CALCULATION OF SUPERSONIC FLOW AROUND V-SHAPED WINGS BY THE FITTING METHOD

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The problem of flow around a V-shaped wing with supersonic leading edges is solved. The method employed is that of fitting with respect to a space variable in which the system of equations of motion is hyperbolic, using the computing scheme of V. V. Rusanov. A comparison between the results of these calculations and experimental data in relation to the pressure distribution along the wing span reveals excellent agreement, except for a limited region, in which the compression jump incident on the plane of the wing interacts with the boundary layer. A comparison between the results obtained by means of the oblique-jump equations and by numerical calculations indicates that the method in question is reasonably accurate.

It has been established experimentally that, when a flow of air passes around a set of V-shaped wings, a complicated system of shock waves is set up in the field of flow, starting from a certain angle of the V [1, 2]. At the present time no methods exist for calculating such flows, except in certain special cases [3, 4].

The only effective method of studying flows characterized by the existence of several shock waves is the fitting method involving difference schemes containing an "artificial viscosity," first introduced by Neumann and Richtmyer [5].

In order to calculate complex spatial flows with a large number of shock waves, the use of the ordinary procedure of the fitting method (in which the stationary or steady-state solution is obtained as a limit of the nonstationary or transient solution for a large number of steps in time t) is practically impossible owing to the limited memory and speed of modern highspeed computers.

There have been only a few examples of the calculation of three-dimensional flow around smooth bodies by the fitting method in the presence of one principal shock wave, and these have involed a fairly smooth change of parameters from the wave to the solid object. The smoothness of the change of flow parameters from the wave to the solid means that a small number of mesh points can be taken in this direction; this fact underlay the success of the calculations conducted in [6-8].

An important class of spatial flows comprises those of the supersonic, steady-state, conical type. The equations of motion are here hyperbolic with respect to one of the spatial coordinates ξ, which may be chosen in such a way that the flow is independent of this coordinate.

Here we are concerned with the idea of calculating such flows by fitting with respect to ξ. The present analysis is devoted to a study of the use of this approach for the case of the flow around a V-shaped wing at a particular angle of attack.

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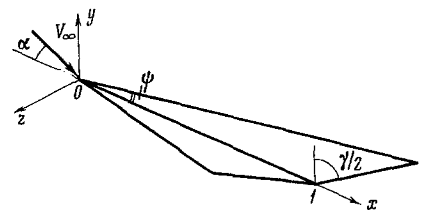

Fig. 1

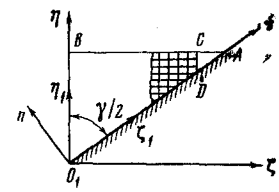

Fig. 2

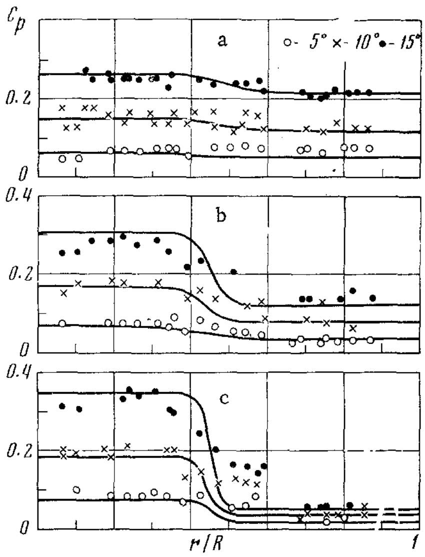

Fig. 3

1. Principal Equations and Difference Scheme. Let us consider the steady-state flow around a V-shaped wing at an angle of attack α (Fig. 1). We shall solve the problem in a mixed coordinat system: The components of the vector velocity u, v, w will be calculated in Cartesian coordinates x, y, z, and the problem will then be solved in the coordinates

$$\xi = \ln x, \qquad \eta = y / x, \qquad \zeta = z / x$$

The equations of motion, written in divergence form in the coordinates ξ, η, ζ, take the form

$$\frac{\partial f}{\partial \xi} + \frac{\partial}{\partial \eta}(F^y - \eta f) + \frac{\partial}{\partial \zeta}(F^z - \zeta f) + 2f = 0 \tag{1.1}$$

$$f = \begin{Bmatrix} \Omega \\ R \\ S \\ T \\ E \end{Bmatrix}, \quad F^y = \begin{Bmatrix} S / u \\ S \\ R - \Omega u + S^2 / \Omega u \\ TS / \Omega u \\ ES / \Omega u \end{Bmatrix}, \quad F^z = \begin{Bmatrix} T / u \\ T \\ TS / \Omega u \\ R - \Omega u + T^2 / \Omega u \\ ET / \Omega u \end{Bmatrix}$$

$$\Omega = \rho u, \qquad R = p + \rho u^2, \qquad S = \rho uv, \qquad T = \rho uw$$

$$E = \rho u (e + p / \rho + (u^2 + v^2 + w^2) / 2)$$

Here u, v, w, are the components of the velocity vector along the x, y, z axes respectively; e is the internal energy of the gas.

Subsequently we shall consider a perfect gas:

$$e = \frac{p}{(k-1)\rho}, \qquad k = \frac{c_p}{c_v}$$

The system (1.1) has been written in dimensionless form: The velocity components are referred to the modulus of the velocity of the unperturbed flow, the pressure to twice the velocity head of the unperturbed flow $\rho_\infty V_\infty^2$, the density to the density of the unperturbed flow ρ_∞.

We also introduce the column vector φ with elements u, ν, w, p, ρ, which are uniquely expressed in terms of the components of the vector f

$$\begin{gathered} u = kR[(k+1)\Omega]^{-1} + \{k^2R^2[(k \\ +1)\Omega]^{-2} - 2(k-1)[E\Omega^{-1} \\ - (S^2 + T^2) / 2\Omega^2](k+1)^{-1}\}^{1/2} \\ v = S / \Omega, \quad w = T / \Omega, \\ p = R - \Omega u, \quad \rho = \Omega / u \end{gathered} \tag{1.2}$$

The x axis is taken in such a way that u > a

$$a^2 = kp / \rho$$

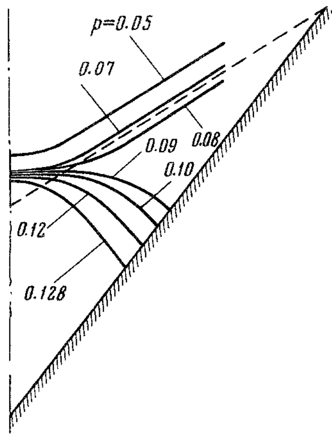

Fig. 4

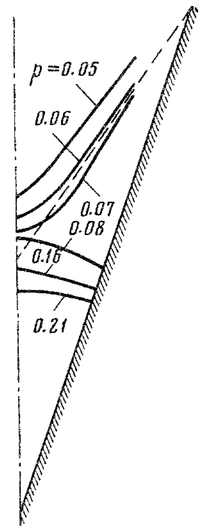

Fig. 5

The boundary conditions take the following form:

on the surface of the wing $\psi(\xi, \eta, \zeta) = \eta - g(\zeta) = 0$ we have the condition that the flow does not pass straight through it

$$(v - u\eta) - (w - u\zeta)g' = 0 \tag{1.3}$$

on the symmetry plane $\zeta = 0$

$$w = 0 \tag{1.4}$$

at infinity in front of the body

$$u = \cos\alpha, \quad v = -\sin\alpha, \quad w = 0, \quad p = \frac{1}{kM_\infty^2}, \quad \rho = 1 \tag{1.5}$$

The boundary problem (1.1), (1,3)-(1.5) was solved by the fitting method with respect to ξ. In our calculations we used the principles laid down by V. V. Rusanov [9]. Analysis of the several differencing schemes of the fitting method [10] shows that the Rusanov and Lax-Wendroff schemes give similar results, the expenditure of machine time being several times smaller in the former case.

Let us introduce the following notation:

$$\Delta\xi = \tau, \quad \Delta\zeta = h_1, \quad \Delta\eta = h_2, \quad h = \sqrt{h_1^2 + h_2^2}$$
$$h_1 = h\cos\chi, \quad h_2 = h\sin\chi, \quad \varkappa_i = \tau / h_i, \quad \varkappa = \sqrt{\varkappa_1^2 + \varkappa_2^2}$$

The value of A at the point with coordinates $(n\tau, lh_2, mh_1)$ we shall call $A_{l,m}^n$.

Let us confine attention to the case in which the surface of the V-shaped wing are flat and the trace of the plane of the wing on the plane x = const passes through the mesh points along the diagonals of the cells (Fig. 2).

We approximate the system (1.1) by a three-point difference scheme in accordance with [9]:

$$f_{l,m}^{n+1} = f_{l,m}^{n} - \frac{\varkappa_2}{2}\{(F^y - \eta f)_{l+1,m} - (F^y - \eta f)_{l-1,m}\}^n - \frac{\varkappa_1}{2}\{(F^z - \zeta f)_{l,m+1}$$
$$- (F^z - \zeta f)_{l,m-1}\}^n + \frac{1}{2}\{\Phi^y_{l+1/2,m} - \Phi^y_{l-1/2,m} + \Phi^z_{l,m+1/2} - \Phi^z_{l,m-1/2}\} - 2\tau^n f_{l,m}^n \tag{1.6}$$

where

$$\Phi_{l+1/2,m}^{y} = {}^1/_2(\beta_{l+1,m} + \beta_{l,m})(f_{l+1,m} - f_{l,m}), \qquad \beta_{l,m} = \varkappa\omega\sigma_{l,m}\cos^2\chi$$
$$(\omega = \mathrm{const})$$
$$\Phi_{l,m+1/2}^{z} = {}^1/_2(\alpha_{l,m+1} + \alpha_{l,m})(f_{l,m+1} - f_{l,m}), \qquad \alpha_{l,m} = \varkappa\omega\sigma_{l,m}\sin^2\chi$$
$$\sigma_{l,m} = \left\{\frac{1}{u^2 - a^2}[u(v^2 + w^2)^{1/2} + a(u^2 + v^2 + w^2 - a^2)^{1/2}]\right\}_{l,m} \tag{1.7}$$

Analysis of the stability of the difference scheme (1.6) was based on the Fourier method. The stability condition takes the form

$$\sigma_0 < \omega < \frac{1-\tau}{\sigma_0} \qquad (\sigma_0 = \max_{l,m}\sigma_{l,m}, \ \sigma_0 < 1)$$

The value of σ_0 may be specified in advance as identical for all the layers, and we may then take ω and in each layer determine $\varkappa^n$ and hence τ^n from the equations

$$\varkappa^n = \sigma_0 / \max_{l,m}\sigma_{l,m}^{n}, \qquad \tau^n = \varkappa^n h_1 h_2 / h$$

In the calculation it is quite easy to automate the choice of $\varkappa^n$ in every layer. In order to ensure stability, it is then necessary that the constant parameters of the scheme σ_0 and ω should satisfy Eq. (1.8).

2. Computing Formulas. The point O_1 (Fig. 2) belongs both to the symmetry axis and to the plane of the wing. It follows from (1.3) and (1.4) that $T_{0,0} = S_{0,0} = 0$ and at this point it is only necessary to determine the first, second, and fifth components of the vector f. In obtaining the computing formula, a transformation to an acute-angled coordinate system was employed (Fig. 2).

$$\xi = \xi_1, \quad \eta = \eta_1 + \zeta_1\sin\chi, \quad \zeta = \zeta_1\cos\chi$$

The formula takes the form

$$(\varkappa^n)^{-1}[f_{0,0}^{n+1} - (1 - 2\tau^n)f_{0,0}^{n}]$$
$$= -\sin\chi(F^z - \zeta f)_{1,1}^{n} - \cos\chi(F^y - \eta f)_{1,0}^{n}$$
$$+ {}^1/_4\omega\cos^2\chi\sin^2\chi(\sigma_{1,1} - \sigma_{0,0})^n(f_{1,1} - f_{0,0})^n$$

A point on the symmetry plane $\zeta = 0$.

It follows from the boundary condition (1.4) that $T_{l,0} = 0$, so that at this point all the components of the vector f except the fourth are determined,

$$(\varkappa^n)^{-1}[f_{l,0}^{n+1} - (1 - 2\tau^n)f_{l,0}^{n}] = -\sin\chi(F^z - \zeta f)_{l,1}^{n} - {}^1/_2\cos\chi\{(F^y - \eta f)_{l+1,0}$$
$$- (F^y - \eta f)_{l-1,0}\}^n + {}^1/_2\omega\sin^2\chi(\sigma_{l,0} + \sigma_{l,1})(f_{l,1} - f_{l,0})^n$$
$$+ {}^1/_4\omega\cos^2\chi\{(\sigma_{l+1,0} + \sigma_{l,0})(f_{l+1,0} - f_{l,0}) - (\sigma_{l,0} + \sigma_{l-1,0})(f_{l,0} - f_{l-1,0})\}^n$$

A point on the plane of the wing.

In deriving the formulas, a transformation was made to a new coordinate system in the neighborhood of the plane of the wing (Fig. 2)

$$\xi_1 = \xi, \qquad s = \zeta\cos\chi + \eta\sin\chi, \qquad n = -\zeta\sin\chi + \eta\cos\chi$$

$$(\varkappa^n)^{-1}[\overset{*}{f}{}_{l,l}^{n+1} - \overset{*}{f}{}_{l,l}^{n}(1 - 2\tau^n)] = -{}^1/_2\sin\chi\cos\chi\{(F^{*z} - \zeta^* f^*)_{l+1,l+1}$$
$$-(F^{*z} - \zeta^* f^*)_{l-1,l-1}\}^n - \{(F^{*y} - \eta^* f^*)_{l,l-1}\sin^2\chi + (F^{*y} - \eta^* f^*)_{l,l+1}\cos^2\chi\}^n$$
$$+ {}^1/_4\,\omega\sin^2\chi\cos^2\chi\{(\sigma_{l+1,l+1} + \sigma_{l,l})(\overset{*}{f}_{l+1,l+1} - \overset{*}{f}_{l,l}) - (\sigma_{l,l} + \sigma_{l-1,l-1})(\overset{*}{f}_{l,l} - \overset{*}{f}_{l-1,l-1})\}^n.$$

All the quantities marked with an asterisk * are computed by the same formulas as the corresponding quantities without the asterisk, except that ν, ω, η, ζ are everywhere replaced by

$$v^* = -w\sin\chi + v\cos\chi, \qquad w^* = w\cos\chi + v\sin\chi$$
$$\eta^* = -\zeta\sin\chi + \eta\cos\chi, \qquad \zeta^* = \zeta\cos\chi + \eta\sin\chi$$

The third component of the vector $f_{l,l}{}^*$ is not calculated, since from the boundary condition (1.3) $S_{l,l}{}^* = 0$.

After determining $f_{l,l}{}^*$ the vector $f_{l,l}$ is obtained from the equations

$$\Omega_{l,l} = \Omega_{l,l}{}^*, \qquad R_{l,l} = R_{l,l}{}^*, \qquad S_{l,l} = T_{l,l}{}^*\sin\chi$$
$$T_{l,l} = T_{l,l}{}^*\cos\chi, \qquad E_{l,l} = E_{l,l}{}^*$$

An internal point in the field of flow:

$$\frac{1}{\varkappa^n}[\overset{n+1}{f_{l,m}} - (1-2\tau^n)\overset{n}{f_{l,m}}] = \frac{-1}{2}\sin\chi\{(F^z-\zeta f)_{l,m+1} - (F^z-\zeta f)_{l,m-1}\}^n$$
$$-{}^1/_2\cos\chi\{(F^y-\eta f)_{l+1,m} - (F^y-\eta f)_{l-1,m}\}^n + {}^1/_4\omega\sin^2\chi\{(\sigma_{l,m+1}+\sigma_{l,m})(f_{l,m+1}-f_{l,m})$$
$$-(\sigma_{l,m}+\sigma_{l,m-1})(f_{l,m}-f_{l,m-1})\}^n + {}^1/_4\,\omega\cos^2\chi\{(\sigma_{l+1,m}+\sigma_{l,m})(f_{l+1,m}-f_{l,m})$$
$$-(\sigma_{l,m}+\sigma_{l-1,m})(f_{l,m}-f_{l-1,m})\}^n$$

The computing region constitutes a right-angled trapezium O_1DCB (Fig. 2). On the boundary BC, the values of the parameters of the unperturbed flow remained intact during the calculation, on the boundary DC the values of the parameters corresponding to flow around the leading edge were retained with an additional slight oblique jump, on the wall O_1D the condition of nonpenetrating flow was maintained, while on the symmetry plane O_1B, $w = 0$. As zero approximation, the field of the flow behind the plane jump attached to the leading edge of the V-shaped wing was employed. The solution was considered as being steady-state if

$$\max_{l,m}\{(\overset{n+1}{f_{l,m}} - \overset{n}{f_{l,m}})/\overset{n}{f_{l,m}}\} \leqslant \varepsilon$$

for all components of the vector f (in our calculations $\varepsilon \approx 10^{-2}$–$10^{-3}$).

The calculations were carried out on the BÉSM-6 computer. The number of points in the field of flow was 1254 in all the versions computed, which corresponded to 50 mesh points on the side of the angle BO_1A (Fig. 2). Depending on the angles γ and α, the number of fitting cycles varied from 500 to 1500. The computing time for one version with 1500 such cycles was 50 min.

3. Results of the Calculations. In order to verify the efficiency of the method, we compared the results of our calculations based on the proposed scheme with the experimental results of A. L. Gonor and A. I. Shvets. The comparison was made for a wing with an angle of ψ = 29°30' at the tip for various values of the V angle and M_∞ = 3.95. The Reynolds number in the experiment was Re = 6.8 · 10^6. Figure 3 shows the pressure distribution $C_p = 2(P - P_\infty)/\rho_\infty V_\infty^2$, obtained by computation (continuous lines) and experimentally. In these graphs, r is the distance from the symmetry plane to the edge of the wing. Figure 3a gives the data for γ = 120°, Fig. 3b the data for γ = 80°, and Fig. 3c the data for γ = 40°. The calculations were carried out for attack angles of α = 5, 10, 15°. We observe excellent agreement between the experimental and computed, data, except for the region in front of the jump of compression falling on the wall, in which experiment indicates a pressure much larger than theory. A similar increase in pressure was noted in [11] and explained as being due to the interacting between the jump of compression falling on the wall and the boundary layer [12]. Figure 4 illustrates the isobars for γ = 80°, α = 10°. The character of the isobars indicates a mode of flow involving Mach reflection of the jump falling from the leading edge by the symmetry plane. Figure 5 shows the isobars for γ = 40°, α = 15°; the character of these indicates a mode of flow with the regular reflection of a plane jump from the symmetry plane. The results of the calculation of a regular reflection from the symmetry plane, with a strong reflected jump, based on the oblique-jump formulas for γ = 40°, α = 15° differ from the results of numerical calculation for p and ρ by no more than 3%. The broken line in Figs. 4 and 5 indicates the position of the plane jump falling on the symmetry plane from the leading edges of the V-shaped wing.

In conclusion, we note that the values of the vector components $\bar{\varphi}$ behind the plane jump formed at the leading edge obtained by the numerical calculation differ from the values obtained by the oblique-jump formulas by no more than 3%. The span of the jump is no greater than five or six mesh points.

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