\langle 0|[Q_{\bar{g}_0 i}(t), Q_{\bar{g}_0 i'}^{+}(t')]|0\rangle, \quad G_{ii'}^{+;+}(\bar{g}_0; t-t')=-i\Theta(t-t')\langle 0|[Q_{\bar{g}_0 i}^{+}(t), Q_{\bar{g}_0 i'}^{+}(t')]|0\rangle,$$
$$G_{fg,h}^{--,+}(t-t')=-i\Theta(t-t')\langle 0|[Q_f(t)Q_g(t), Q_h^{+}(t')]|0\rangle, \quad G_{fgh,l}^{---,+}(t-t')=-i\Theta(t-t')\langle 0|[Q_f(t)Q_g(t)Q_h(t), Q_l^{+}(t')]|0\rangle \quad (29)$$

etc. In Eqs. (29), $|0\rangle$ denotes the ground state of the Hamiltonian H_M, i.e., $H_M|0\rangle=0$.

Proceeding in the standard manner [10,11], for the Fourier transforms of the functions in (29) we obtain a hierarchy of equations. If we ignore the higher functions of the type $G^{---,+}$, which describe processes with the participation of more than three phonons (H_{vq} has cubic structure with respect to the phonon operators), we arrive at the system of equations

$$(\omega_{\bar{g}_0 i}-\eta)G_{ii'}^{-;+}(\bar{g}_0,\eta)-\sum_{ff'}\left[U_{\bar{g}_0 i}^{ff'}G_{ff',\bar{g}_0 i'}^{--,+}(\eta)+\frac{3}{4}V^{ff'\bar{g}_0 i}G_{ff',\bar{g}_0 i'}^{++,+}(\eta)\right]=-\delta_{ii'},$$
$$(\omega_g+\omega_{g'}-\eta)G_{gg',\bar{g}_0 i'}^{--,+}(\eta)-\sum_{j=1}^{m_0}U_{\bar{g}_0 j}^{gg'}G_{ji'}^{-,+}(\bar{g}_0;\eta)-\frac{3}{2}\sum_{j=1}^{m_0}V^{g\bar{g}_0 jg'}G_{ji'}^{+,+}(\bar{g}_0,\eta)=0,$$
$$(\omega_g+\omega_{g'}+\eta)G_{gg',\bar{g}_0 i'}^{++,+}(\eta)+\frac{3}{2}\sum_{j=1}^{m_0}V^{\bar{g}_0 jgg'}G^{-,+}(\bar{g}_0;\eta)+\sum_{j=1}^{m_0}U_{\bar{g}_0 j}^{gg'}G_{ji'}^{+,+}(\bar{g}_0;\eta)=0, \quad (30)$$
$$(\omega_{\bar{g}_0 i}+\eta)G_{ii'}^{+;+}(\bar{g}_0;\eta)-\sum_{ff}\left[\frac{3}{4}V^{gg'\bar{g}_0 i}G_{gg',\bar{g}_0 i}^{--,+}(\eta)+U_{\bar{g}_0 i}^{gg'}G_{ff',\bar{g}_0 i'}^{++,+}(\eta)\right]=0.$$

Functions of the type $G^{-,+}$ and $G^{--,+}$ can be associated with the coefficients C and D (see Eq. (1)), respectively. Apart from these functions, the system (30) contains functions of the type $G^{+,+}$ and $G^{+++,+}$ and the combinations $(\omega_g+\omega_g+\eta)$ appear. These quantities arise because of the circumstance that in the method of equations of motion for the two-time Green's functions the correlations in the ground state are effectively taken into account. Because of the terms of the type V in H_{vq} the phonon vacuum $|\Psi_0\rangle$ is not an eigenstate of H_M with zero energy. If $|0\rangle\equiv|\Psi_0\rangle$, as is assumed in Eq. (1) (and this corresponds to $V^{gg'g''}\equiv 0$), then $G^{+,+}\equiv 0$, $G^{++,+}=0$, the last two rows in (30) become identities, and we arrive at the system of equations (identical in form to the system (6) but with right-hand side)

$$(\omega_{\bar{g}_0 i}-\eta)G^{-,+}_{ii'}(\bar{g}_0;\eta)-\sum_{gg'}U^{gg'}_{\bar{g}_0 i}\,G^{--,+}_{gg',\bar{g}_0 i'}(\eta)=-\delta_{ii'},\qquad (\omega_g+\omega_{g'}-\eta)G^{--,+}_{gg',\bar{g}_0 i'}-\sum_{j=1}^{m_0}U^{gg'}_{\bar{g}_0 j}G^{-,+}_{ji'}(\bar{g}_0;\eta)=0. \tag{31}$$

Solving (31) explicitly, we have $G^{-,+}_{ii'}(\bar{g}_0;\eta)=-(-1)^{i+i'}M^{ii'}(\bar{g}_0;\eta)/d(\bar{g}_0;\eta)$ and by virtue of Eq. (27) we obtain

$$G^{-,+}_{ii'}(\bar{g}_0;\eta)=G^{M}_{ii'}(\bar{g}_0;\eta). \tag{32}$$

In deriving (32) we assumed that processes with the participation of more than three phonons and correlations in the ground state are not particularly important.

5. Conclusions

By applying the standard diagram method we have established the fact that restriction in the operator wave function of even–even deformed nuclei to single- and two-phonon components corresponds to allowance for only single-loop diagrams. This is the main result of the present paper, and it was obtained in two ways. We have given an analytic expression for the contribution of a two-loop graph. It is worth noting that by using the diagram technique it was possible to show that $U^{g'g''}_{g}$ and $\sum_{gg'}\frac{U^{gg'}_{\bar{g}_0 i}U^{gg'}_{\bar{g}_0 i'}}{\omega_g+\omega_{g'}-\eta}$ play the part of a vertex and the polarization operator of the QPM. This indicates the convenience of using Green's functions in the model (1).

I am very grateful to Professor V. G. Solov'ev for valuable advice and great assistance in the present work, to R. A. Broglia, A. Bulgak, R. V. Dzholos, G. N. Afanas'ev, L. A. Malov, and V. V. Voronov for helpful discussions, and also to A. I. Vdovin for reading the manuscript and critical comments.

LITERATURE CITED

1. V. G. Solov'ev, Fiz. Elem. Chastits At. Yadra, **9**, 580 (1978).
2. A. B. Migdal, Theory of Finite Fermi Systems, Interscience, New York (1967).
3. S. T. Belyaev, Yad. Fiz., **1**, 3 (1965).
4. S. P. Kamerdzhiev, Izv. Akad. Nauk SSSR Ser. Fiz., **41**, 1230 (1977).
5. P. F. Bortignon, R. A. Broglia, D. R. Bes, and R. Liotta, Phys. Rep., **30C**, 307 (1977).
6. H. Reinhardt, Nucl. Phys. A, **251**, 317 (1975).
7. J. P. Blaizot and E. R. Marshalek, Nucl. Phys. A, **309**, 453 (1978); V. Paar, Verh. Dtsch. Phys. Ges., **3**, 79 (1979); G. Kyrchev, Nucl. Phys. A, **349**, 416 (1980).
8. P. F. Bortignon and R. A. Broglia, "Damping of the nuclear motion," Preprint NBI-NT-311, Niels Bohr Institute, Copenhagen (1981).
9. S. P. Kamerdzhiev, Present State of Investigations of "New" Giant Resonances. Electromagnetic Interactions of Nuclei at Low and Medium Energies. Proc. Fourth Seminar [in Russian], Nauka, Moscow (1979), pp. 93-124.
10. D. N. Zubarev, Usp. Fiz. Nauk, **71**, 71 (1960).
11. A. A. Abrikosov, L. P. Gorkov, and I. E. Dzyaloshinskii, Methods of Quantum Field Theory in Statistical Physics, Englewood Cliffs (1963).
12. N. H. March, W. H. Young, and S. Sampanther, The Many-Body Problem in Quantum Mechanics, CUP, Cambridge (1967).
13. G. Ochirbat, Teor. Mat. Fiz., **26**, 358 (1976).
14. V. G. Solov'ev, Theory of Complex Nuclei [in Russian], Nauka, Moscow (1971).
15. A. I. Vdovin, G. Kyrchev, and Ch. Stoyanov, Teor. Mat. Fiz., No. 1, 137 (1974).
16. G. Kyrchev, "Anharmonicity effects in even–even deformed nuclei," Author's Abstract of Candidate's Dissertation, JINR, Dubna (1978).