THE 1/N EXPANSION IN THE O(N) × O(N) SCALAR THEORY AND THE PROBLEM OF THE RESTORATION OF SYMMETRY AT HIGH TEMPERATURES

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The possibility of restoration of symmetry in the model (1) in spaces of dimensions d = 3 and 4 at high temperatures is investigated in the framework of the 1/N expansion. In the case d = 4 spontaneous symmetry breaking does not occur at T = 0, and the problem remains open. In the case d = 3, the symmetry spontaneously broken at T = 0 is necessarily restored in the limit $T \to \infty$.

1. Introduction

It is only comparatively recently that quantum theories of fields in contact with a thermal reservoir at temperature T began to be studied. The first work in this direction was the paper of Kirzhnitz and Linde [1]. Almost at once Weinberg [2] noted that the O(N) × O(N) scalar field theory had a behavior at large T unusual from the point of view of quantum-mechanical statistics. The Lagrangian of this theory has the form (spacetime is Euclidean)

$$L=\sum_i \frac{1}{2}\partial_\mu\varphi_i\,\partial_\mu\varphi_i+\sum_{i,j}(\varphi_i^2-N\gamma_i)\frac{g_{ij}}{N}(\varphi_j^2-N\gamma_j), \tag{1}$$

where $i,j=1,2$, φ_i is the i-th N-dimensional vector, each component of which is a real scalar field, g_{ij} is a real symmetric matrix of coupling constants, and γ_i are mass parameters. Weinberg found the corrections to the effective potential of the model (1) in accordance with the usual perturbation theory and showed that one can find values of the coupling constants for which the spontaneously broken O(N) × O(N) symmetry will not be restored at any arbitrarily high temperature (the Weinberg effect). The Weinberg effect was used by various authors in their physical theories. For example, in [3] it was the basis of an explanation of the baryon-antibaryon asymmetry of the universe, in [4] it was used to construct a mechanism to suppress the production of superheavy monopoles, etc.

Of course, the Weinberg effect confounds our physical intuition, according to which a spontaneously broken symmetry must be restored at sufficiently high T. It was therefore suggested by Fujimoto et al. [5,6] that the symmetry in models of the type (1) is not restored only because ordinary perturbation theory with respect to the coupling constants is used. But if one works in the framework of some nonperturbative method, the Weinberg effect should be absent and the O(N) × O(N) symmetry restored as $T \to \infty$.

The authors of [5,6] used the 1/N method. In the leading order in 1/N they considered equations of Dyson-Schwinger type for the squares of the particle masses and showed that at sufficiently high T the equations have positive solutions. This means that the point $\varphi_i=0$ is a stable, in general local, minimum of the potential as $T \to \infty$. On this basis, Fujimoto et al. concluded that in the leading order in 1/N in the model (1) the original symmetry is necessarily restored in spacetime of dimensions d = 3 and 4.

There are at least two reasons why these results are not fully established and are taken with a large measure of skepticism. They are: a) Fujimoto et al. themselves recognize that the method of the Dyson-Schwinger equations can be used to investigate the properties of the effective potential only at the origin. In [5,6] there is no proof of the absence of a deeper absolute minimum of V_{eff} away from the origin. In our view, it is necessary to investigate in the leading order in 1/N the entire effective potential and show that in the limit $T \to \infty$ the point $\varphi_i=0$ is not only a local but also a global minimum of it;

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b) Fujimoto et al. implicitly assume that at T = 0 the initial symmetry can be spontaneously broken in the model (1) in the leading order of the 1/N expansion. But this property should first be verified, so that one can then conclude there is restoration of the original symmetry.

The existence of such a property for the model (1) cannot be accepted as an article of faith, it must be proved, since even in the four-dimensional O(N) model with one vector multiplet of scalars the original symmetry cannot be spontaneously broken in the leading order in 1/N [7].

The aim of this paper is to reexamine in the framework of the 1/N method the problem of restoration of symmetry in the model (1) at high T with allowance for remarks a) and b). We shall investigate the theory in spaces of dimensions d = 3 and 4. The method that we use is based on examination of the properties of the effective potential in the leading order of the 1/N expansion. In particular, particular attention is devoted to the case T = 0.

In Sec. 2 we consider the model (1) for d = 4 and T = 0. We show that in the leading order in 1/N spontaneous symmetry breaking does not occur. Therefore, one cannot speak of restoration of the original symmetry at high T. In the case d = 3 (Sec. 3) the situation is quite different in the leading order of the 1/N expansion. At T = 0, the O(N) × O(N) symmetry can be spontaneously broken, and in the limit $T \to \infty$ the symmetry is restored. Thus, the assertion of Fujimoto et al. that there is no Weinberg effect in the model (1) is proved by the 1/N method only for the case d = 3. In the case d = 4 it is necessary to use other nonperturbative methods.

2. The Model (1) in Four Dimensions

In this section, we consider the model (1) in the leading order of the 1/N expansion in Euclidean space of dimension 4 at zero temperature. For this, it is convenient to introduce O(N)-singlet fields $\sigma_i(x)$ (i = 1, 2), in terms of which the Lagrangian of the model takes the form

$$L_\sigma=\sum_i\left\{\frac{1}{2}\partial_\mu\varphi_i\,\partial_\mu\varphi_i+{}^1/{}_2\sigma_i(\varphi_i^2-N\gamma_i)\right\}-\frac{N}{16}\sum_{i,j}\sigma_i\lambda_{ij}\sigma_j, \tag{2}$$

where the matrix λ_{ij} is the inverse of the matrix g_{ij}. By virtue of the O(N) × O(N) invariance of the theory it can be assumed that the first N − 1 components of each multiplet have vanishing vacuum expectation values, i.e., we can assume $\langle\varphi_i^\alpha\rangle=0$ $(\alpha<N)$. We introduce the notation $\varphi_i^\alpha(x)=\pi_i^\alpha(x)$ $(\alpha<N)$, $\varphi_i^N(x)=\psi_i(x)\sqrt{N}$. Since we are mainly interested in the phase structure of the model, we preclude from consideration the fields π_i^α. For this, we go over from the action determined by the Lagrangian L_σ to a new action S_{eff} in accordance with the following procedure. In the generating functional of the Green's functions of the fields $\psi_i(x)$ we integrate over the fields $\pi_i^\alpha(x)$. We obtain

$$Z(J)=\int D\sigma_i\,D\psi_i\exp\{-NS_{\rm eff}(\sigma,\psi)+J_i\psi_i\},$$

where in the limit $N \to \infty$

$$S_{\rm eff}(\sigma,\psi)=\int d^4x\sum_i\left\{\frac{1}{2}\partial_\mu\psi_i\,\partial_\mu\psi_i+\frac{1}{2}\sigma_i(\psi_i^2-\gamma_i)\right\}-\sum_{i,j}\frac{1}{16}\sigma_i\lambda_{ij}\sigma_j+\frac{1}{2}{\rm Tr}\ln(-\Box+\sigma_1)(-\Box+\sigma_2), \tag{3}$$

in which $\Box$ is the Laplacian in four-dimensional Euclidean space. The action (3) is the effective action of the original model in the leading order in 1/N (if necessary, the auxiliary fields can be eliminated from (3) by means of the equations of motion). In the case when all the fields are constant, we can obtain from (3) the effective potential in the leading order of the 1/N expansion in accordance with the formula $S_{\rm eff}=\int d^4x V$. Here

$$V=-\frac{1}{16}\sum_{i,j}\sigma_i\lambda_{ij}\sigma_j+\frac{1}{2}\sum_i\left\{\sigma_i(\psi_i^2-\gamma_i)+\int\frac{d^dk}{(2\pi)^d}\ln(k^2+\sigma_i)\right\}, \tag{4}$$

where d = 4. The expressions (3)–(4) contain ultraviolet divergences when expressed in terms of bare quantities. They can be eliminated by means of the following renormalization procedure (we use dimensional regularization, D = 4 − ε, with M the normalization point):

$$\lambda_{rii}(M)-M^{4-D}\lambda_{ii}=4M^{4-D}\int\frac{d^Dk}{(2\pi)^D}[k^2+M^2]^{-2},\quad \gamma_{ri}(M)-M^{-2}\gamma_i=-M^2\int\frac{d^Dk}{(2\pi)^D}\frac{1}{k^2},\quad \lambda_{r12}=\lambda_{12},$$

where λ_r and γ_r are the renormalized parameters. The vacuum expectation values of the fields σ_i and ψ_i ($\langle\sigma_i\rangle\equiv m_i^2$, $\langle\psi_i\rangle\equiv\Phi_i$) satisfy the stationarity equations $\partial V/\partial\sigma_i=\partial V/\partial\psi_j=0$ $(i,j=1,2)$:

$$\frac{m_1^2}{4\pi^2}\ln\frac{m_1^2}{M^2}=4\gamma_{r1}M^2+m_1^2\lambda_{r11}+m_2^2\lambda_{12}-4\Phi_1^2,$$
$$\frac{m_2^2}{4\pi^2}\ln\frac{m_2^2}{M^2}=4\gamma_{r2}M^2+m_1^2\lambda_{r22}+m_1^2\lambda_{12}-4\Phi_2^2,\quad m_1\Phi_1=m_2\Phi_2=0. \tag{5}$$

The system of equations (5) has solutions of three types: A) $\Phi_{1,2}\neq 0$, $m_{1,2}=-$; B) $\Phi_1\neq 0$, $m_1=0$, $\Phi_2=0$, $m_2\neq 0$; C) $\Phi_{1,2}=0$, $m_{1,2}\neq 0$. They correspond to the following symmetry groups of the theory: A) O(N − 1) × O(N − 1), B) O(N − 1) × O(N), C) O(N) × O(N).

We now show that the physical vacuum of the theory cannot be a state for which the expectation values of the fields σ and ψ are solutions of Eqs. (5) of types A or B. This means that the original symmetry of the model (1) cannot be spontaneously broken in the leading order of the 1/N expansion. To this end, we shall use the following necessary condition for a certain state to be a vacuum of a theory, namely, in such a theory there must not be tachyons, i.e., particles with negative square of the mass. Therefore, the two-point Green's functions of the fields σ and ψ, constructed over the physical vacuum, must not have singularities for $p^2 > 0$ (the momenta are Euclidean).

In practice, we shall not deal with the Green's functions themselves but only with the single-particle-irreducible (1PI) Green's functions $\Gamma_{\sigma\psi}$, which form a matrix Γ that is the inverse of the matrix of two-point Green's functions. In order to construct $\Gamma_{\sigma\psi}$ over a state that is supposed to play the part of the vacuum, it is sufficient to find $\delta^2S_{eff}/\delta\sigma\delta\psi$ on the corresponding solution of the system (5), i.e., for $\sigma_i = m_i^2$, $\psi_i = \Phi_i$. It is easy to obtain in the momentum Euclidean space the following nonvanishing 1PI Green's functions in the leading order of the 1/N expansion:

$$\Gamma_{\sigma_i\sigma_i}=-{}^1/_8\lambda_{ii}-{}^1/_2B(p,m_i),\quad \Gamma_{\psi_i\psi_i}=p^2+m_i^2,\quad \Gamma_{\sigma_1\sigma_2}=\Gamma_{\sigma_2\sigma_1}=-{}^1/_8\lambda_{12},\quad \Gamma_{\psi_i\sigma_i}=\Gamma_{\sigma_i\psi_i}=\Phi_i. \tag{6}$$

The remaining elements of the matrix Γ are zero. In Eqs. (6),

$$B(p,m)=\int\frac{d^4k}{(2\pi)^4}[(k^2+m^2)[(p+k)^2+m^2)]^{-1}.$$

In terms of the renormalized quantities, the matrix element $\Gamma_{\sigma_i\sigma_i}$ has the form

$$\Gamma_{\sigma_i\sigma_i}=-\frac{\lambda_{rii}}{8}-\frac{1}{32\pi^2}\left[\ln(e^2M^2/m_i^2)+\sqrt{1+\frac{4m_i^2}{p^2}}\ln\left[\frac{\sqrt{p^2+4m_i^2}-p}{\sqrt{p^2+4m_i^2}+p}\right]\right]\equiv-\frac{1}{8}A_i(p,m_i). \tag{7}$$

<u>Impossibility of an O(N − 1) × O(N − 1) Phase.</u> Suppose that as vacuum in the model (1) we have a state for which the vacuum expectation value of the fields σ, ψ is a solution of type A of Eqs. (5), i.e., suppose

$$\langle\sigma_i\rangle=m_i^2=0,\quad \langle\psi_i\rangle=\Phi_i=\sqrt{\gamma_{ri}M^2},\quad \gamma_{ri}>0.$$

This state is invariant with respect to O(N − 1) × O(N − 1).

Substituting these values for m_i and Φ_j in Eqs. (6)–(7), we can obtain the matrix Γ of the 1PI Green's functions. It is obvious that the elements of the matrix that is the inverse of Γ, i.e., the ordinary Green's functions of the fields σ and ψ, will be proportional to $(\det\Gamma)^{-1}$, which in our case has the form

$$\frac{64}{p^4}\det\Gamma=\left[A_1(p,0)+\frac{8\gamma_{r1}M^2}{p^2}\right]\left[A_2(p,0)+\frac{8\gamma_{r2}M^2}{p^2}\right]-\lambda_{12}^2. \tag{8}$$

We can readily obtain $A_i(p, 0)$ from (7):

$$A_i(p,0)=\lambda_{rii}+\frac{1}{2\pi^2}\ln(eM/p). \tag{9}$$

It can be seen from (8) that for all admissible values of the parameters $\det\Gamma$ must necessarily vanish at at least one point $p_0 > 0$. This means that the theory with such a vacuum is unphysical, since it will contain tachyons. Thus, the physical vacuum cannot be $O(N-1) \times O(N-1)$ invariant.

Impossibility of an $O(N-1) \times O(N)$ Phase. We show that the vacuum of the theory cannot be $O(N-1) \times O(N)$ invariant. Suppose otherwise. Then the vacuum expectation values of the fields must be solutions of type B of Eqs. (5). Then

$$m_1=\Phi_2=0, \quad \Phi_1^2=M^2\gamma_{r1}+\frac{m_2^2}{4}\lambda_{12}, \tag{10}$$

and m_2 satisfies the equation

$$\frac{m_2^2}{4\pi^2}\ln(m_2^2/M^2)=4\gamma_{r2}M^2+m_2^2\lambda_{r22}. \tag{11}$$

Substituting the parameters m_i and Φ_j from (10)-(11) in Eqs. (6)-(7), we find the matrix of 1PI Green's functions, the determinant of which has the form

$$\frac{64}{p^2(p^2+m_2^2)}\det\Gamma=A_2(p,m_2)\left[\frac{8\Phi_1^2}{p^2}+A_1(p,0)\right]-\lambda_{12}^2\equiv C(p), \tag{12}$$

where the functions A are given in (7) and (9). The expression in the square brackets on the right-hand side of (12) necessarily vanishes at some point $p_0 > 0$, where $C(p_0) = (-1)\lambda_{12}^2$. It follows from the form of the functions A that in the limit $p \to \infty$ the function $C(p) \to \infty$. Therefore, in the interval (p_0, ∞) there exists a point p_1 at which $C(p_1) = 0$. At this point, the two-point Green's functions will be singular, and this corresponds to a tachyon instability of the theory with the chosen vacuum. Thus, the original assumption is false, and in the model (1) the vacuum cannot be $O(N-1) \times O(N)$ symmetric.

Thus, we have shown that at $T = 0$ in the four-dimensional model (1) in the leading order of the 1/N expansion spontaneous breaking of the original symmetry does not occur.* The conclusions drawn in [5] about the restoration of the original symmetry at high T are incorrect because $O(N) \times O(N)$ cannot be spontaneously broken at $T = 0$ (moreover, we believe that spontaneous symmetry breaking cannot occur at any values of T in the leading order in 1/N in the model (1) for $d = 4$). Therefore, in order to prove in four-dimensional space the assertion of Fujimoto et al. — that the Weinberg effect is absent — it is necessary to use nonperturbative methods different from the 1/N method.

3. The Model (1) in Three Dimensions

In three-dimensional spacetime, in contrast to four dimensions, the original symmetry can be spontaneously broken in the leading order in 1/N in the model (1). Since this result was obtained in [8], we shall not here analyze in detail the case $T = 0$ but merely give the most important information about this model that is needed after a nonzero temperature is assumed.

The Case $T = 0$. The effective potential of the model (1) in the leading order of the 1/N expansion has the form (4) for $d = 3$. In three dimensions, the bare parameters γ_i are, in general, divergent quantities. However, if dimensional regularization is used, γ_i will be finite. Therefore, we shall not here introduce renormalized parameters. From (4) for $d = 3$ we can obtain

$$V(\sigma,\psi)=-\frac{1}{16}\sum_{i,j}\sigma_i\lambda_{ij}\sigma_j+\frac{1}{2}\sum_i\left\{\sigma_i(\psi_i^2-\gamma_i)-\frac{1}{6\pi}\sigma_i^{3/2}\right\}. \tag{13}$$

Not all values of the coupling constants in (13) are allowed physically. We obtain conditions on λ_{ij} by using the requirement that the potential be bounded below. To find the effective potential $v(\psi)$, which depends only on the fields ψ_i, we must eliminate from (13) the auxiliary fields σ_i by means of the equations $\partial v/\partial\sigma_i = 0$:

$$\lambda_1\sigma_1+\lambda_3\sigma_2+4(\gamma_1-\psi_1^2)+(1/\pi)\sqrt{\sigma_1}=0, \quad \lambda_2\sigma_2+\lambda_3\sigma_1+4(\gamma_2-\psi_2^2)+(1/\pi)\sqrt{\sigma_2}=0, \tag{14}$$

where $\lambda_{ii} \equiv \lambda_i$, $\lambda_{12} \equiv \lambda_3$. Solving these equations for σ_i and substituting them in (13), we can find the potential $v(\psi)$. Let us see if $v(\psi)$ is bounded below. Let ψ_i^2 take fairly large values. Then in (14) we can ignore the terms γ_i and $\sqrt{\sigma_j}$. The resulting equations

*One can show [8] that the vacuum in the model (1) for $d = 4$ is an $O(N) \times O(N)$-invariant state. In this case the theory contains no tachyons.

can be solved:

$$\sigma_1(\psi)=\frac{4}{\Delta}(\lambda_2\psi_1^2-\lambda_3\psi_2^2),\quad \sigma_2(\psi)=\frac{4}{\Delta}(\lambda_1\psi_2^2-\lambda_3\psi_1^2), \tag{15}$$

where $\Delta = \lambda_1\lambda_2 - \lambda_3^2$. Substituting (15) in (13) and retaining only the terms of order ψ, we have

$$v(\psi)\approx\frac{1}{\Delta}\{\lambda_2\psi_1^4+\lambda_1\psi_2^4-2\lambda_3\psi_1^2\psi_2^2\}. \tag{16}$$

The expression (16) is positive definite if and only if (this is equivalent to the complete $v(\psi)$ being bounded below)

$$\frac{\lambda_1}{\Delta}>0,\quad \frac{\lambda_2}{\Delta}>0,\quad \lambda_3\,\mathrm{sign}(\Delta)<\sqrt{\lambda_1\lambda_2} \tag{17}$$

(for more details about the conditions for various Higgs potentials to be bounded below, see [9]). The stationarity points of the potential $v(\psi)$ satisfy the equations

$$\psi_1\sigma_1(\psi)=\psi_2\sigma_2(\psi)=0, \tag{18}$$

which are derived from the relation

$$\frac{\partial v}{\partial\psi_i}=\sum_{k=1}^{2}\frac{\partial V}{\partial\sigma_k}\frac{\partial\sigma_k}{\partial\psi_i}+\frac{\partial V}{\partial\psi_i}$$

with allowance for the fact that $\partial V/\partial\sigma_k = 0$. Equations (18) also have three types of solutions, A, B, and C, which correspond to $O(N-1)\times O(N-1)$, $O(N-1)\times O(N)$, and $O(N)\times O(N)$ invariance groups of the vacuum of the model (see the previous section).

Suppose the coupling constants belong to the set (17). It was shown in [8] that within this set a global minimum of the potential $v(\psi)$ can be an extremal point of the type A, B, or C depending on the values of the coupling constants. Thus, in the leading order of the 1/N expansion at $T = 0$ in three dimensions spontaneous breaking of the original symmetry can occur in the model (1).

The Case $T \neq 0$. If in the model (1) for $d = 3$ we take into account temperature on the basis of the single-loop approximation then we can show that for certain admissible values of the coupling constants the symmetry cannot be restored as $T \to \infty$ [6], i.e., the Weinberg effect is present in the three-dimensional version of the model (1). We shall now show that in the framework of the 1/N expansion the Weinberg effect is absent.

To take into account the influence of temperature on the model, we shall use the imaginary-time formalism. For this, the measure of integration in the Euclidean momentum space must be modified as follows:

$$\int\frac{d^3k}{(2\pi)^3}(\dots)\to T\sum_n\int\frac{d^2k}{(2\pi)^2}(\dots),\quad k_0\to 2n\pi T, \tag{19}$$

where $n = 0, \pm1, \pm2, \dots$. Applying the rule (19) to the expression (4) for $d = 3$, we obtain an auxiliary effective potential that depends on the temperature:

$$V^T(\sigma,\psi)=-\frac{1}{16}\sum_{i,j}\sigma_i\lambda_{ij}\sigma_j+\frac{1}{2}\sum_i\Big\{\sigma_i(\psi_i^2-\gamma_i)+T\sum_n\int\frac{d^2k}{(2\pi)^2}\ln(\sigma_i+k^2+4n^2\pi^2T^2)\Big\}. \tag{20}$$

Using in (20) the relation [10]

$$\sum_{n=-\infty}^{\infty}\ln(E^2+4n^2\pi^2T^2)=\frac{E}{T}+2\ln(1-e^{-E/T}),$$

we reduce V^T to the form

$$V^T(\sigma,\psi)=V(\sigma,\psi)+\sum_{i=1}^{2}\frac{T}{4\pi}\int_0^\infty dx\ln(1-\exp(-\sqrt{\sigma_i+x}/T)), \tag{21}$$

where $V(\sigma,\psi)$ is the potential (13) for $T = 0$. Since we are interested in the behavior of the model at high T, we make in (21) a high-temperature expansion on the basis of the formula [11]

$$\frac{T}{4\pi}\int_0^\infty dx\ln(1-\exp(-\sqrt{\sigma+x/T}))=\mathrm{const}\cdot T^3-\frac{T\sigma}{8}\left\{\ln\left(\frac{\sigma}{T^2}\right)-1\right\}+\frac{\sigma^{3/2}}{12\pi}+O\left(\frac{\sigma}{T}\right). \tag{22}$$

Substituting (22) in (21) and omitting the terms of the form $O(\sigma/T)$ and $\mathrm{const}\cdot T^3$ (the last expression does not depend on the fields), we obtain

$$V^T(\sigma,\psi)=-\frac{1}{16}\sum_{i,j}\sigma_i\lambda_{ij}\sigma_j+\frac{1}{2}\sum_i\left\{\sigma_i(\psi_i^2-\gamma_i)-\frac{T}{4\pi}\sigma_i[\ln(\sigma_i/T^2)-1]\right\}. \tag{23}$$

To find the effective potential $v^T(\psi)$, which depends only on the fields ψ_i, we eliminate the fields σ_i from (23) by means of the equations $\partial V^T/\partial\sigma_i = 0$:

$$\sigma_2=\frac{1}{\lambda_{12}}f_1(\sigma_1,\psi_1),\qquad \sigma_1=\frac{1}{\lambda_{12}}f_2(\sigma_2,\psi_2), \tag{24}$$

where

$$f_i(\sigma_i,\psi_i)=-\lambda_{ii}\sigma_i-\frac{T}{\pi}\ln\left(\frac{\sigma_i}{T^2}\right)+4(\psi_i^2-\gamma_i). \tag{25}$$

The point of the absolute minimum of the potential $v^T(\psi)$ satisfies the equations

$$\psi_1\sigma_1(\psi)=\psi_2\sigma_2(\psi)=0, \tag{26}$$

which can be derived in the same way as (18).

Equations (26) have the trivial solution $\psi_{1,2} = 0$. It corresponds to the fact that at high T the ground state of the O(N) × O(N) model is symmetric. It is readily seen that the system (26) does not have other solutions. For otherwise the stationarity equations would have solutions of the form A: $\sigma_{1,2}(\psi)=0$, $\psi_{1,2}\neq 0$, or B: $\psi_1\neq 0$, $\sigma_1(\psi)=0$, $\sigma_2(\psi)\neq 0$, $\psi_2=0$. However, it follows from (24)–(25) that σ_i cannot vanish for any fixed values of the fields in the interval $(0, \infty)$ and of the coupling constants in the set (17).

Thus, the global minimum of the potential $v^T(\psi)$ at sufficiently large T can only be O(N) × O(N) symmetric. Therefore, the original symmetry is necessarily restored in the three-dimensional model (1) in the framework of the 1/N expansion.

4. Conclusions

This paper has been devoted to the problem of the restoration of symmetry with increasing temperature in the field-theory model (1) (in other field theories with several multiplets of scalars this problem is also present). The point is that in the model (1) there exists a set of coupling constants such that [2]: 1) at T = 0 spontaneous breaking of the O(N) × O(N) symmetry occurs; 2) at arbitrarily high T restoration of the original symmetry does not occur. To draw this conclusion, Weinberg used the single-loop approximation to find the effective potential of the model.

However, it is believed that the model (1) does not in fact have such a property and that it is an artefact of using ordinary perturbation theory with respect to the coupling constants. But if certain nonperturbative methods of calculation are used, the symmetry is necessarily restored (Fujimoto et al. [5,6]).

To test this suggestion, we have used the 1/N expansion and shown that: 1) in the three-dimensional field theory (1) the symmetry spontaneously broken at T = 0 is restored at sufficiently high T; 2) for the investigation of the problem in the four-dimensional model (1) the 1/N method is not suitable, since even in the case T = 0 in the leading order in 1/N spontaneous symmetry breaking does not occur in the theory.

The four-dimensional model (1) was recently investigated [12] by the nonperturbative method of a Gaussian effective potential, which does not have the shortcomings of the 1/N expansion. Results that confirm the conclusion of Fujimoto et al. about the restoration of symmetry were obtained. In view of this, we believe that the high-temperature behavior in the model (1) obtained by Weinberg is an artefact of ordinary perturbation theory and in reality the symmetry is restored in the limit $T \to \infty$.

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BEHAVIOR OF MASSLESS FEYNMAN INTEGRALS NEAR SINGULAR POINTS

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For a certain class of massless Feynman amplitudes without internal vertices, including single-loop integrals, it is shown that on the leading Landau surface these integrals have singularities of only the type of a pole, square root, or logarithm; the corresponding critical points (in the sense of the theory of singularities of differentiable mappings) are simple. Diagrams with nonisolated critical points are considered. The question of the possibility of identity of the leading Landau surfaces for the graph and its subgraphs and factor graphs is investigated.

The aim of this paper is to attempt to generalize to massless integrals the well-known result on the behavior of massive amplitudes on the Landau surfaces, namely, they have there a branch point of the type of a square root or a logarithm. This aim is achieved only partly, and the obtained result encompasses only purely massless diagrams, which are here called simple (definition in Sec. 3). They include all single-loop diagrams.

The methods of investigation are largely from the theory of the singularities of differentiable mappings. The standard arguments lead to the investigation of the critical points of the projection of the set of singularities of the integrand onto the space of the external momenta. One of the main results shows that for simple diagrams these critical points are simple (a simple pinch in a different terminology). After this the Picard–Lefschetz theorem makes it possible to obtain the necessary information about the branch points of the amplitude. This is done in Sec. 3, where, using the theory of holonomic systems of pseudodifferential equations, we obtain a more accurate formula that takes into account the growth near the Landau surface. In Sec. 2 we obtain the results we need about the possible coincidence of two Landau surfaces of one graph. It can be shown that for diagrams without internal vertices in the purely massive and purely massless cases without multiple lines this is impossible; we also establish the importance of the conditions. In Sec. 4 we study some examples of isolated critical points; they are encountered in diagrams with multiple lines, and also on $\alpha = 0$ Landau surfaces. We also make some comments about diagrams with internal vertices. In Sec. 1 we give the notation and mention some known results.

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1. Let Γ be a purely external (unless stated otherwise) Feynman diagram with set of

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